# On Node Isolation in Directional Sensor Networks

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## 1 Introduction

Wireless ad-hoc sensor networks (WSNs) consist of randomly and densely deployed nodes which self-organize to cooperatively maintain multi-hop network connectivity [1]. The nodes act as both environmental sensors and network routers. The ability to set up distributed sensor networks inexpensively, in large-scale, quickly, and without fixed infrastructure makes them a promising candidate for a host of applications, including military surveillance and disaster relief.

Recently, there has been increased interest in an emerging subclass of WSNs known as directional sensor networks (DSN) [1, 2] consisting of nodes that employ a directional communication paradigm. Two common scenarios where DSNs occur are; (1) directional radio frequency employing directed antennas, and; (2) free space optical employing directional line-of-sight lasers.

Directional communication has a number of advantages over omni-directional communication in ad hoc networking, including significantly increasing network lifetime by reducing communication energy and multi-path components [1]. Randomly deployed DSNs however pose additional challenges to network connectivity, due to link directionality.

Connectivity, defined to mean that a path exists between any pair of network nodes, is of particular significance in order to maintain communication among nodes. One important approach to connectivity analysis of ad hoc networks studies the conditions under which isolated nodes occur in the network. Even though ensuring that no isolated network node occurs is a necessary albeit insufficient condition for connectivity, recent studies show that for dense networks with a large number of nodes $n$, with high probability the network is connected at the moment it achieves no isolated node [3].


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This key result implies that the probability that no isolated node occurs $p_d$ provides a *tight upper bound* for the probability that the network is connected $p_c$, for large $n$ and probabilities close to one [3], and motivates our study of the relationship amongst network parameters that guarantee with high probability a "no isolated node" property for DSNs. The fundamental question we address in this paper, and which is of practical importance in network-level simulation of DSNs, is: How may network parameters be chosen so that with high probability $p_d$, there is no isolated node?

## 2 Directional Sensor Network Model

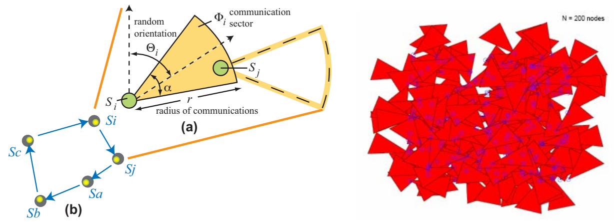
We assume a set $\mathcal{S}_n = \{s_i : i = 1, 2, \cdots n\}$ of $n$ directional nodes, are randomly and densely deployed in a given two dimensional region according to a Uniform distribution. Each node $s_i \in \mathcal{S}_n$ has an equal and independent likelihood of falling at any location $\Upsilon_i \sim \text{Uniform}(0,1)^2$ and facing any orientation $\Theta_i \sim U(0, 2\pi]$. The resulting $n$-node multi-hop DSN, defined by parameters $n, r$ and $\alpha$ has been modeled [2].

DSN nodes employ a directional transmitter. Consequently, each node $s_i$ can send data within a contiguous, randomly oriented sector $\frac{-\alpha}{2} + \Theta_i \le \Phi_i \le \frac{+\alpha}{2} + \Theta_i$ of radius $r$, and fixed angle $\alpha \in [0, 2\pi]$ radians, as depicted in Figure 1(a). Therefore a sector area of communication $\Phi_i$ defined by the 4-tuple $(\Upsilon_i, \Theta_i, r, \alpha)$ is associated with each node $s_i$.

The receiver is omni-directional so that $s_i$ may directly talk to $s_j$ (denoted $s_i \to s_j$) if and only if $\Upsilon_j \in \Phi_i$. However, $s_j$ can only talk to $s_i$ via a multi-hop back-channel or reverse route, with other nodes acting as routers along the reverse path (unless of course $\Upsilon_i \in \Phi_j$). Figure 1(b) illustrates the reverse route: $s_j \to s_a \to s_b \to s_c \to s_i$. The case $\alpha = 2\pi$ represents the conventional omni-directional communication paradigm, modeled as a random geometric graph (RGG).

We model the DSN topology as a directed graph $G_n(\mathcal{S}_n, \mathcal{E})$ consisting of a vertex node set $\mathcal{S}_n$ and edge set $\mathcal{E}$, where every edge is an ordered pair of distinct nodes. The edge matrix $\mathcal{E}(i, j)_{1 \le i,j \le n} = 1$ if $\Upsilon_j \in \Phi_i$ (i.e., $s_i \to s_j$); and 0 otherwise, represents the $n \times n$ *Adjacency matrix* of $G_n(\mathcal{S}_n, \mathcal{E})$ with one row and one column for every node. Figure 1(c) depicts a simulated 200-node DSN.

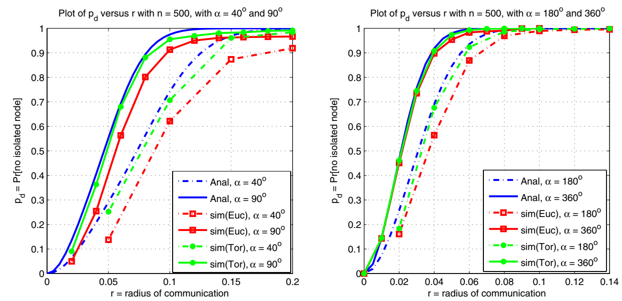
Unlike the RGG, the directional paradigm requires that two distinct sets of neighbors be defined for each node. The set of $s_i$'s *successors* $\mathcal{S}_i =: \{s_k\}, \forall k : \mathcal{E}(i, k) = 1$ of cardinality $\delta_i^+$ consisting of nodes $s_i$ transmits to, and the set of $s_i$'s *predecessors* $\mathcal{P}_i =: \{s_h\}, \forall h : \mathcal{E}(h, i) = 1$ of cardinality $\delta_i^-$




(a) Node $s_i$ transmits within sector $\Phi_i$ defined by the 4-tuple $(\Upsilon_i, \Theta_i, r, \alpha)$. Node $s_i \to s_j$ if $s_j \in \Phi_i$, but $s_j \to s_a \to s_b \to s_c \to s_i$.

(b) A simulation scenario 200-node DSN in a square region area $1m^2$, with $r = 0.2$m and $\alpha = 40^o$

**Figure 1.**




(a) $\alpha = 40^o$ and $90^o$

(b) $\alpha = 180^o$ and $360^o$

**Figure 2.** $p_d$ **for varying** $r$ **and** $\alpha$ **with** $n = 500$

consisting of nodes $s_i$ receives from. Node $s_i$ is termed *f-isolated* if $\delta_i^+ = 0$, otherwise it is *f-connected*; and similarly it is *b-isolated* if $\delta_i^- = 0$, otherwise it is *b-connected*. A node is *connected* if it is both f-connected and b-connected, else it is isolated (i.e., either f-isolated, b-isolated or both).

## 3 Analysis on Probability of Node Isolation

We denote $p_f^i = \Pr[\delta_i^+ > 0]$ and $p_b^i = \Pr[\delta_i^- > 0]$ as the probability that node $s_i$ is not f-isolated and b-isolated respectively. Node $s_i$ is connected (not isolated) if it is both f-connected *and* b-connected with probability

$$p_d^i = p_f^i \cap p_b^i = p_f^i . p_{b|f}^i \qquad (1)$$

where $\cap$ is the intersection operator and $p_{b|f}^i$ is the probability that $s_i$ is b-connected, given that it is f-connected.

***Evaluating*** $p_f^i$: Employing spatial point statistical methods [3], it is known that the number of nodes $k$ located in $\Phi_i$ follows a Poisson $\sim \frac{n\alpha r^2}{2}$, so that $\delta_i^+$ has the pdf (for $n$ large):

$$\Pr[\delta_i^+ = k] \quad = \quad \frac{e^{\frac{-n\alpha r^2}{2}} \left(\frac{n\alpha r^2}{2}\right)^k}{k!}$$

$$\text{and} \quad p_f^i = \Pr[\delta_i^+ \geq 1] = 1 - \Pr[\delta_i^+ = 0] = 1 - e^{\frac{-n\alpha r^2}{2}} \qquad (2)$$

***Evaluating*** $p_{b|f}^i$: Assume $s_i$ is f-connected ($\Upsilon_j \in \Phi_i$) so that $\mathcal{S}_i \neq \emptyset$. For independently deployed nodes, let us fix a node $s_j \in \mathcal{S}_i$, $j \in \{1, 2, \cdots n\}$, $j \neq i$, and consider two events (A and B) such that $p_{b|f}^i = \Pr[A] \cup \Pr[B]$.

(1) Pr[A]: Considering nodes outside $\Phi_i$, $p_b^i$ equals $p_f^i$.

(2) Pr[B]: Given $s_i \to s_j$, then $\Pr[s_j \to s_i] = \frac{\alpha}{2\pi}$, and Pr[at least one of $k$ nodes in $\mathcal{S}_i$ is in $\mathcal{P}_i$] = $1 - (1 - \frac{\alpha}{2\pi})^k$. Then Pr[B] = Pr[$k$ nodes $\in \mathcal{S}_i$]$\times$ Pr[at least one of the $k$ nodes $\in \mathcal{P}_i$] is:

$$\Pr[B] = \sum_{k=1}^{n-2} \Pr[\delta_i^+ = k] - \sum_{k=1}^{n-2} \Pr[\delta_i^+ = k](1 - \frac{\alpha}{2\pi})^{k+1}$$

$$= 1 - \left(1 - \frac{\alpha}{2\pi}\right)^{n-2} - e^{\frac{-n\alpha r^2}{2}} \left(1 - \frac{\alpha}{2\pi}\right) \left[e^{\left(\frac{n\alpha r^2}{2} - \frac{n\alpha^2 r^2}{4\pi}\right)} - 1\right]$$

Substituting $p_{b|f}^i = \Pr[A] + \Pr[B] - \Pr[A].\Pr[B]$ we obtain:

$$p_{b|f}^i \quad = \quad 1 - e^{\frac{-n\alpha r^2}{2}} \left(1 - \frac{\alpha}{2\pi}\right)^{n-2}$$

$$- e^{-n\alpha r^2} \left(1 - \frac{\alpha}{2\pi}\right) \left[e^{\left(\frac{n\alpha r^2}{2} - \frac{n\alpha^2 r^2}{4\pi}\right)} - 1\right] \qquad (3)$$

***Evaluating*** $p_d^i$: Substituting Eqn. 2 and 3 into Eqn. 1 yields:

$$p_d^i \quad = \quad \left[1 - e^{\frac{-n\alpha r^2}{2}}\right] \left[1 - e^{\frac{-n\alpha r^2}{2}} \left(1 - \frac{\alpha}{2\pi}\right)^{n-2} \right.$$

$$\left. - e^{-n\alpha r^2} \left(1 - \frac{\alpha}{2\pi}\right) \left(e^{\left(\frac{n\alpha r^2}{2} - \frac{n\alpha^2 r^2}{4\pi} - 1\right)}\right)\right] \qquad (4)$$

Concluding, $p_d = (p_d^i)^n$, where $p_d^i$ is derived in Eqn. 4. Our result extends the work of [3], yielding the relationship between $n, r, \alpha$, and $p_d$ for general sensor networks $G_n(\mathcal{S}_n, \mathcal{E})$.

## 4 Simulations and Discussions

For our simulations, we employ a uniform random generator to randomly position and orient $n = 500$ nodes in a unit area square region. With $r$ varying and $\alpha$ set at representative values of $40^o, 90^o$ (Fig. 2(a)) and $180^o, 360^o$ (Fig. 2(b)), we employ Euclidean distance metric to obtain $\mathcal{E}$, and count the number of isolated nodes $n_I$ of the resulting $G_n(\mathcal{S}_n, \mathcal{E})$. We then compute $p_d$(Euc) as $1 - n_I/n$. Repeating the random topology simulations 1000 times and averaging yields an acceptable confidence for the empirical $p_d$. To eliminate border effects [3], we conduct a second set of similar simulations using Toroidal metric to compute $\mathcal{E}$ and $p_d$(Tor).

Fig. 2 depicts the analytical and simulation (Euc. and Tor.) $p_d$ plots. We observe that our simulations follow the analytical curves closely, while the Toroidal metric mitigates border effects quite well. However, we note that as $\alpha \to 0$ the disparity between analytical and simulation $p_d$ grows.

PROPOSITION 1. Let $r_c$ and $r_d$ denote the minimum $r$ at which $G_n(\mathcal{S}_n, \mathcal{E})$ is connected and attains no isolated node respectively. Then $Pr[r_c = r_d] \to 1$ as $n \to \infty$ and $\alpha \to 2\pi$.

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