Enhanced Availability in Randomly Deployed Wireless Sensor Networks via Hybrid Uni/Omni-Directional Antennas

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Abstract*—Availability is a critical requirement in any modern information system. For many classes of sensor networks, random deployment cannot be avoided and the associated wireless network connectivity characteristics significantly impact system availability. A highly available network must be able to sustain services such as data gathering and event-detection even during link losses owing to reasons that include intermittent interference and network attacks. Improved availability is often achieved by rerouting the traffic along alternative paths. Accordingly, k-connectivity is a key property of highly available networks. In this paper, we analyze k-connectivity of hybrid wireless networks that incorporate both omni- and sectorized uni-directional antennas. We present results that help understand the relationship between node density, transmission radius, antenna beam width and the existence of disjoint paths between the network's member nodes and a sink. Analytical and simulation-based results are provided to motivate the use of this emerging paradigm. We demonstrate that under a broad class of conditions, a hybrid approach significantly improves k-connectivity and has a larger average number of disjoint paths between member nodes and the network sink even at lower node densities and transmission radii.

I. Introduction

Modern day requirements for continuous surveillance, especially of remotely located areas, that are dangerous or just not economically feasible for the active presence of human observers strongly motivates the use of wireless sensor networks (WSNs) that consist of a network of tiny nodes capable of sensing, collecting and organizing data. The nodes also communicate significant information to a central base. Sensor networks are expected to cooperatively monitor physical and environmental parameters in a broad set of applications that include battlefield surveillance and reconnaissance, environment and habitat surveying, healthcare, home automation and border security [1] [2]. A defining characteristic that usually separates WSNs from other ad-hoc networks is that of self-organization and unattended operation. Sensor networks are expected to organize, sense and communicate on their own once they are deployed in a target environment.

The most fundamental sensor network architecture consists of randomly distributed nodes in a target area. Each node is typically equipped with multiple sensors, depending on the physical quantity that needs to be measured along with a radio frequency (RF) or optical transmitter/receiver arrangement and a small microprocessor. The power source for each node is usually a battery that has a limited life time. The nodes sense physical activity and report significant information to a data collection center called the $sink$. The sink may directly communicate information to human operators and necessarily has higher capabilities than a member node in the network in terms of energy, memory, computational power and bandwidth. It is a trusted entity in the network. The sink can also be envisioned as a mobile entity or a mobile base station [3] that is useful in cases when nodes collect data and transmit readings to a base station when available.

Although the variety of applications for which sensor networks are proposed is diverse, a common requirement of all scenarios is a high level of connectivity. Connectivity is a fundamental requirement in wireless sensor and ad-hoc networks, representing the capability of a member node to communicate with any other node in the network either by direct transmission or via multi-hop relays. It impacts every aspect of network performance including comprehensive monitoring, the capability of self-organization, energy consumption, network longevity and network capacity.

Owing to a variety of reasons including intermittent interference, network attacks and topological disturbances, established links in a WSN may be damaged. The damage may either be temporary or in certain cases, permanent. In such scenarios, it is necessary to improve the resilience of a network through redundancy by providing multiple paths between member nodes and the sink, so that a timely alternative path exists in case one of the links on a current path is damaged beyond repair. A highly available network is one that is capable of sustaining critical network services such as gathering of surveillance data by the sink even over common periods of link failure.

Traditionally, omni-directional antennas have been used on sensor network motes as the fundamental communication model; significant contributions have been made to characterize network connectivity under such physical layer transmission models. In contrast, this paper addresses the issue of improvements in k-connectivity and network availability

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through the existence of multiple independent disjoint paths between member nodes and the sink. Using a non-traditional communication paradigm, the $Hybrid$ approach that involves the use of directional antennas and utilizes the gains are achieved from using them in all sectors of a mote's pattern.

This paper is organized as follows. First, related work in the field of directional antennas and connectivity is reviewed. Section III introduces common antenna models employed in this work and essential graph theoretic concepts. Section IV provides details of the novel $Hybrid$ paradigm. Analysis and simulations of network connectivity are provided in Section V followed by conclusions and avenues for future research.

II. Related Work

Research into network connectivity can be classified into three categories, graph theoretic research dating back to the 1970s, more practical routing and link layer protocol analysis work, and application-specific proposals using unconventional communication paradigms such as directional antennas.

Some of the most influential work in connectivity analysis is by Gupta and Kumar [4] in which they derive the critical transmission range of nodes placed randomly in a disc of unit area, so that the resulting network is connected with a probability of one as the number of nodes tends to infinity. Betstetter, in part, extends these results in [5] to consider an omni-directional communication model and includes results addessing the minimum node degree for connectivity with an emphasis on the importance of metrics like node density. Betstetter also presents significant results on k-connectivity and the probability of node isolation.

In [6], Saha and Johnson suggest the use of directional antennas in the context of routing in mobile ad-hoc networks when receiving nodes temporarily move out of the transmission range of a transmitting node. The work emphasizes exploiting the extended reach that is available via directional antennas to compensate for disconnection of critical network links due to node mobility. Okorafor and Kundur [7] study network-level security advantages gained through the use of directional communication links. Given that most common sensor network attacks [8] assume bidirectional links, the authors in [7] demonstrate how directional communications provides an additional degree of freedom for network routing that can be exploited for inherent resilience against common classes of networking attacks.

Hu and Evans [9] analyze the wormhole attack in sensor networks. The authors use directional antenna information to share sector information about neighboring nodes to identify adversary nodes masquerading as false neighbors. Wormhole endpoints are blacklisted via the directional information that is shared between neighbors. The use of directional antennas for improved security largely increases the probability of detecting the wormhole attack. The $hybrid$ work by Milner and Davis in [10] motivates the use of omni-directional RF communication along with the use of a uni-directional FSO transceiver. The RF communication is enabled in cases when LoS is unavailable.

The work presented in this paper uses a somewhat similar communication model that has not been studied in terms of connectivity improvements. We use directional communication, activated in one sector that is randomly oriented based on deployment. Our focus in this paper is on characterizing the degree of improvement on network availability (through connectivity) that this new paradigm facilitates.

III. Background

A. Omni-Directional vs Uni-Directional Antennas

Ideally, an omni-direcitonal antenna radiates or receives equally well in all directions. It is an antenna system that radiates power uniformly in one plane (say, the horizontal plane) and has a directive pattern in a plane perpendicular to the first one. Traditionally sensor networks are modeled using motes that use omni-directional antennas for communication. The omni-directional radiation pattern is typically approximated as a circle with the radiating element at the center.

In contrast, the energy in a uni-directional antenna is focused in one direction and hence these types of antennas are typically characterized by transmissions that reach much farther than their omni-directional counterparts. The *main lobe* in the uni-directional antenna's pattern is the direction of maximum radiation (or reception, if reception is also modeled using directional antennas). There are also extra *minor lobes* ($side$ and $back$ lobes). These lobes represent lost energy, that is energy spent on directions away from the direction of interest, where the antenna intends to transmit. Uni-directional antenna designers always attempt to minimize these lobes. The radiation pattern of a uni-directional antenna is approximated as a sector.

The *beamwidth* of a uni-directional antenna is a measure of its directivity which is the width of the main lobe measured in degrees. Beamwidth is usually measured between the -3 dB points, the points on the main lobe where the signal strength drops by -3 dB (one-half) from the point of maximum signal intensity. This is also called the *half-power beamwidth*. The beamwidth is represented by α.

$$P = C \cdot \pi r^2 \tag{1}$$

$$P = C' \cdot \frac{\alpha r'^2}{2} \tag{2}$$

From [11], the energy required by a sensor node to reach all neighboring nodes within its transmission range is proportional to the area covered by its radiation pattern and is given in Eqs. (1) and (2) where r is the omni-directional antenna transmission radius, r' is the uni-directional range in the direction of peak gain, α is the antenna beamwidth, P is the transmission power drawn at each antenna and C and C' are appropriate constants. In Eq. (2) the sidelobes and backlobes are considered to be negligible and the power is radiated entirely through the primary main lobe.

To compare the capabilities of uni-directional and omni-directional antennas, the area covered by each respective

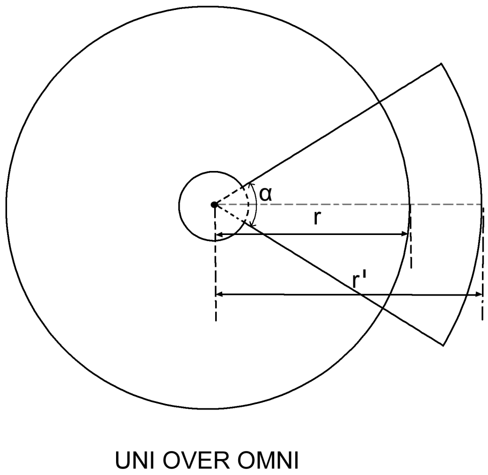

Fig. 1. Uni-Directional Antennas vs. Omni-Directional Antennas

Fig. 2. Multiple paths in a random WSN deployment

antenna's radiation pattern is fixed to be equal. This assumption is justified since the drawn power of each antenna is well known to be proportional to the subsequent area of the radiation pattern. Then, the ratio $k = r'/r$ quantifies the additional reach possible by a uni-directional antenna over its omni-directional counterpart, which in turn depends on the uni-directional antenna beamwidth.

$$k = r'/r = \sqrt{\frac{2\pi}{\alpha}} \geq 1 \tag{3}$$

Eq. (3) shows that a uni-directional antenna with a narrower beamwidth will be able to transmit signals that reach a longer distance, i.e. the narrower the beamwidth, the higher the antenna gain. The above equations also assume that the antennas involved have 100% efficiency, meaning that all the power delivered to the antenna circuit is radiated during transmission, the power fed being effectively converted into radiated power.

Fig. 1 compares the perfect radiation patterns of uni-directional and omni-directional antennas. It is to be noted that although it may appear that a very narrow antenna beam would have phenomenal effects on increasing antenna gain there are practical limitations in extending transmission range using this approach. Sensor networks have very strict requirements on form factor and size of motes, since antenna size needs to be equivalent to the wavelength λ of operation for power efficient functioning.

B. Graph Theory Basics

As the analysis on connectivity and security presented in this work uses concepts from graph theory, some basics are presented first [12].

An ad hoc network can be represented as an undirected graph G. A graph $G = G(V, E)$ consists of a set of n nodes or vertices and a set of m node pairs or edges. $V = \{1, \ldots, n\}$, is the set of vertices that actually represents the motes deployed in the WSN; and the set of edges, denoted by E, represents the communication links between these sensor motes. As the assumptions in this work include that of a symmetric channel and as every member node is assumed to have similar capabilities, a network can be modeled as an undirected graph.

A $neighbor$ of a node is any node that has a direct link or an edge with the node being considered. The $degree$ of a node x, denoted as $d(x)$, is the number of neighbors of node x. A node of degree d = 0 is called an $isolated\ node$. Such nodes have no neighbors.

The minimum node degree of a graph G is denoted as

$$d_{min}(G) = \min_{\forall\ u \in G} d(u) \tag{4}$$

A graph is connected, if for every pair of nodes there exists a path, consisting of one or more edges, connecting them. The graph is deemed disconnected otherwise. For WSNs, this may relate to cases in which one more more nodes cannot communicate with a larger subnetwork. For a truly connected network, all nodes in the deployment must be able to communicate with each other either via direct or multi-hop communication.

Another important metric is *k-connectivity* that is significant to WSN applications in which robust network availability in the face of link failure is required. A graph has k-connectivity, where $k = \{1, 2, 3, 4, 5, \ldots\}$ if for every pair of nodes there exists at least k disjoint and mutually independent paths connecting them. Another related definition states that if a graph is still connected after disabling any $(k-1)$ links, then the graph is k-connected. It can equivalently be shown that if the failure of any $(k-1)$ nodes still results in a connected network, then the graph is k-connected.

Fig. 2 gives an example for the existence of multiple paths for a random deployment of four motes. As can be seen in the figure, between motes A and C three paths exist. All the other motes have two paths to each other. For this reason, the example shown is 2-connected, as breaking any two links would deem the network disconnected.

IV. The Hybrid Approach

We assume a set of n network nodes where n is any natural number. The nodes are independently and randomly distributed over a region A. A uniform distribution is employed so that a constant node density $\rho = \frac{n}{A}$ can be defined. The node density

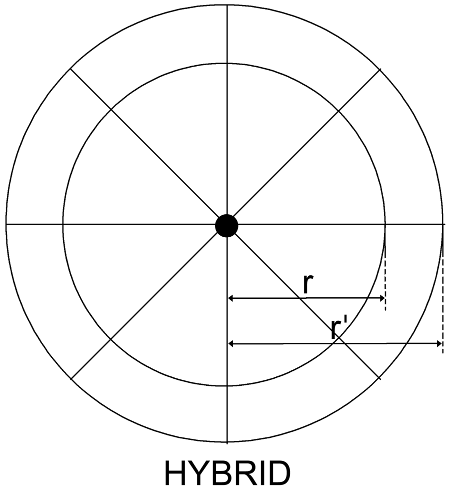

Fig. 3. The Hybrid Approach

is a representation of the average number of nodes per unit area. If we consider A to be a unit square then, $\rho = n$.

We represent a wireless sensor network as an undirected graph $G = G(V, E)$ where V represents the set of member nodes in the network and E is the set of edges between nodes that are able to communicate with each other. Undirected graphs are assumed because we consider a unit disk model to approximate omni-directional communications, where the existence of a link between any two nodes u and v in the network is dependent on the euclidean distance between them. If $\| u - v \| \leq r$, then according to this model both u and v are capable of sending and receiving information from each other. r is the transmission radius of the omni-directional antenna in the network nodes.

Each sensor node in this approach is capable of both omni-directional and sectorized uni-directional communications. Depending on the antenna beamwidth α used, the number of sectors N_s varies. The node is capable of transmitting in N_s non-overlapping sectors where the antenna beamwidth α has an angle span $\frac{2\pi}{N_s}$ radians.

The node is capable of transmitting to a maximum range r' defined by Eq. (3) in each sector when compared with an omni-directional antenna of transmission radius r. Nodes are also capable of transmitting omni-directionally at a radius r. Reception at each node is modeled to be omni-directional.

According to the relationship in Eq. (3), nodes modeled using a hybrid approach as presented in this work are designed to consume the same power per transmission when compared with a traditional omni-directional antenna equipped sensor mote. Thus, in terms of battery life and system longevity, node following this communication model would see very similar performance to those adhering to an omni-directional model.

Fig. 3 shows a node following the approach mentioned. A perfect radiation pattern is shown such that in each sector the node has an extra transmission reach of $r' - r$.

TABLE I
TRANSMISSION RANGE COMPARISONS FOR OMNI-DIRECTIONAL AND HYBRID MOTES - LOW r

r	$r'_{\frac{\pi}{3}}$	$r'_{\frac{\pi}{4}}$	$r'_{\frac{\pi}{6}}$
0.05	0.122474	0.141421	0.173205
0.10	0.244949	0.282843	0.346410
0.15	0.367423	0.424264	0.519615
0.20	0.489898	0.565685	0.692820
0.25	0.612372	0.707107	0.866025
0.30	0.734847	0.848528	1.039231

Table. I compares the normalized transmission range of motes with omni-directional capability and those equipped with hybrid capable antennas. This table specifically lists the comparison for lower, more practical values of transmission radii, in contrast to the area of interest where the motes will be deployed. More details of the design of this communication model is available in [13].

V. ANALYSIS AND SIMULATIONS

The aim of this section is to characterize how using the $Hybrid$ approach, k-connectivity and average number of disjoint paths for the network improves when compared to a traditional omni-directional antenna based network.

We are interested, in part, in demonstrating that the hybrid approach, which involves using sensor motes capable of omni-directional and sectorized uni-directional transmission discussed in section IV, is capable of improved performance when compared to a network that uses motes with only omni-directional capability. In particular, we look at the improvements in the probability of k-connectivity, defined in previous sections, for the entire network of nodes.

An upper bound for such a probability is computable by considering the probability that the minimum degree of each node in the network graph is greater than or equal to k. In topological terms, this is equivalent to every node in the network having n_{neigh} neighbors such that $n_{neigh} \geq k$. Thus, the probability of $d_{min} \geq k$ would give us the upper bound we are looking for.

Lemma 1: For a random undirected graph of n nodes if edges are added to the empty graph in an order chosen randomly and uniformly from the $\binom{n}{2}!$ possibilities, then almost surely the graph that results from the edge additions becomes k-connected when it achieves a minimum degree of k. For large n,

$$Prob(\text{G is k-connected}) = Prob(d_{min} \geq k) \qquad (5)$$

where d_{min} is the minimum degree (defined in previous sections) per node.

The above has been proven for random graphs in [14] and [15] for graphs with pathloss models.

For the sake of application interest in WSNs with low node densities, an upper bound for a probability of k-connectivity is computable by considering the probability that the minimum degree of each node in the network graph is greater than or equal to k. In topological terms, this is equivalent to every node in the network having n_{neigh} neighbors such that $n_{neigh} \geq k$. Thus, the probability of $d_{min} \geq k$ would give the upper bound that is needed.

Results for the same exist in [5] in the context of wireless multi-hop networks with nodes capable of omni-directional communication. Following the nearest neighbor methods approach employed in that work and using standard graph theoretical results the upper bound can be computed.

Theorem 1: If $P_{HYB}(d_{min} \geq k)$ is the probability of the average minimum degree being greater than k for a network with hybrid-enabled motes and $P_{OMNI}(d_{min} \geq k)$ is that for an omni-directional network then,

$P_{HYB}(d_{min} \geq k) \geq P_{OMNI}(d_{min} \geq k)$

Proof:

The minimum degree probability as a function of node density and transmission radius is known from [5].

$$P_{OMNI}(d_{min} \geq k) = \left(1 - \sum_{N=0}^{k-1} \frac{(n\pi r^2)^N}{N!} \cdot e^{-n\pi r^2}\right)^n \quad (6)$$

Here $\rho = n$, since by definition $\rho = \frac{n}{A}$ but in this case $A = 1$.

The approximation for computing the required bounds for k-connectivity via computing the probability for a minimum degree requirement on each node is expressed below.

$$P(\text{G is } k\text{-connected}) \leq P(d_{min} \geq k) \quad (7)$$

As justified earlier, the use of the hybrid approach enables activation of all sectors, thus extending the reach of the sensor mote along all directions. While analytically evaluating this approach, the capability of all sectors to be activated depending on unicast traffic awaiting transmission helps extend Eq. (6) by substituting for the transmission radius r with r' in accordance with the relationship in Eq. (3). In the following equations, the minimum degree probability in the omni-directional is denoted by $P_{OMNI}(d_{min} \geq k)$ and in the hybrid case as $P_{HYB}(d_{min} \geq k)$. Eq. (6) can now be rewritten as,

$$P_{HYB}(d_{min} \geq k) = \left(1 - \sum_{N=0}^{k-1} \frac{(n\pi r'^2)^N}{N!} \cdot e^{-n\pi r'^2}\right)^n \quad (8)$$

Using Eq. (3) substituting r' as $r\sqrt{\frac{2\pi}{\alpha}}$ so that

$$\begin{aligned} P_{HYB}(d_{min} \geq k) &= \left(1 - \sum_{N=0}^{k-1} \frac{(n\pi \frac{2\pi}{\alpha} r^2)^N}{N!} \cdot e^{-n\pi\frac{2\pi}{\alpha}r^2}\right)^n \\ &= \left(1 - \sum_{N=0}^{k-1} \frac{(\frac{2n\pi^2 r^2}{\alpha})^N}{N!} \cdot e^{\frac{-2n\pi^2 r^2}{\alpha}}\right)^n \\ &= \left(1 - \sum_{N=0}^{k-1} \frac{(2n\pi^2 r^2)^N}{\alpha^N N!} \cdot e^{\frac{-2n\pi^2 r^2}{\alpha}}\right)^n \end{aligned} \quad (9)$$

From Eq. (3) and with the expansion in Eq. (9) it can be concluded that,

$$P_{HYB}(d_{min} \geq k) \geq P_{OMNI}(d_{min} \geq k) \quad (10)$$

■

The hybrid case is equivalent to the omni-directional case when hypothetically, a beamwidth setting of 2π is used. For all other settings, the hybrid case will thus have a higher probability of disjoint paths in the network deployment.

The simulations (using MATLAB) below explicitly support this claim. The nodes are assumed to be static, with uniform random distribution and capable of both omni-directional and directional communications. Directional communications is modeled via sectorized uni-directional antennas, dividing the entire omni-directional region of 2π radians into a number of sectors according to the antenna beamwidth. Each sector can be activated, one at a time so that at any instant the node may appear to be equivalent to a uni-directional antenna and that reception is omni-directional. In the omni-directional mode, each node is capable of transmitting at a radius r. When switched to the uni-directional mode, each node is capable of transmitting at a radius r' related to r by Eq. (3), in each sector.

The results shown below are based on a randomly distributed network of nodes in a unit square. There is a centrally located sink at coordinates (0.5, 0.5). The interest of these simulations is in studying the effect of node density, transmission radii and uni-directional antenna beamwidth on the k-connectivity of a randomly deployed network of sensor nodes. The attempt begins by computing the probability of 2-connectivity, or the probability that every node in the network deployment will have at least 2 disjoint mutually independent paths to the centrally located sink. We generated 1000 random topologies to be able to compute the probability. Mutually independent paths are computed using standard disjoint path algorithms, using min-cut/max-flow techniques and link reversals that provide optimal sets of disjoint paths as mentioned in [16] and [17]. To understand the relationship with node density and transmission radius empirically, the normalized r was varied between 0.05 and 0.45 and n, the node density, between 10 and 100. This is basically the probability of 2-connectivity. The effects of varying the beamwidth from $\pi/6$ to $\pi/3$ was also demonstrated by appropriate configurations for the simulations. These plots are shown below.

Fig. 4 and Fig. 5 describe the probability of 2-connectivity over varying transmission radii, node density and antenna beamwidth.

It can be seen from the first plot with a constant n and varying r that again, the hybrid approach provides a very substantial non-zero probability even between the lower transmission radii settings of 0.15 and 0.25. At a setting of 0.25, the hybrid approach out performs the omni-directional setting by almost 40%. When the operational transmission radius is set to a high 0.4, the improvement is almost around 80% as can be seen.

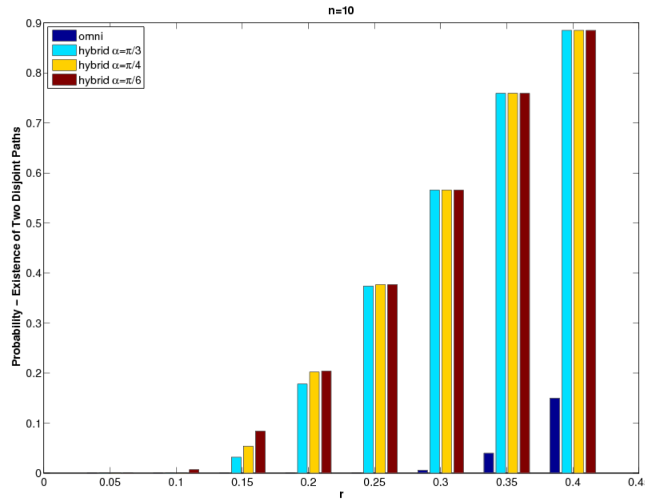

Fig. 4. Probability of Existence of Two Mutually Disjoint Paths for all Nodes in the Network- Varying Transmission Radius r

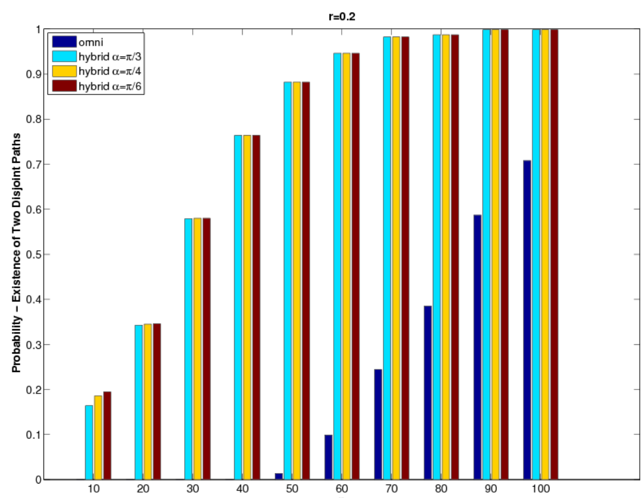

Fig. 5. Probability of Existence of Two Mutually Disjoint Paths for all Nodes in the Network- Varying Node Density n

The second plot in Fig. 5 describes the effect of varying node density n for a constant r of 0.2. Intuitively with increasing node density, the omni-directional setting is able to climb to higher probabilities, as seen for the maximum node density of 100 that is considered for these simulations, the probability for an omni-directional configuration reaches around 0.7. In contrast, the hybrid approach was at a probability of more than 0.7 around a node density of just 40. This emphasizes on the improved performance available when the hybrid approach is employed even at lower node densities. At the interim node density of 50, the hybrid approach out performs an omni-directional only setting by more than 90%.

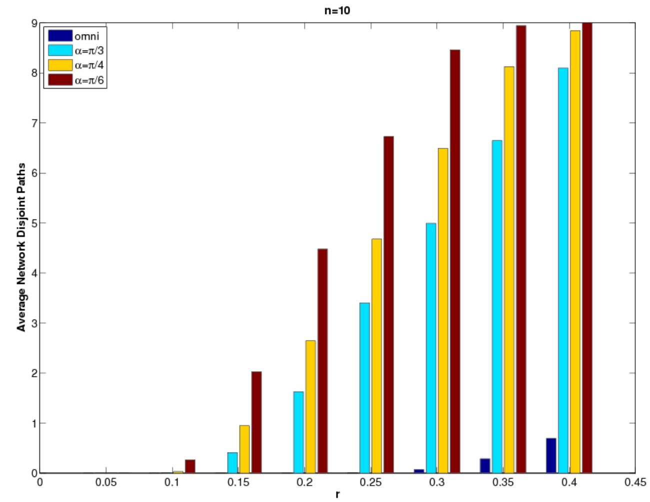

Fig. 6. Average Number of Mutually Disjoint Paths for all Nodes in the Network - Varying Transmission Radius r

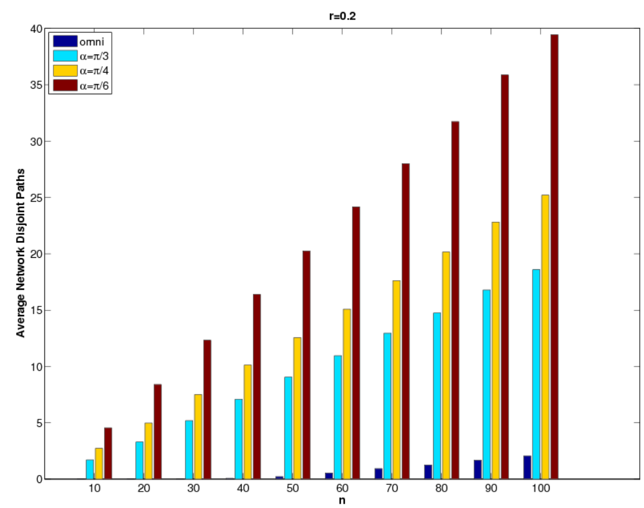

Fig. 7. Average Number of Mutually Disjoint Paths for all Nodes in the Network - Varying Node Density n

To further demonstrate the improvements in terms of the availability of disjoint paths for each node, another set of simulations are presented that use the metric *Average Number of Disjoint Paths for the Network*. This metric represents the average number of paths all member nodes in the network deployment possesses towards the centrally located sink.

Results for varying n and r are presented in Fig. 7 and Fig. 6. For the first plot, a very low node density of n = 10 was considered. 10 nodes distributed over a unit square, is usually a very sparse deployment even for a normalized radius of say, 0.2 for an omni-directional configuration. Interestingly enough, the hybrid setting with r at 0.2, meaning that for α = $\frac{\pi}{6}$, r' is around 0.69, the average number of disjoint paths was around 7. For the omni-directional setting, the network was able to even reach 1-connectivity.

For varying n, there is an almost linear relationship in terms of the incremental gains achievable from using the hybrid approach. At the maximum setting of node density 100, the hybrid approach provides around 17, 25 and 40 disjoint paths on an average for the network at the beamwidth settings of $\frac{\pi}{3}$, $\frac{\pi}{4}$ and $\frac{\pi}{6}$ respectively. The omni-directional setting even at the maximum node density of 100 could barely make an average value of around 3 mutually independent paths.

VI. Conclusion and Comments

In this work we have presented results on k-connectivity and multiple disjoint path availability improvements using a hybrid approach towards antennas in sensor network motes. We have cited significant related work in this area, but to the best of our knowledge there is no work that looks specifically at the improvements that have been considered in this paper. The improvements in k-connectivity are almost as high as 80% when compared to the traditional communication model.

The hybrid mode will only be used in the transmit section and the omni-directional setting is retained. The reason for the same being the compatibility of using this paradigm in FSO networks where line of sight is lost and also to be able to support the broadcast mode of transmission. It may be noted that the basic communication mode in sensor networks is unicast and individual activation of sectors towards a next-hop node permits increasing gains.

Future work will look at the improvements possible by using directional reception in terms of connectivity. Also, a detailed analysis of security improvements will be considered. We also intend to look at the effect of interference and the improvements that may be obtained by using the hybrid approach.

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