

Distributed Sustainable Generation Dispatch via Evolutionary Games

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Abstract—Today’s power grid is provisioned conservatively for rarely occurring demand peaks. These peaks are served by flexible generation systems that are typically costly and have significant carbon footprint. Distributed power sources such as wind turbines and solar panels are sustainable but unreliable as these have inherently variable generation capacities. An effective power dispatch management system is necessary to harness the significant generation potential of these intermittent systems. In this work, a novel scheme is proposed which leverages upon the recent cyber-enablement in the power grid to distributively dispatch a large number of strategically interacting small-scale variable generators. We incorporate evolutionary game theoretic techniques into the formulation of the dispatch strategy as it provides an opportunity to model the aggregate behaviour of tactical agents making inter-dependent decisions and aids with establishing deterministic steady state predictions of the system state. Numerical and theoretical results presented in this work show that the proposed strategy is highly scalable and enables real-time power dispatch of intermittent systems while maintaining low computational overhead.

I. INTRODUCTION

The power grid is composed of a diverse mix of energy generation systems designed to provision for all types of consumer power demands. Highly fluctuating peak demands are typically served by natural gas or coal plants as these can rapidly ramp up or down generation as needed. Unfortunately, these flexible generation systems typically incur high capital costs, fuel costs and carbon emissions [5]. Alternate renewable systems such as wind and solar power generators are green energy sources with lower levelized costs [4]. These Distributed Generation (DG) systems can be deployed in large numbers at close proximity to consumers - increasing power efficiency due to reduced transmission line losses. Many positive attributes such as these indicate that DGs will play a significant role in future power systems. For example, in Ontario, the provincial government has aimed to reduce CO₂ emissions from a high of 35 MegaTonnes in 2005 to 6 MegaTonnes in 2025 [11]. In order to achieve this goal, the provincial government has currently phased out all coal plants and constructed a long term energy plan that mandates renewable generation to account for 46% of overall generation capacity in 2025 [11].

In order to harness the full generation potential of DGs, it is imperative for the Electric Power Utility (EPU) to utilize an effective power dispatch management strategy that can dynamically manage and balance aggregate demand with variable generation capacity in a real-time manner. Many innovative

power dispatch algorithms have been proposed in the existing literature which can be classified as centralized, decentralized or distributive techniques. Centralized dispatch strategies in [1], [8], [17] and [16] construct a dispatch optimization problem which grows exponentially in complexity as the number of DGs increase in the system. Decentralized and distributed schemes such as [7] and [6] have convergence characteristics that render these unsuitable for real-time dispatch.

In this paper, we leverage upon the recent cyber-enablement and open framework of the power grid to propose a distributed power dispatch algorithm that enables DGs to intelligently configure their own power dispatch based on interactions with other DGs and simple downlink signals broadcasted by the EPU. This scheme completely abstracts the EPU from local conditions of the DGs and relieves it from the arduous task of allocating power dispatch to every participating DG with variable generation capacities. Issues such as computational intractability and slow convergence identified in schemes proposed in the existing literature are overcome in this dispatch scheme. Comprehensive theoretical and numerical results presented in this work indicate that our scheme is highly *scalable*, *real-time* and entails *low computational* overhead for participating entities. The proposed scheme is a novel application of Evolutionary Game Theory (EGT) for distributed power control. EGT is typically employed in the energy management literature for pricing scheme adoption analyses (e.g. [2], [9]). EGT is suitable for the dispatch problem considered in this work as it offers a comprehensive framework to model the aggregate behaviour of strategically interacting entities in a system from which deterministic static and dynamical properties of the system can be established.

The remainder of this paper is organized as follows. In Section II, the system model is presented. Formulation and theoretical analysis of the proposed dispatch algorithm is detailed in Section III. In Section IV, numerical implementation of the proposed scheme in a realistic environment is presented to further strengthen theoretical results developed in Section III. Finally, the paper is concluded in Section V.

II. SYSTEM MODEL

An EPU will aim to minimize dependence on peak-serving generation systems when sufficient sustainable generation capacity exists to meet a subset if not all aggregate demand in the system. Entities that can participate in the dispatch

management scheme include all DGs that are part of the power distribution system managed by the EPU. Following assumptions are made in this work:

- 1) Aggregate demand and generation capacities of each DG remain constant for at least 10 minutes.
- 2) The distribution system is connected to the main grid which contains negative spinning reserves, synchronized generation resources and storage systems.
- 3) The EPU can implicitly measure the deficit or surplus generation in the system.
- 4) DGs are equipped with programmable modules, cellular transmitters and receivers. EPU can transmit downlink cellular signals.
- 5) DGs can communicate with one another in an anonymous and encrypted manner.
- 6) Only energy constraint considered is the balance between demand and generation.
- 7) Number of DGs in the dispatch system is large.

The first assumption is valid as today's power dispatch models are based on constant hourly generation and demand forecasts [10]. The second assumption is necessary to ensure that all demand can be fully served when aggregate generation capacity in the system is insufficient. The third assumption is feasible as the EPU can monitor the power purchased or sold from the distribution system. The fourth assumption, also made in [7], is necessary to allow DGs to receive signals from the EPU and communicate with one another to make intelligent distributive dispatch decisions. The fifth assumption ensures that security and privacy is maintained in all communications. The sixth assumption, also made in many dispatch works such as [6], simplifies the problem by not taking into account network flow constraints and system physics. We expect that the final assumption is practical. For example, in the specific case of the residential neighbourhood, many DGs can be deployed as rooftop installations.

Next, an overview of the basic notations used in this work is presented. Given that sufficient generation capacity is available, a DG can select a strategy e_i from the set Ω of size M and these strategies map to the power commitment levels defined in $y = [y_1 \dots y_M]^T$ kW. The empirical distribution of all strategies in the system is denoted by $x = [x_1 \dots x_M]^T$ where x_i is the frequency at which strategy e_i is used in the system.

III. DISTRIBUTED DISPATCH ALGORITHM

Game theory provides a useful set of tools to model interactions between distributive entities and is therefore well suited for the dispatch problem considered in this work. Traditionally, *normal form games* have been used to model how *rational* players interact with one another based on inter-dependent costs assigned to the actions taken by each participant. In order to make a rational strategy selection, each player will need to compute the cost associated with all strategy permutations which maybe employed by other players and this can be a laborious task for a large system. In the dispatch context, EGT offers a theoretical foundation to model a large number of strategically interacting DGs playing a *population game* [14].

The population game anonymizes participating DGs and is only concerned with the aggregate behaviour x of the system which represents the population state.

Many difficulties encountered in the normal form game due to the presence of a large number of DGs are eliminated in the EGT setup. For instance, as there is a large number of DGs in the system, the decision made by a small set of DGs to switch strategies will not cause a noticeable impact on the costs of other strategies. Also, due to the strong law of large numbers, the stochastic effects of DGs' actions will be averaged out and the evolution of the aggregate behaviour x of the system will be very close to a deterministic system. At equilibrium, given that sufficient generation capacity exists in the system, each DG strives to dispatch minimal individual power while ensuring that the aggregate power dispatched by all DGs meets overall system demand.

A DG participating in the population game will revise its current strategy at a random time using an *imitative protocol* to make *myopic* decisions using local interactions with another randomly selected DG in the system. These interactions result in the deterministic evolution of the population state x in a manner similar to the biological selection process of behavioural traits in a population of species. The resulting state evolution is referred to as the Replicator Dynamics (RD). Many interesting steady state natural phenomena resulting from distributive imitative interactions between biological entities have been observed in the past [12] and we show in this work that desirable stationary results can also be obtained for the dispatch problem by applying a similar approach.

A. Cost Function

The penalty a DG incurs for selecting a particular strategy e_i depends on the population state x as follows:

$$J(e_i, x) = y_i(n x^T y - C) \quad (1)$$

where y_i is the power dispatched when strategy e_i is selected, $n x^T y$ is the aggregate power dispatched by all n DGs in the population and C is the aggregate power demand in the system. Let the cost for every pure strategy be represented by the vector $J(x) = [J(e_1, x) \dots J(e_M, x)]^T$. As strategies are present in the population with a frequency x , the average cost of the game is $\bar{J}(x) = x^T J(x)$.

B. Revision Protocol

Based on these costs, a DG performs imitative revisions of its current strategy at randomly chosen time. During the revision process, the DG examines its current strategy and the strategy of another randomly selected DG. It imitates its opponent with a probability proportional to $\rho_{i,j}(x)$ which is the conditional switch rate of DGs from strategy e_i to strategy e_j given that the population is at state x and is defined as [13]:

$$\rho_{i,j}(x) = x_j [J(e_i, x) - J(e_j, x)]_+ \quad (2)$$

The intuition behind this particular definition is that if the cost of using one strategy is lesser than the current strategy, then the rate at which the DG will switch to it will increase in proportion to the difference in costs and the strategy frequency.

C. Dynamical System Evolution

The resulting dynamical population state evolution when all DGs use imitative revisions is derived here. The average rate of DGs switching into strategy e_j is $\sum_{i=1}^M x_i \rho_{i,j}(x)$. The average rate of DGs switching from e_j is $\sum_{i=1}^M x_j \rho_{j,i}(x)$. The randomness in the above expression is not considered without loss of generality as the strong law of large numbers states that the behaviour of the system will tend to average when the population size is large. The change dx_j in the population state x_j due to these revisions is the net proportion of DGs switching to and from the strategy e_j according to:

$$\begin{aligned}\dot{x}_j &= \sum_{i=1}^M x_i \rho_{i,j}(x) - x_j \sum_{i=1}^M \rho_{j,i}(x) \\ \dot{x}_j &= \sum_{i=1}^M x_i x_j [J(e_i, x) - J(e_j, x)]_+ \\ &\quad - x_j \sum_{i=1}^M x_i [-J(e_i, x) + J(e_j, x)]_+ \\ \dot{x}_j &= -x_j [J(e_j, x) - \bar{J}(x)] \quad \square\end{aligned}$$

The first line is obtained by simplifying the net proportion of transitions to and from strategy e_j . This expression is known as the *mean dynamics* [3]. The second line results from substituting Equation 2 into $\rho_{i,j}(x)$. The last line is the dynamic resulting from imitative revisions and is derived by combining the first and last terms of the second line. This system is also referred to as *Replicator Dynamics* (RD) [15]. According to the RD, if the individual cost of selecting strategy e_j is lesser than the overall cost of the game, then the frequency of that particular strategy will increase in the system.

D. Static Equilibrium Properties

The game theoretic equilibrium considered in this work is the Nash Equilibrium (NE). When NE is attained no DG can deviate from its current strategy without incurring more or equal cost and this is formally defined in Definition 1.

Definition 1. [12] x is an NE and in Δ_{NE} if:

$$x^T J(x) \leq y^T J(x) \quad \forall y \in \Delta \quad (3)$$

Stationary points of the RD dynamical system occur when $\dot{x}_i = 0 \quad \forall i \in M$ and form the set $x^{eq} \in \Delta_{RD}$. There exists a relationship between points in Δ_{NE} and Δ_{RD} .

Theorem 1. All $x \in \Delta_{NE}$ are Lyapunov stable points of Δ_{RD} .

Proof: According to Sandholm, a strict Lyapunov function V for the dynamical system \dot{x} exists if $\Delta V(x)^T \cdot \dot{x} \leq 0 \quad \forall x \in \Delta$ with equality occurring only for x satisfying $\dot{x} = 0$ [14]. It is shown in the following that $V(x) = \frac{1}{2n}(nx^T y - C)^2$ is the strict Lyapunov function for the RD system. First, it should

be noted that $\Delta V(x) = F(x)$. Using this and the generalized mean dynamic, the following is derived:

$$\begin{aligned}\Delta V(x)^T \cdot \dot{x} &= \sum_{j=1}^n F_j(x) \left(\sum_{i=1}^n x_i \rho_{i,j}(x) - x_j \rho_{j,i}(x) \right) \\ &= \sum_{j=1}^n \sum_{i=1}^n (x_i \rho_{i,j}(x) F_j(x) - x_j \rho_{j,i}(x) F_j(x)) \\ &= - \sum_{j=1}^n \sum_{i=1}^n x_i \rho_{i,j}(x) (F_i(x) - F_j(x)) \\ &\leq 0\end{aligned}$$

Line 1 is obtained by substituting the cost function F into $\Delta V(x)$ and the generalized mean dynamic into \dot{x} . Since Line 2 contains a double summation, the indices can be re-arranged to obtain Line 3. According to the definition of the imitative revision protocol, $\rho_{i,j}(x) \geq 0$ if $F_i - F_j \geq 0$ and 0 otherwise. As the two positive terms in Line 3 are negated, this expression is negative. Hence, this shows that $V(x)$ is indeed a strict Lyapunov function for this system. Next, we demonstrate that Lyapunov stability implies NE:

$$\begin{aligned}\Delta V(x)^T \cdot \dot{x} \leq 0 &\rightarrow J(x)^T \cdot \Delta x \leq 0 \\ J(x)^T \cdot (x - y) \leq 0 &\rightarrow x^T \cdot J(x) \leq y^T \cdot J(x)\end{aligned}$$

The final expression in the above is exactly the condition for NE listed in Definition 1 \square .

With the RD system, if the initial population state x^0 does not lie in the interior of the simplex Δ then all unused strategies will become extinct as $x_i = 0 \rightarrow \dot{x}_i = 0$. The resulting rest point of the state trajectory may not be an NE. Work in [14] shows that these rest-points are not Lyapunov stable. Hence as long as the initial states x^0 lie in the interior of Δ , the rest-points of the dynamical system are Lyapunov stable and therefore NEs. So far, the dynamical and static system properties have been investigated in this section. Next, the distributive implementation of the proposed dispatch scheme by DGs is discussed in the following.

E. Distributive Algorithm

Each DG in the population will compute the time τ at which it will re-examine its current strategy using an exponential distribution with rate $2s$ which is the expected communication delay in the system [10]. The EPU implicitly computes the surplus or deficit power in the system by measuring the inflow or outflow of power from the synchronous generators in the main grid to obtain $D = (nx^T y - C)$. The value of D is sent to all DGs in a periodic manner every $2s$. The DG that is currently re-evaluating its strategy will send a broadcast requesting for an opponent. Other DGs receiving this request will choose to respond in a random manner. In order to preserve security and privacy, responding DGs can encrypt and anonymize messages. If the initiating DG receives more than one response, it will select one randomly and compute $\rho_{i,j}$ using D . Based on this switching rate, the DG will randomly determine whether or not to imitate strategy e_j if its generation

capacity is sufficient. In order to ensure that no extinction occurs in the system, DGs will once in a while select a strategy randomly instead of imitating. The distributive implementation is summarized in Table I.

Distributed Algorithm for DG

Initialization:

- Select a strategy e_i randomly from M

Algorithm:

- 1) Compute τ using exponential distribution with rate R .
 - 2) After τ sec, broadcast request for an opponent and randomly select one of the responses.
 - 3) Calculate $\rho_{i,j}$ based on the strategy used by the selected responding DG.
 - 4) If current generation capacity is greater than or equal to y_j , switch to strategy e_j with probability proportional to $\rho_{i,j} > 0$ or switch to a randomly selected strategy e_k that meets available generation capacity with probability ϵ .
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TABLE I: Summary of distributed implementation of dispatch

IV. RESULTS

Numerical results illustrating the effectiveness of the proposed distributed dispatch strategy summarized in Table I is presented in this section. Unless otherwise mentioned, the simulation environment contains 1000 agents each representing a DG with generation capacities that can be one of 1kW, 2kW or 3kW. All numerical results presented in this work are obtained from implementations in MATLAB.

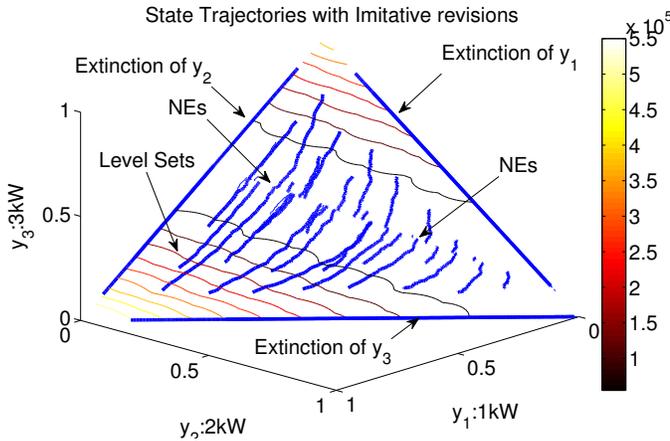


Fig. 1: Solution trajectories superimposed with cost contours

Fig 1 illustrates the solution trajectories of the system for various initial population states. These trajectories are superimposed with the average cost of all agents in the population. All solution trajectories with initial states in the interior of the simplex reach NE rest-points at which the average cost of the entire system is the lowest. Non-interior initial states

result in trajectories that are non-Nash and it is evident from this figure that initially unused strategies have become extinct as expected. This diagram can also be used to gauge the convergence speed of the solution trajectories to the NE states. Solution trajectories that are orthogonal to the level sets of the average cost of the system have the fastest descent to the NE states [14]. From Fig 2, it is evident that DGs exhibit the most orthogonal behaviour when trajectories are closer to the NE states.

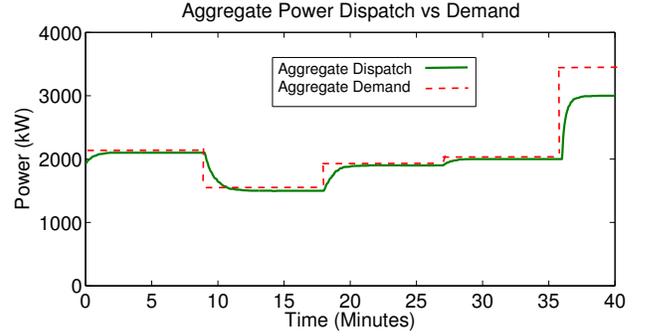


Fig. 2: Aggregate power dispatched by DGs

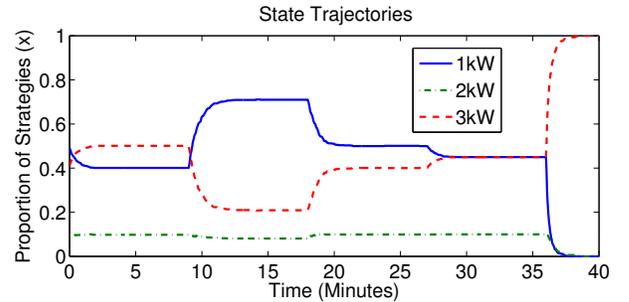


Fig. 3: Solution trajectory for dispatch

In the next set of experiments, the ability of agents to distributively respond to changes in aggregate demand or generation capacity in the system is investigated. Fig 2 illustrates the aggregate dispatch of all DGs in the system versus the aggregate demand C in the system. Overall C and generation capacity of each DG changes in a random manner. It is evident that the aggregate dispatch of all DGs are able to rapidly converge to meet the aggregate demand in the system in a distributive manner. Between 35 and 40 minutes, the overall demand is 3500 kW and this is much higher than the maximum possible generation capacity in the system (1000 DGs * 3kW). In this case, all DGs operate at maximum capacity. Fig 3 presents the solution trajectories of the three levels of power dispatch in the system. The aggregate dispatch of all agents exhibit no oscillatory or divergent behaviour as expected.

In the EGT formulation of the distributed dispatch algorithm, we make a strong assumption that a “large” number of DGs are present in the system. Next, the impact of the number of participating DGs on the convergence characteristics of

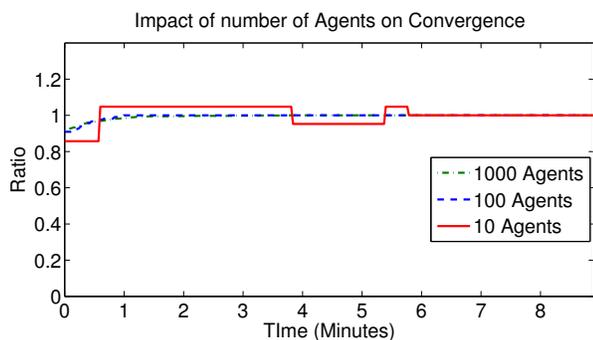


Fig. 4: Impact of number of DGs on dispatch convergence

overall dispatch is presented in Fig 4. Three systems are considered in which the first one contains 1000 agents with $C = 2100\text{kW}$, second contains 100 agents with $C = 210\text{kW}$ and the last system contains 10 agents with $C = 21\text{kW}$. Results in Fig 4 indicate that larger the number of agents, the faster and smoother is the convergence rate. Systems with 100 and 1000 agents have similar convergence trends while the system with 10 agents displays more chaotic behaviour. This experiment indicates that as long as the number of agents in the system is not too low (i.e. 10 agents), theory and results from the original EGT formulation can be preserved.

Next, the convergence and computational overhead properties of the proposed dispatch scheme are compared with other dispatch schemes proposed in the literature. For centralized optimization techniques, the size of the problem increases as the number of DGs n increase in the system. Centralized optimization techniques such as the interior point method has a computational complexity of $O(n^3)$ while our scheme has a complexity of $O(1)$ as all participating entities perform simple arithmetic computations. According to the work in [7], the proposed decentralized algorithm requires at least $n(n-1)$ exchanges of information for convergence. For a 1000 DG system, given that the communication delay is about 2s, the time required for convergence far exceeds the 10 minute period. Simulations indicate that our scheme requires at most 2 minutes for convergence.

These results show that when the initial population state is an interior, the proposed distributed dispatch algorithm enables participating DGs to effectively converge to NE rest-points which incur the least average population costs. Although it is fairly simple for the EPU to compute NE population state in a centralized manner, a major challenge is assigning strategies to each DG so that this NE population state can be achieved. The EPU needs to be aware of the local generation conditions and availability of each DG. The proposed algorithm abstracts all these details from the EPU. Even if a subset of agents are unable to participate in the dispatch process, the algorithm enables the system to react appropriately to changes.

V. CONCLUSIONS

In this work, we propose a distributed dispatch strategy based on EGT techniques that can be used to effectively

manage DGs so that these can replace some of the traditionally used costly and unsustainable peak serving generation systems. We demonstrate that EGT provides a comprehensive framework to distributively control DGs and model aggregate behaviour of the system using a deterministic dynamical process. The imitative revision protocol used by participating DGs entails low computational overhead and the dynamics of the system is such that DGs are able to rapidly converge to the desired aggregate dispatch level when sufficient generation capacity exists in the system which renders the proposed strategy suitable for real-time deployment. As future work, other types of revision protocols which have various degrees of communication requirements and privacy implications should be explored as alternatives for the dispatch problem.

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