

# WATER-FILLING FOR WATERMARKING?

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## ABSTRACT

In this paper we address the question: Is water-filling appropriate for watermarking? The water-filling paradigm is a traditional solution to the capacity maximization of parallel zero-mean additive white Gaussian channels subject to a signal energy constraint. In this work, we take an information theoretic approach to analyze the watermark communication problem in the presence of perceptual coding. Our effective watermark channel is modeled as a set of parallel independent zero-mean uniformly distributed additive noise channels. Energy allocation principles are identified to maximize the capacity results. Our findings are compared to the traditional water-filling solution and shed light on strategies to maximize the data hiding rate in the presence of compression.

## 1. INTRODUCTION

The research area of watermarking has resulted in an incredible number of algorithmic proposals suited to the diverse needs of multimedia industries. One can easily be overwhelmed by the sheer number of models, measures, paradigms, principles, and standards applied in these fields of work.

In this paper, we focus on identifying general rules of thumb for reliable high capacity watermarking in the presence of compression. Our intent is to take a communication analogy for watermarking and answer the fundamental questions:

- Where should I place most of the watermark energy so that it is robust to perceptual coding.
- How much information can I reliably hide?

These questions lead us to a more analytic and information theoretic approach to addressing the problem of data hiding in the presence of compression. We hope that the insights gained through this work may be applied directly to previously proposed and future robust watermarking algorithms to enhance performance.

We will specifically concentrate on the issue of watermark energy allocation to determine where and to what degree within a host multimedia signal the hidden data should be embedded. Traditionally, the degradations on the watermark from incidental and intentional attacks have been modeled conservatively as additive

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white Gaussian noise (AWGN). We can model the overall effective watermark communication channel as a set of parallel AWGN channels to account for non-stationarity of the noise [1]. Assuming that these channels are independent, the watermark energy allocation to maximize capacity in the presence of fixed signal energy must obey the well-known water-filling paradigm [2]. That is, more signal energy should be placed in channels with less noise energy. Figure 2 elucidates the process. Specifically, the signal energy should be allocated to the individual channels as if water was being poured on the power spectral density plot of the noise.

Thus, with respect to our problem of watermarking in the presence of perceptual coding, we need to embed the watermark in regions of the host signal less susceptible to quantization noise.

Although this initial insight is useful, its relevance to the watermarking problem is highly dependent on the accuracy of the assumptions. The following limitations of the basic formulation hold:

1. the water-filling problem constrains the energy of the signal to be fixed; however, in practice, the watermark is constrained to be imperceptible which is more sophisticated than a general mean square error constraint and is related to fundamental models of human perception.
2. the noise on the watermark, itself, is assumed to be Gaussian which is conservative and attempts to encompass a broad class of distortions, but may not be appropriate to model perceptual coding effects in particular.

Thus, it appears that the optimal watermark energy allocation should somewhat resemble the water-filling problem, and must account for these additional issues.

Our objective is to establish new insights for effective watermarking in the presence of lossy compression for a class of non-spread spectrum (SS) watermarking techniques in which watermark extraction or detection does not suffer from host signal interference. In such methods, the source of degradation on the watermark is due solely to quantization noise from perceptual coding which allows us to more accurately model the noise using non-Gaussian statistics. We go beyond our previous work in [3] which derives capacity bounds for watermarking in the presence of lossy compression and focus on watermark energy allocation issues.

The contributions of this paper are to:

- present previously derived capacity measures for a class of watermarking approaches.
- using these quantities, derive unique watermark energy distribution principles to maximize the capacity for data hiding.

- compare and contrast our energy allocation results with the traditional water-filling problem.

## 2. MODELS AND MEASURES

The analysis in this work is applicable to the following situations: A) The perceptual coding occurs through quantization of coefficients in a specified transform  $T$  domain. Specifically, the JND perceptual paradigm is employed as discussed in the next section and B) watermarking occurs in this same domain prior to compression; the watermark embedding procedure, although not completely specified, occurs in such a manner that no “self-noise” interferes with the hidden information (several quantization-based algorithms such as [4, 5, 6] fall within this class). It should be noted that we assume the perceptual model used for JND compression is conservative, and there exists “perceptual room” to hide the watermark information without causing notice.

### 2.1. The JND Perceptual Model

Many models exist to describe the masking characteristics of the human perceptual system [7]. Of these, one of the most popular is based on a just noticeable difference (JND) paradigm [8]. A set of JNDs is associated with a particular invertible transform  $T$ . Given that a multimedia signal is transformed using  $T$ , the JNDs provide an upper bound on the extent that each of the coefficients can be perturbed without causing perceptual changes to the signal quality. The set of signal and transform dependent JNDs can be derived using complex analytic models or through experimentation.

In this paper we use the variable  $JND_i$  to denote the associated JND for a class of coefficients in a sub-channel  $c_i$  (which we describe later). The changes imposed on the signal from both watermarking and compression must both perturb the associated coefficients in  $c_i$  such that the combined effect causes a magnitude change in the coefficients below  $JND_i$ . Given that perceptual coding is pre-set to make a maximum change of  $J_i$  (this is directly related to the degree of quantization imposed during lossy compression), the available room for watermark signal changes is given by  $\bar{J}_i = JND_i - J_i$  to ensure imperceptibility. It is assumed that these values are implicitly available for watermark energy allocation as they will constrain the changes to be below perceptible range.

### 2.2. Quantization as Uniformly Distributed Noise

To model a broad spectrum of possible attacks on the watermarked signal, the degradation on the watermark itself has traditionally been assumed to be in the form of additive Gaussian noise as discussed in the introduction. This representation, although global, is somewhat conservative for assessing the effects of a specific type of degradation such as perceptual coding.

In this paper, we propose employing zero-mean uniformly distributed white noise independent of the watermark signal for coefficient quantization effects from perceptual coding. We justify this assumption by applying Bennett’s Theorem [9] which is commonly applied for the analysis of data converters. Although this approximation is valid for a narrow set of conditions, it does provide useful insights into the behaviour of A/D converters [9]. In the same way we believe that use of this model will provide better understanding into watermarking in the face of JND perceptual coding.

### 2.3. Parallel Additive Noise Channels

Traditionally, communication systems have been broken down into simpler components through identification of sub-channels [2]. For simplicity, these sub-channels are often assumed to be independent of one other and involve stationary additive noise degradations.

In the same way, in a multimedia signal, a group of coefficients  $G_i$ , quantized to the same degree  $J_i$  for the purpose of compression, form what we call a sub-channel  $c_i$ . This sub-channel is used to communicate a part of the energy of the watermark information. The watermark is embedded to a specific degree within these coefficients just as a portion of the signal would be transmitted through a communication sub-channel. Figure 1 elucidates the analogy. From our discussion in the previous section, the effect of quantization due to perceptual coding can be modeled as zero mean additive white uniformly distributed noise. Specifically, the watermark signal in  $c_i$ , denoted  $W_i$ , experiences the following degradation

$$\hat{W}_i = W_i + Z_i \quad (1)$$

where  $\hat{W}_i$  is the degraded watermark signal, and  $Z_i$  is noise with probability density

$$f(z_i) = \begin{cases} \frac{1}{2J_i} & \text{for } |z_i| < J_i \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

Assuming there are a total of  $N$  sub-channels, we can model our overall watermark communication system as a set of parallel, independent sub-channels  $c_i$  for  $i = 1, 2, \dots, N$ , for which each  $c_i$  experiences additive white zero-mean uniformly distributed noise with maximum absolute amplitude  $J_i$ .

The challenge is now the design of  $W_i$  to maximize the overall capacity of the system.

## 3. MAXIMIZING CAPACITY FOR HEAVY JND COMPRESSION

Using the watermark sub-channel model of Equation (1), it is shown in [3] that the capacity of  $c_i$  can be maximized by employing antipodal watermark signalling. The capacity of the  $i$ th sub-channel is given by

$$\mathbb{C}_i = \frac{\alpha_i}{J_i} \quad (3)$$

where  $\alpha_i$  is the antipodal watermark signal magnitude, and  $\alpha_i < J_i$ . In effect, we assume that the degree of coefficient quantization from perceptual coding is larger than the watermark signal amplitude. This corresponds to situations in which there is relatively aggressive lossy compression in comparison to data hiding.

### 3.1. Optimal Watermark Energy Allocation

To optimize the capacity of the overall system  $\mathbb{C}$ , we must consider the appropriate energy allocation of the watermark to each of the sub-channels  $c_i$ .

Specifically, we find  $\mathbb{C}$  by maximizing the mutual information between  $\hat{W}_i$  and  $W_i$  subject to the following assumption (**A1**), and constraints (**C1**, **C2**):

**A1**  $Z_i$ , the watermark noise, is uniformly distributed and independent of the noise in any other sub-channel.

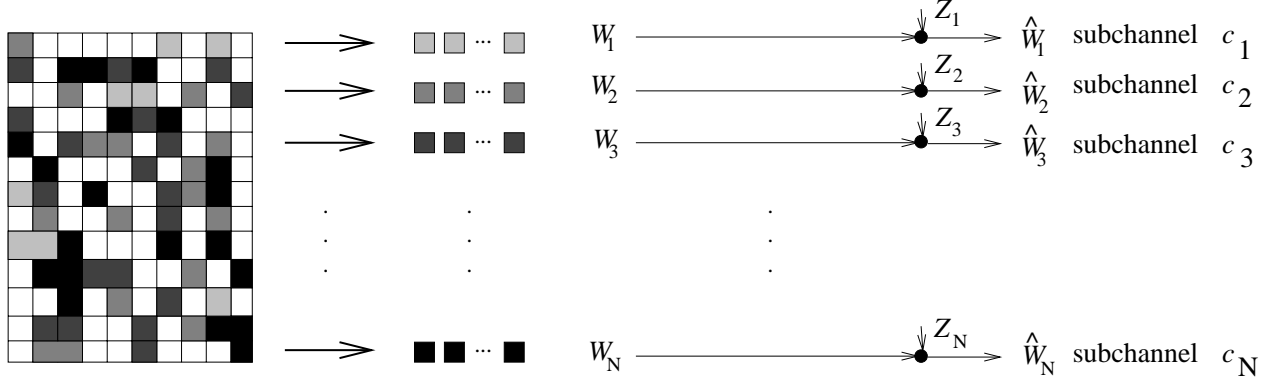


Figure 1: The two-dimensional signal on the left represents the transform  $T$  domain coefficients containing the watermark information. After the watermark is embedded, they are quantized to varying degrees for perceptual coding. Coefficients that experience the same level of quantization are represented by the same colour in the above diagram. We consider these coefficients to be associated with a given sub-channel. This sub-channel communicates the watermark information in the presence of additive noise. The degree of noise experienced by the watermark signal in sub-channel  $c_i$  is a function of the level of quantization noise experienced by the associated group of coefficients.

**C1** The energy of all  $\{\alpha_i\}$  are constrained such that

$$\sum_{\forall i} \alpha_i^2 \leq E_w \quad (4)$$

This constraint controls the maximum overall peak signal to noise ratio (PSNR) due to watermarking, so that subsequent processing such as compression, band-pass filtering or addition of other watermarks does not become worse or noticeable due to the existence of this watermark. Also, many watermarking algorithms vary robustness by controlling the PSNR, so this constraint is implicitly part of many techniques.

**C2** The amplitudes of  $\{\alpha_i\}$  are constrained such that

$$|\alpha_i| \leq \bar{J}_i \quad (5)$$

The parameter  $\bar{J}_i$  represents the maximum change that can be imposed on the associated coefficients by watermarking to guarantee that the joint effect of both watermarking and compression is imperceptible; the above inequality is directly based on our use of the JND perceptual paradigm to model masking.

It can be shown that the overall capacity is calculated by finding

$$\mathbb{C} = \max I(W_1, W_2, \dots, W_N; \hat{W}_1, \hat{W}_2, \dots, \hat{W}_N) \quad (6)$$

where the maximization is with respect to the joint pdf of  $W_1, \dots, W_N$ .

Applying assumption **A1**, it can be shown [3] that Equation (6) reduces to

$$\mathbb{C} = \sum_{i=1}^N \frac{\alpha_i}{J_i} \quad \text{bits/channel use} \quad (7)$$

To maximize the overall capacity, we wish to find the appropriate assignment for  $\alpha_i$  subject to the constraints **C1**, **C2**.

Table 1: Optimal watermark energy allocation algorithm for high capacity data hiding.

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Initialize Variables
  Set maximum watermark energy  $E_w$ 
  Initialize the dummy variable set  $\mathcal{L} = \{\emptyset\}$ 
Loop
   $\bar{\mathcal{L}} := \{1, 2, \dots, N\} - \mathcal{L}$ 
   $flag := 0$ 
  For  $i \in \bar{\mathcal{L}}$ 
     $\alpha_i := \frac{\sqrt{E_w - \sum_{j \in \mathcal{L}} \bar{J}_j^2}}{\sqrt{\sum_{j \in \bar{\mathcal{L}}} (J_i/J_j)^2}}$ 
    if  $\alpha_i > \bar{J}_i$ 
       $flag := 1$ 
       $\alpha_i := \bar{J}_i$ 
       $\mathcal{L} := \mathcal{L} \cup \{i\}$ 
    end
  End
Until  $flag = 0$ 

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Using the method of Lagrange multipliers, it can be shown that the algorithm in Table 1 may be deployed for the appropriate assignment of  $\alpha_i$ . It is the opinion of the author that there is no closed form expression for  $\alpha_i$  in terms of  $E_w$  in general.

#### 4. DISCUSSION

As previously mentioned, the solution to the energy allocation problem presented in Table 1 resembles the traditional water-filling solution in that

- more watermark energy is allocated to channels with less quantization noise from perceptual coding.
- the capacity grows monotonically with watermark energy.

Thus, we see that use of *complementary* host signal regions than those used for compression provides greater data hiding capacity. This motivates the use of different perception models for

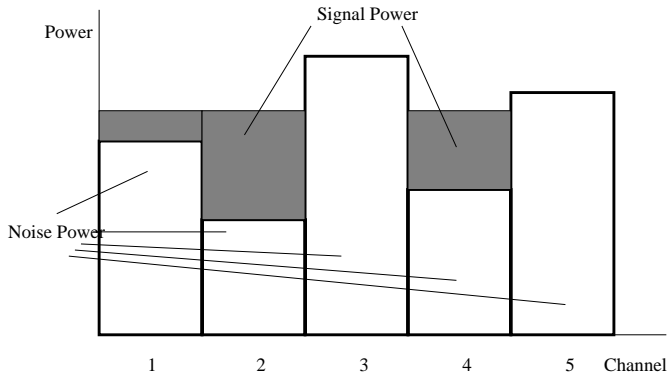


Figure 2: Water-filling Solution for Capacity Maximization. When the signal power  $E_w$  is increased from zero, the channel(s) with the least noise are first allotted signal power. As  $E_w$  is increased even more, some power is put to noisier channels.

watermarking and coding. The fact that higher capacity is achieved for greater energy watermark signals is intuitively clear and once again verifies the obvious need for more efficient perceptual masking models which will allow the embedding of higher energy watermarks.

Our use of non-Gaussian statistics and an additional JND amplitude constraint for  $\{\alpha_i\}$  lead to a number of distinctions with the water-filling problem:

- the maximum amplitude increase of the watermark signal must stop after the perceptual threshold  $\bar{J}_i$  is reached to guarantee imperceptibility.
- as the total watermark signal energy  $E_w$  is increased the signal energy increases at different rates depending on  $J_i$  in each channel; the energy allocation may be considered to occur for each isolated channel through a distinct valve with a specified throughput. Figure 3 provides an overview.

## 5. FINAL REMARKS

The work presented in this paper is intended to help identify novel techniques to improve the capacity of data hiding in the presence of lossy compression. Future work involves designing watermarking algorithms incorporating these energy allocation principles to verify whether they help in improving the robustness of the hidden information.

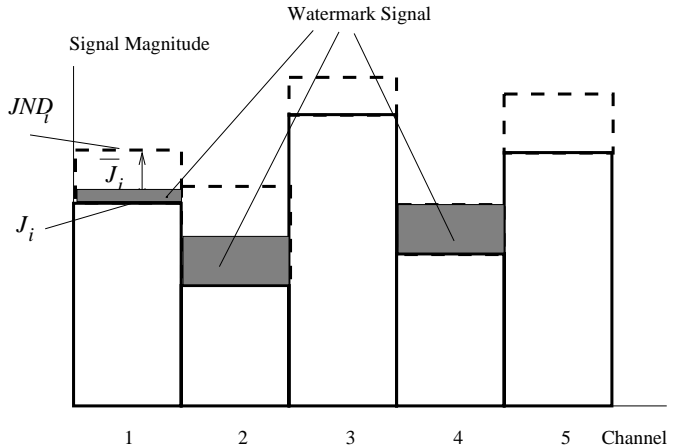


Figure 3: Optimal Watermark Signal Allocation Strategy. In contrast to Figure 2, the watermark component can only be increased up to a magnitude of  $\bar{J}_i$ . The rate of signal increase is different for each channel and is approximately inversely proportional to  $J_i$ . Thus, channels with less quantization noise are given watermark signal components much faster as  $E_w$  is increased.

## 6. REFERENCES

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