Blind Image Deconvolution Revisited

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The objective of this article is to discuss the major approaches we overlooked in the May 1996 article of *Signal Processing Magazine* titled "Blind Image Deconvolution." We would like to discuss them here for completeness along with some other works found in more recent literature. As the area of blind image restoration is a rapidly growing field of research, new methods are constantly being developed. Although it is difficult to include all of the newer works here, we have tried to address the most successful techniques.

Projection-Based Blind Deconvolution

This method, proposed by Yang et al. [1], belongs to the class of nonparametric deterministic image constraints methods. The motivation in designing this method is to improve upon the poor convergence properties of other older methods of its class. The degradation is assumed to be represented by the following linear model:

$$g(x,y) = f(x,y)*h(x,y)$$

where $f(x,y)$ is the true undistorted image, $h(x,y)$ is the point spread function (PSF), and * represents the two-dimensional linear convolution operator. The true image and PSF are assumed to be square integrable and additive noise is neglected.

The approach attempts to simultaneously restore the true image and PSF by restricting them to lie in the following intersection of sets (Please note that bold variables represent lexicographically ordered vectors of their two-dimensional counterparts):

$$(\hat{\mathbf{f}},\hat{\mathbf{h}}) \in C_O = C_g \cap C_f \cap C_h$$

where

$C_g = \{(\tilde{\mathbf{f}},\tilde{\mathbf{h}}): \tilde{f}(x,y)*\tilde{h}(x,y) = g(x,y)\}$

$C_f = \{(\tilde{\mathbf{f}},\tilde{\mathbf{h}}): \tilde{f}(x,y),$ satisfies prior knowledge of the true image},

$C_h = \{(\tilde{\mathbf{f}},\tilde{\mathbf{h}}): \tilde{h}(x,y)$ satisfies prior knowledge of the PSF}.

Examples of C_f and C_h are constraints of support, intensity range, and bandlimitedness.

To avoid the mathematical complications of using generalized projection operators to estimate the true image and PSF, a more practical approach to blind deconvolution is proposed. The following cost function is defined:

$$J(\tilde{\mathbf{f}},\tilde{\mathbf{h}}) = \sum_{\forall(x,y)} \left(g(x,y) - \left[\hat{f}(x,y)*\tilde{h}(x,y)\right]\right)^2.$$

The procedure begins by making educated initial estimates of the true image and PSF. The technique is cyclical in nature. Each cycle is comprised of two parts. First, J is minimized with respect to $\tilde{\mathbf{h}}$ subject to the constraint that $\tilde{\mathbf{h}} \in C_h$ (keeping $\tilde{\mathbf{f}}$ constant at the last image estimate) to produce a PSF estimate. Second, J is minimized again by reversing the rolls of $\tilde{\mathbf{h}}$ and $\tilde{\mathbf{f}}$ to produce an image estimate. The cycles are continued until a prespecified convergence criterion is met. The complexity of the algorithm is of order $O(N_f^2)$ where N_f is the total number of pixels in the image estimate.

Simulation results in [1] demonstrate the performance of the method. It has been shown to outperform Lane's conjugate gradient algorithm, and it appears to be robust to inaccuracies of support size and is sensitive to noise. The major disadvantage is that the solution is not necessarily unique and that the method is subject to erroneous solutions associated with the local minima of J. Modifications of the method have been proposed to improve the speed of convergence [2] and to improve the quality of restoration for the multiple frame situation [3]. The most probable application area for this method is astronomy.

Maximum Likelihood Restoration

This technique is a more general approach to the maximum likelihood (ML) ARMA parameter estimation methods discussed in the May 1996 issue of *Signal Processing Magazine*. The true image and additive noise of the imaging system are assumed to be multivariate Gaussian processes. The degradation model in matrix-vector notation is:

$$\mathbf{g} = \mathbf{Hf} + \mathbf{n}$$

where $\mathbf{g}$, $\mathbf{f}$, and $\mathbf{n}$ are the lexicographically ordered vectors corresponding to the blurred image, true image, and additive

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noise, respectively. **H** is the PSF matrix. It is assumed that **f** and **n** are uncorrelated with covariance matrices Λ_F and Λ_N, respectively. It is also assumed that the additive noise is white, so that $\Lambda_N = \sigma_n^2 \mathbf{I}$ where **I** is the identity matrix.

The problem of blind deconvolution becomes that of estimating the unknown parameters $\phi = \{H, \Lambda_F, \sigma_n^2\}$ from **g**. A maximum likelihood estimate of these parameters is obtained. That is, we find the parameter set such that the probability density function (pdf) of **g** conditioned on ϕ is maximized. This pdf is given by:

$$p(\mathbf{g}|\phi) = \frac{1}{\sqrt{|2\pi(\mathbf{H}\Lambda_F\mathbf{H}^H + \sigma_n^2\mathbf{I}|}} \exp\left\{-\frac{1}{2}\mathbf{g}^H(\mathbf{H}\Lambda_F\mathbf{H}^H + \sigma_n^2\mathbf{I})^{-1}\mathbf{g}\right\}$$

where $(\cdot)^H$ is the conjugate transpose of a matrix. Due to the high degree of nonlinearity, the optimization is difficult and is conducted by using the expectation-maximization (EM) algorithm [4, 5].

Since only second order statistics are used in the estimation procedure, the algorithm only applies to the identification of non-minimum phase PSFs. In addition, the PSF estimate may not be unique. By making additional assumptions on the symmetry, sign, support and/or scaling of the PSF a unique solution may be obtained.

The simulation results of the algorithm show that there is a trade-off between the sharpness of the true image estimate and noise amplification. The estimation of the noise variance σ_n^2 is usually smaller than the true value which produces a

Erratum

The objective of this erratum is to correct and help clarify some of the typographical errors and other oversights found in the article "Blind Image Deconvolution" in the May 1996 issue of *Signal Processing Magazine*. The following is a list of corrections:

- At the top of page 47, O(Ng4) should be $O(N_g^4)$.
- In Eq. (3), Φf_i should be Φ_{fi}.
- In Eq. (5), Φg should be Φ_g.
- In Eq. (16), $\det|Q_n$ should be $\det|Q_n|$.
- In Eq. (17), $2\pi^{N2} \det|Q_v$ should be $2\pi^{N^2} \det|Q_v|$, and $(1-A)^T$ should be $(I-A)^T$.
- In Eq. (18) g^{Tp-1g} should be $g^T P^{-1} g$.
- On page 50, the order of the ML algorithm should be $O(N_f)$ and on page 51, the order of the GCV algorithm implementation is $O(N_f \log(N_f))$.
- In the top of Table 3 θ_i $\{\{h(l,m),...$ should be $\theta_i = \{\{h(l,m),....$
- In Eq. (6) of Table 3 $E^k(\hat{f},\theta_i =$ should be $E^k(\hat{f},\theta_i) =$.
- The end of Table 3 has been cut-off. The word "estimate" should follow the last line shown in the table.
- In Eq. (23), $u(s,y)$ should be $u(x,y)$.
- In the second column, fourth row of Table 5, "if $\hat{E} \le E$" should be "if $\hat{E} \le E$ and if $\hat{f}(x,y) \ge 0$". Also, $\sqrt{\frac{E}{E}}\hat{f}(x,y)$ should be $\sqrt{\frac{E}{\hat{E}}}\hat{f}(x,y)$.
- On page 55, in the section "Nonparametric Methods Based on High Order Statistics," the nonlinearity $g[\cdot]$ should not be confused with the blurred image $g(x,y)$.
- In the top of Table 6, $<\cdot.\cdot>$, should be $<\cdot,\cdot>$.
- On the bottom of Table 6, **u***k* should be $\mathbf{u}_k$.
- In Table 6, all instances of u_k should be in bold (i.e., $\mathbf{u}_k$). In addition a reference is missing; the program `dlirmin.c` can be found in Ref. [64] of the article.
- In the first row of Table 9, "ARMA Parameter Extraction" should be "ARMA Parameter Estimation".
- In the Complexity row of Table 9, all algorithm orders are missing. We list them here:
  1. Zero Sheet Separation: $O(N_f^4)$.
  2. ARMA Parameter Estimation: ML–$O(N_f)$, GCV–$O(N_f \log(N_f))$.
  3. Nonparametric Deterministic Constraints Algorithms: SA–$O(N_f^4)$, IBD–$O(N_f \log(N_f))$, NAS-RIF–$O(N_f N_u N_{ls,k})$.
  4. HOS Methods: $O(N_f^2 + N_f N_u)$.

noisier image estimate than that given by a true Wiener filter. It has also been found that the proper incorporation of constraints improves image restoration significantly.

The main advantage of the technique is that no parametric form of the PSF or its support size is required and the algorithm has low computational complexity. The complexity of the algorithm is $O(N_g)$ where N_g is the total number of pixels in the original image. In addition, the algorithm allows one to obtain explicit equations for the blur, noise, and power spectra of the true image. The main disadvantage is that the EM algorithm might converge to a local optima of the cost function. A multichannel extension of the algorithm for multispectral images has been proposed [6]. It has been shown to result in a more reliable restoration. The most probable applications for the algorithm is photography, medical imaging, and multispectral image processing.

Other Recent Developments

With the surge of research in the area of blind image restoration, a great deal of work in the area has been recently published. Many of the newer contributions are extensions or refinements of previously cited work and place emphasis on applications. We briefly discuss them in this section.

Some recent work is applied in particular to medical imaging. ML estimation using the EM algorithm applied to three-dimensional light microscopy has been proposed [7]. The technique is an extension of references [5] and [15] of our May 1996 *Signal Processing Magazine* article. Another technique has been applied to improve the spatial resolution of ultrasound images [8]. High-order spectra are used for the identification of possibly nonminimum-phase PSFs. The algorithm is successful in suppressing additive Gaussian noise in the image.

Other blind deconvolution techniques that make use of high-order statistics can be found for the identification of texture images [9, 10]. They assume that the blurring process is represented by an AR filter, and use iterative identification algorithms to estimate the true image and AR parameters. These techniques fit into the class of nonparametric methods based on high-order statistics.

New work has also been done for the class of zero sheet separation techniques. Algorithms have been designed to make blind deconvolution using zero sheets less sensitive to noise [11].

Other recent work applies to the class of nonparametric deterministic image constraints methods. An extension of the iterative blind deconvolution algorithm (reference [39] of *Signal Processing Magazine*, May 1996) to situations where many differently blurred versions of the same object are available is suggested [12]. A modification of the iterative blind deconvolution algorithm using the Richardson-Lucy algorithm is also proposed for solar images [13]. In addition, a modification of Lane's conjugate gradient algorithm (reference [46] of *Signal Processing Magazine*, May 1996) has been applied to adaptive optics compensated data [14]. Analysis of conditions to guarantee a unique solution for the NAS-RIF algorithm (reference [43] of *Signal Processing Magazine*, May 1996) has also been done. Other researchers have examined the importance of the selection of initial conditions and the accuracy of the support constraints for this class of methods [15].

A statistical technique for situations in which an inaccurate noisy PSF estimate is available has been developed. In this method image restoration is performed using a regularized constrained total least-squares approach [16].

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