

A Game Theoretic Approach to Real-Time Robust Distributed Generation Dispatch

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Abstract—Power demands are rising at an exponential pace due to the increasing proliferation of high-energy consuming devices such as plug-in hybrid electric vehicles. It is well known that scaling traditional power generation systems to accommodate these soaring demands will be excessively costly and may lead to negative environmental ramifications. One approach to supplement increasing energy needs involves diversifying the generation mix to incorporate a large number of local distributed generators (DGs) for economical and sustainable operation. However, such an approach remains an open challenge due to the inherent generation variability of DGs. In this paper, we propose a distributed generation dispatch strategy that can effectively coordinate a large number of DGs to meet consumer demand in real time. Through theoretical analysis based on population games and simulation studies, we demonstrate that our dispatch strategy is scalable and allows for the seamless integration of alternative energy resources into the grid in a robust and an optimally cost-effective manner.

Index Terms—Economic dispatch, game theory, sustainable generation.

I. INTRODUCTION

THE energy mix in today's power grid is largely comprised of unsustainable generation systems. For instance in the United States, as of 2014, nearly 70% of primary energy production is commissioned from carbon-fueled generation systems such as coal, natural gas, and crude oil [1]. Many of these modes of generation are supported by imported fuels that are subject to external volatilities, including political upheaval and natural disasters, and this dependence can pose a significant threat to a nation's energy security [1]. Moreover, these traditional synchronous plants and supporting transmission lines are nearing the end of their operational life span. The cost of maintaining this aging infrastructure to support increasing consumer demands is projected to be very high [2]. Hence, in support of the smart grid vision [3], it is imperative to diversify today's

generation mix to include alternative energy sources such as distributed generators (DGs).

DGs are typically small-scale generation systems such as photovoltaics (PVs), wind turbines, and storage devices that are located at close proximity to consumers. Significant penetration of DGs at the distribution substation level will eliminate the need for an expensive transmission infrastructure and associated line losses, while promoting sustainability as many DGs are renewable. However, a major deterrent for widespread DG integration is their inherent generation variability. Optimally dispatching a large number of highly variable energy sources largely remains an open challenge to date. Proposed dispatch strategies in the existing literature can be generally categorized into centralized and distributed schemes.

Centralized dispatch solutions are typically computed offline (for, say, day-ahead markets), where access to vast computational resources such as the cloud is readily available. These strategies rely extensively on the prediction models to forecast highly variable demand and supply. Even though sophisticated optimization methods such as stochastic models [4], heuristics [5], and simulated annealing [6] are engaged to solve the optimal dispatch problem, significant error margins are introduced due to prediction inaccuracies. Real-time optimal dispatch can avert these errors. However, centrally computing optimal online solutions is intractable for larger systems. Moreover, centralized solutions are subject to privacy and single point of failure issues [7]. In order to address cyber security in dispatch and data aggregation, solutions applying sophisticated authentication and encryption techniques like that in [8] have been proposed. These, however, entail high overhead due to the additional processing and require extensive changes to the existing metering infrastructure.

The recent push toward cyber enablement by electric power utilities (EPUs) has equipped various components in the power grid with bidirectional communication capabilities [9]. *Distributed* dispatch strategies capitalize on this information-enriched paradigm to enable iterative local dispatch adjustments via repeated exchange of data between DGs until the system converges to optimality. In the literature, these schemes are based on theoretical constructs that include dual decomposition [10], gradients [11], dynamic programming [12], and consensus protocols [13]. Completely decentralized solutions, such as in [14], utilize local measurements such as frequency variation to make dispatch adjustments. Other decentralized solutions such as [15] and [16] propose solutions based on decomposition techniques for economic dispatch to manage heterogeneous energy sources

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in order to avert the peaking of aggregate load in the system. Noncooperative game theory is applied in [17] to compute optimal renewable dispatch using linearized power flow equations. As the convergence speed of these methods is proportional to the system size, these do not scale well for a real-time dispatch of a large number of DGs. In [18], an evolutionary game theory is applied to accommodate a large number of DGs. However, as the proposed algorithm in [18] results in nonunique solutions, the system will be subject to significant ringing and instability with minor perturbations. Srikantha and Kundur [19] leverage population games in the context of demand response and the protocols utilized involve information exchanges between participating consumers. This is not suitable in a generation dispatch environment from a security perspective. Another proposal in the context of game theory applied to demand response is given in [20], which incorporates dual decomposition and best response decision-making obtained via the gradient projection method. We differ from this proposal as we utilize the population game theory and state dynamics to prove system convergence. In this work, we examine the use of revision protocols based on the traditional game theory for dispatch where interactions between participants are not necessary.

In this paper, we present a dispatch solution that is a novel departure from existing dispatch strategies. In our distributed strategy, the EPU broadcasts light-weight unidirectional signals that foster coordination amongst a large number of DGs in real-time. Principles from the population game theory are leveraged to provide insights on the strong static, dynamic, and convergence properties of the algorithm. Comprehensive simulations based on realistic models validate these findings. Our contributions in this paper are fivefold:

- 1) We propose a novel dispatch strategy with two distinct implementations and provide analytical and practical convergence studies of the two techniques;
- 2) We show that one particular implementation is highly scalable and enables asymptotic convergence to optimality when there exists sufficient generation capacity in the system;
- 3) We demonstrate the robustness of the system to cyber attacks;
- 4) We study the impact of various generation mix on policy initiatives; and
- 5) We highlight differences between our proposal and strategies in the recent literature.

The remainder of this paper is structured as follows. In Section II, an overview of the system setting is provided. We present our proposed strategy in Section III. Section IV contains our implementation and results. Finally, we conclude in Section V.

II. SYSTEMS SETTINGS

We consider the dispatch problem in which a *large* number of interspersed DGs connect to a common distribution substation that supplements local consumer demands (analogous to a grid-connected microgrid as illustrated in Fig. 1).

A. Assumptions

The following assumptions are made to facilitate the design of the distributed DG dispatch strategy proposed in this paper:

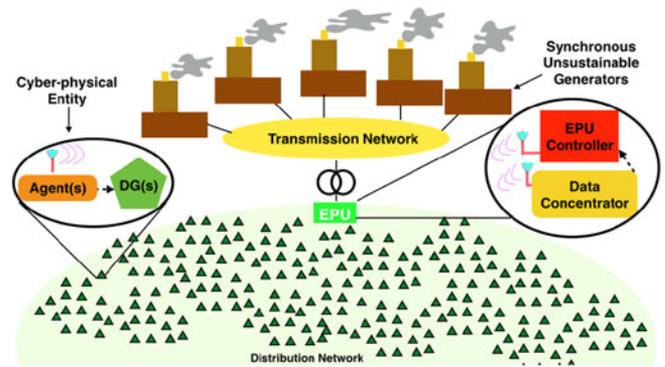


Fig. 1. System model.

- 1) The EPU can measure the aggregate cost of various dispatch strategies currently in use by DGs in the system via data concentrators;
- 2) The grid consists of a significant amount of DGs that include storage, renewable sources (roof-top solar panels, micro wind turbines), and diesel generators;
- 3) The EPU can broadcast signals to DGs every 3 s;
- 4) Every DG consists of a cyber-physical *agent* composed of an intelligent controller and a receiver;
- 5) The active power setpoint of every DG can be selected by its intelligent controller from a finite set of values;
- 6) Demand and DG generation capacity remains almost constant for every 60 s interval;
- 7) The system is equipped with ample storage and negative spinning reserves; and
- 8) Dispatch cost of DGs is quadratic.

The first four assumptions support the cyber-physical vision of a diversified smart grid [3]. Assumption 1 is facilitated by active monitoring systems consisting of phasor measurement units (PMUs) and data aggregators. Assumption 3 ensures that sufficient communication latency margin is incorporated into the signaling period. In Assumption 4, cyber-physical agents (referred to as *DG agents* in the remainder of the paper) are the representatives of the EPU residing at every DG to make local dispatch decisions based on signals transmitted by the EPU and local generation capacity which are then reported to data concentrators. Assumption 5 states that an intelligent controller resides at every DG and it can select from a set of discrete power generation levels given that there exists a sufficient generation capacity. This effectively integrates heterogeneous energy sources such as batteries with discrete power output. Assumption 6 reinforces the real-time nature of the dispatch problem as considered in this paper. In the literature, dispatch algorithms such as that proposed by [21] assume demand and supply to be constant for 1 h intervals, whereas we consider a much shorter period to alleviate major prediction errors so that more accurate prediction models can be evoked instead [22]. The next assumption is necessary to maintain reliable operation in case the system is islanded from the main grid. The final assumption is applied in primary energy markets [23].

B. Original Dispatch Problem

There are m DG agents and k power consumers in the system. DG agents form the population \mathcal{P} . A DG agent can select

from one of n active power levels (synonymously referred to as *strategies*) from the set $y = [y_1 \dots y_n]^T$. Based on Assumptions 2 and 5, $n \ll m$. Power dispatched by DG i is constrained by the local generation capacity c_i . The classical dispatch problem solved by the EPU attempts to match overall demand with available supply in a cost effective manner according to \mathcal{P}_D

$$(\mathcal{P}_D) \min_z f_o(z) = \frac{1}{m} \sum_{i=1}^n Y_i T_i(z)^2$$

$$\text{s.t. } \sum_{i=1}^m z_i = \sum_{j=1}^k w_j \quad \text{and} \quad 0 \leq z_i \leq c_i$$

$\forall i = 1 \dots m$, where optimization variable $z_i \in y$ is the set-point at which DG i supplies power to the system, $z \in \mathbb{R}^m$ is the dispatch vector representing the power dispatched by all m DG agents in the system, c_i is the current generation capacity available to DG i , and w_j is the power demand from consumer j . The objective $f_o(z)$ is a quadratic function (also used in primary energy markets such as that of [23]). We select $T_i(z) = \frac{1}{\sqrt{2}y_i} \sum_{k \in \mathcal{P}_i} z_k$, where \mathcal{P}_i represents all DGs using strategy y_i . Y_i is a strictly positive value that increases with i (i.e., $Y_i \leq Y_{i+1}$) so that the higher power generation is associated with greater cost. The first constraint enforces balance between overall power supply and demand that favors tractability in lieu of incorporating detailed physical constraints (also adapted in existing work such as [11]). The second constraint confines power supplied by DG i to be within its current generation capacity c_i limits. As the optimization variable takes discrete values (i.e., Assumption 5), \mathcal{P}_D is a discrete optimization (DO) problem. Since DO is a nondeterministic polynomially (NP) hard problem, solving \mathcal{P}_D becomes computationally intractable as the number of DGs increases in the system. Moreover, as variations in generation capacities (typical with DGs such as solar and wind sources subjected to irregularities due to cloud cover and wind speed) and consumer demands are taken into account at high granularity, optimally solving \mathcal{P}_D at such a small time scale is not trivial. Specifically, due to Assumption 6, the demand w_j and generation parameters c_i are updated in the problem \mathcal{P}_D every 1 min. Solving \mathcal{P}_D every minute to account for flux in generation and supply is not feasible.

III. DISTRIBUTED DISPATCH STRATEGY

To overcome the challenges imposed by highly fluctuating generation constraints, consumer demands and integer optimization variables, we apply a series of transformations to \mathcal{P}_D . As a result, the original centralized dispatch problem can be solved in a distributed manner whereby the EPU broadcasts signals $F = [F_1 \dots F_n]^T$ containing the costs of all y strategies in a periodic manner (Assumption 3) at every signaling iteration and every DG agent reacts to F by revising its current dispatch strategy (if necessary) according to the available local generation capacities. We show that our distributed solution fits a population game theoretic framework. Using well-defined theoretical constructs from the field of population game theory, we are able to establish that the distributed decision-making by DG agents always allows the system to converge to an equilib-

rium which is also the global minimum of the economic dispatch problem under consideration. We also demonstrate theoretically and practically that our solution is robust to perturbations.

A. Signals Transmitted by the EPU

At each signaling iteration, the EPU computes F by solving a modified version of \mathcal{P}_D . In the \mathcal{P}_D transformation process, all local generation capacity constraints are first removed; these are subsequently incorporated by DG agents into their local decision-making as discussed in Section III-B. Then, a change of variables from z to x is applied. Variable x is an n -dimensional vector whose i th component represents the proportion of DG agents in the population that are currently using strategy y_i . The relationship between z and x_i is as follows: $x_i = \sum_{k \in \mathcal{P}_i} \frac{z_k}{y_i m}$. \mathcal{P}_i represents all the DG agents currently using strategy y_i . The cumulative power dispatched by these DG agents divided by y_i provides information about the total number of DG agents in the system using strategy y_i . This value is then divided by m , where m is the total number of DG agents in the system. Hence, the resulting value x_i represents the fraction of DG agents using strategy y_i . Hence, x belongs to the simplex $\Delta = \{x \in \mathbb{R}^n \mid \sum_{i=1}^n x_i = 1, x_i \geq 0 \forall i = 1 \dots n\}$. Constraints associated with the simplex are naturally satisfied as the DG agents will distributively select one of n power dispatch levels. These changes result in \mathcal{P}'_D :

$$(\mathcal{P}'_D) \min_{x \in \Delta} \frac{m}{2} \sum_{i=1}^n x_i^2 Y_i \quad \text{s.t. } m \sum_{i=1}^n x_i y_i = \sum_{j=1}^k w_j.$$

As $m \rightarrow \infty$, it is important to note that x can be considered to be continuous. With the decoupling of local constraints, change of variables, and Assumption 2, \mathcal{P}_D has now been transformed into a continuous strictly convex optimization problem \mathcal{P}'_D , which can be solved in polynomial time. Moreover, as the variable space of \mathcal{P}'_D is in the order of $O(n)$, which is the number of dispatch strategies available to each DG and $n \ll m$, this problem can be solved very quickly. As the accuracy of the solution is within $\pm 1/m$, the larger the number of DGs agents m in the system, the more accurate will be the equivalence between \mathcal{P}_D and \mathcal{P}'_D . Assumption 2 ensures that this is the case. In Section IV, we explore the impact of relaxing Assumption 2. As such, the EPU has access to aggregate demand measurements (i.e., $\sum_{j=1}^k w_j$) made available by active monitoring systems due to Assumption 1. This information can be used by the EPU to readily solve for \mathcal{P}'_D and to obtain the unique global optimal value x^* as \mathcal{P}'_D is a problem that is much smaller than \mathcal{P}_D . Also, as local constraints are no longer considered in \mathcal{P}'_D , the EPU does not need to be aware of individual generation capacity information to compute x^* . The main challenge, then, lies in allocating power dispatch levels to each DG in order to produce the optimal strategy composition x^* while heeding individual generation capacity constraints. To facilitate this, the EPU must provide broadcast signals that indirectly guide DG agents to iteratively revise their local strategies so that these result in the optimal solution, and one more transformation is necessary for the construction of these signals. If the aggregate power generated by DGs does not match overall demand, then the signal computed by the EPU must incorporate an appropriate

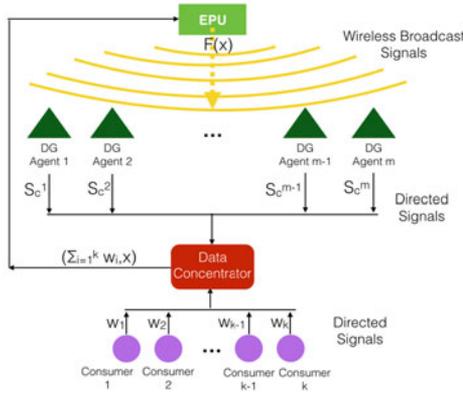


Fig. 2. Communication exchanges.

penalty to reflect the imbalance. This is realized through the construction of a Lagrangian dual \mathcal{L}'_D of \mathcal{P}'_D as follows:

$$(\mathcal{L}'_D) \max_v \min_{x \in \Delta} f(x, v) = \frac{m}{2} \sum_{i=1}^n x_i^2 Y_i + v \left(\sum_{j=1}^k w_j - m \sum_{i=1}^n x_i y_i \right)$$

where the Lagrangian multiplier $v \in \mathbb{R}$ corresponds to the constraint that balances supply with demand. When x is fixed to x^* , optimal v^* can also be easily computed by the EPU as this is the only unknown in the problem. As facilitated by Assumptions 1 and 4, DG agents will transmit information about current dispatch to data concentrators in real-time. This will then be aggregated and sent to the EPU. Hence, from this process, the EPU will be able to infer x that is the proportion of DG agents currently using each dispatch strategy in y . Equipped with v^* and x , the EPU computes F_i according to (1), which is in fact the gradient of $f(x, v^*)$

$$F_i(x) = m Y_i y_i \left(\frac{x_i}{y_i} - \frac{v^*}{Y_i} \right) \quad (1)$$

where F_i is the cost of the dispatch level $y_i \in y$ and $F = [F_1 \dots F_n]$. The complexity of computing the cost of the dispatch strategies for the EPU is now $O(n)$ as the original problem with m variables has been converted to one with n variables due to the change of variables. As $n \ll m$, this cost is very low. Changes in demands (i.e., $\sum_{j=1}^k w_j$) that occur every minute (i.e., Assumption 6) are incorporated into the computation of v^* which is obtained when \mathcal{L}'_D is solved for v after fixing x to be x^* . As the computation of the dispatch strategy costs in (1) includes v^* , demand changes are implicitly incorporated into the strategy costs calculated by the EPU.

Fig. 2 summarizes the communication exchanges involved between the EPU, DG agents, and data concentrators for the computation of $F(x)$. The EPU computes $F(x)$ and broadcasts this to all DG agents via a wireless medium every 3 s (Assumption 3). Since the cost signals are common to all agents, point-to-point links need not be established between the EPU and every DG agent. The latency of a broadcast over a network

as large as a DN is typically in the order of microseconds [13]. In order to compute the cost signals, the EPU requires information about the aggregate demand and current strategy distribution in the system. The former is updated only once every minute (Assumption 6). For updates on the latter, whenever a DG agent revises its strategy based on the most recently received cost signal (as detailed in Section III-B), it transmits this information to a data concentrator which immediately mirrors this to the EPU. As the cost signals capture changes in generation and demands, the DG agents are able to adapt appropriately to these changes using just these signals. These communications are directed signals and latencies are typically in the order of microseconds. As the EPU requires this information every 3 s, this latency is negligible. Hence, the communication overhead incurred for computing the cost signals is negligible.

The structure of the cost signal in (1) leads to many interesting system properties that we highlight next from a game theoretic perspective. Given that the DG agents are strategic entities that respond to F rationally, we assert that interactions amongst agents can be completely specified by the population game \mathcal{G} . This game consists of a large number of players (i.e., Assumption 2) which are the DG agents. These players/agents have at their disposal the strategy set y consisting of the various power dispatch levels available to the DG agents. The associated costs F_i of these strategies are the functions of the system state x and are computed by the EPU based on (1). The players/agents make rational strategy decisions based on the current state x of the system and actions of other players as reflected by the cost signals transmitted by the EPU. The resulting population game anonymizes individual players/agents and allows for the analysis of aggregate system behavior (e.g., proportion of players choosing a particular strategy and the evolution of strategy frequency and costs in the population overtime to static equilibria if any). A *potential* game results when the partial derivatives of the cost vector F satisfy $\frac{\partial F_i}{\partial x_j} = \frac{\partial F_j}{\partial x_i}$, where i and j are the available strategies [24]. This is the case for our game setup as the partial derivatives of F are $\frac{\partial F_i}{\partial x_j} = 0, \frac{\partial F_j}{\partial x_i} = 0 \quad \forall y_i \neq y_j$, where $y_i, y_j \in y$.

In potential games, the incentive of each player to change strategies can be expressed as a singular global potential function which facilitates tractable analyses. Intuitively, DG agents participating in a potential game will most likely switch from a more expensive to a less expensive strategy and these rational strategy revisions will continue to reduce the potential of the system until any more revisions will escalate the cost incurred by DG agents. We present two such rational decision-making protocols in Section III-B. This equilibrium point x_{NE} in a game theoretic setup is referred to as the *Nash Equilibrium* (NE) and one formal definition of NE is [24]

$$x_{NE} = \{x \in \Delta | x_j > 0 \rightarrow F_i(x) \geq F_j(x) \quad \forall y_i, y_j \in y\}. \quad (2)$$

Hence, the population game setup describes how the system evolves (due to players/agents rationally revising local strategies based on the current system state and strategy costs) to a static equilibrium defined in a classical normal-form game. At this point, when two players are repeatedly randomly drawn from

the population and allowed to play a normal-form game, the resulting outcome will be the mixed NE x^* . This x_{NE} actually coincides with the global minimum of \mathcal{P}'_D as the Karush Kuhn Tucker (KKT) conditions [24] required for x^* to be the optimal solution of the convex problem \mathcal{P}'_D are exactly the conditions listed in (2) for an NE. From this, it is evident that the system will eventually converge to the global minimum of the dispatch problem \mathcal{P}'_D when DG agents make *rational* strategy revisions given that there exists sufficient generation capacity. This minimum is also the optimal solution of the original dispatch problem \mathcal{P}_D .

Another important criterion, in addition to the convergence of the system to the global minimum, is the robustness of the equilibrium. In the population game theory, a system state x^* is referred to as a stable state if it is robust to perturbations. When deviations from the system state x^* result in a modified state y , then strategy costs become higher. This results in players/agents revising their local strategies to offload the additional cost incurred by the perturbation. This will result in the system moving back to the stable state x^* . In Section IV, we present a bound on the maximum possible perturbation that can be automatically countered by the agents. According to [25], two conditions are required for x^* to be classified as a stable state. First is that x^* should be an NE. This has already been ascertained in our previous analysis. The second condition is that of the local superiority of x^* where if $(x^* - y)F(x^*) = 0$ then $(x^* - y)F(y) < 0$. This is the case for our cost function as it is the gradient of a strict convex function $f(x, v^*)$. One main property of a strictly convex function $f(\cdot)$ with the gradient $F(x)$ is $(y - x)'(F(y) - F(x)) > 0 \forall x \neq y$ [26]. When $(x^* - y)F(x^*) = 0$ is substituted into this relation, the result is $(x^* - y)F(y) < 0$, which is exactly the second condition required to qualify x^* as a stable state. Hence, once the system reaches the global minimum x^* , which is also the NE, it is robust to perturbations.

Next, we present two types of rational strategy *revision protocols* and the induced state dynamics.

B. Strategy Revisions by DG Agents

A *revision protocol* dictates how a DG agent should respond to cost signals F broadcast by the EPU. Suppose that a DG agent is currently using strategy y_i . A revision protocol defines the probability $\rho_{ij}(F(x), x)$ at which an agent should switch from strategy y_i to y_j as a function of F and the current state x . Such revisions induce the following state dynamic:

$$\dot{x}_i = R_{in}(x, \rho_{j,i}) - R_{out}(x, \rho_{j,i}) \quad (3)$$

where the first term $R_{in}(x, \rho_{j,i}) = \sum_{j=1}^n x_j \rho_{j,i}(F(x), x)$ specifies the rate at which DG agents switch into strategy y_i and the second term $R_{out}(x, \rho_{j,i}) = x_i \sum_{j=1}^n \rho_{i,j}(F(x), x)$ represents the rate at which DG agents switch out of strategy y_i . The stochastic effects are essentially eliminated due to the presence of a large number of DG agents in the system (Assumption 2) as this brings the strong law of large numbers into effect. If a DG agent cannot switch to the selected dispatch strategy due to the limited local generation capacity, we translate this to be a

reduction in the population size. The impact of population size on the system behavior is analyzed in Section IV.

It is evident from (3) that the type of revision protocol adapted by DG agents will directly affect system dynamics. A suitable revision protocol should enable rapid convergence to the global minimum of \mathcal{P}_D when there is sufficient generation capacity. There must also be no cycling behavior as oscillations can lead to system instability. We evoke one version of Lyapunov theory with game theoretic considerations to assess these properties; the state dynamic induced by a revision protocol must satisfy the following two properties for asymptotic convergence to the NE [24]:

- (1) When $\dot{x} \neq 0$, then $\dot{L}(x) = \dot{x}'F(x) < 0$
- (2) When $\dot{x} = 0$, then $x \in x_{NE}$

where $L(x)$ is a strict Lyapunov function. The first condition requires that the energy in the system decreases for all x that are not NE. The second condition requires that the equilibrium of the state dynamic is the NE of the system. We have already shown that the equilibrium in our system is an NE, which is also the global minimum of \mathcal{P}_D with sufficient generation capacity. As long as the existence of a strict Lyapunov function satisfying the above conditions can be proven, it will be possible to establish strong convergence characteristics of the corresponding revision protocol.

Next, the two revision protocols considered in this paper [i.e., best response (BR) and perturbed best response (PBR) revisions] and the associated state dynamics are presented. BR is an approach used by players involved in a classical game to make decisions on how to best select a strategy given the current state of the system. Hence, the BR revision protocol serves as a natural extension for distributed strategy selection by DG agents where the current state of the system is reflected by cost signals transmitted by EPU. We show in this section that BR revisions can result in instability and discontinuities. In order to overcome these issues, we consider the PBR revision protocol that applies perturbations to the BR revision protocol. This allows for a more tractable theoretical analysis.

First, we consider BR revisions. Best response correspondence is the decision-making technique used in classical games consisting of multiple players. Players choose the strategy that results in the least cost given the strategies of other players. Similarly, in BR revisions, each DG agent makes a strategy switch based on the probability $\rho^{BR}(F(x), x)$ [24] as

$$\rho^{BR}(F(x), x) = \underset{y \in \Delta}{\operatorname{argmin}} y'F(x). \quad (4)$$

The state dynamic induced by BR revisions is

$$\dot{x}_i = \rho^{BR}(F(x), x) - x_i$$

which is derived by substituting $\rho^{BR}(F(x), x)$ into (3). This dynamic is a differential inclusion as $\rho^{BR}(F(x), x)$ is a discontinuous set. Thus, analyzing this dynamic via conventional methods is not straightforward [24]. However, in general, some oscillatory behavior will be evident as the system will cycle in and out of equilibrium due to nonuniqueness.

TABLE I
DISPATCH BY EVERY AGENT

Summary of Distributed Dispatch by DG Agent i
Start Algorithm:
<ul style="list-style-type: none"> Initialize at $t \leftarrow 0$, $t_{\text{next}} \leftarrow 0$. Initiate dispatch: $s_c \leftarrow \text{rand}(y)$.
Repeat the following:
<ol style="list-style-type: none"> Calculate next revision time τ_i with a Poisson arrival rate of $1/\mu$. The next revision occurs at $t_{\text{next}} \leftarrow t + \tau_i$. When $t > t_{\text{next}}$: <ul style="list-style-type: none"> Utilize last received F and x if necessary (from EPU) to select strategy s_i according to $\rho_{i,j}$ defined by: Eq. 4, or Eq. 5. Update strategy: $s_c \leftarrow \{\max\{y_{s_i}\} y \leq \min\{c_i, s_i\}\}$ If s_c has been revised, then transmit an update on this change to the closest data concentrator

In order to eliminate issues associated with nonuniqueness and discontinuity in BR, a perturbation is applied in (4) and this results in the PBR revision protocol as follows [24]:

$$\rho^{\text{PBR}}(F(x), x) = \underset{y \in \Delta}{\text{argmin}} \left(y' F(x) - \eta \sum_{i=1}^n y_i \log(y_i) \right) \quad (5)$$

where the KKT conditions can be evoked to show that y is a unique solution. The perturbation is a negative entropy function, where η represents the degree of perturbation. The resulting dynamic obtained by substituting (5) into (3) is

$$\dot{x}_i = \rho^{\text{PBR}}(F(x), x) - x_i. \quad (6)$$

A strict Lyapunov function for this perturbed dynamic is $L(x) = f(x, v^*) - f(x_p^*, v^*) + v(x) - v(x_p^*)$, where $v(x) = -\eta \sum_{i=1}^n x_i \log(x_i)$. The existence of this $L(x)$ indicates that the system dynamic will asymptotically converge to the perturbed equilibrium x_p^* due to the second term in (5). Selecting appropriate values for ν will reduce the effect of the perturbation. A proof of this has been included in the Appendix.

Using one of the revision protocols introduced in the above, every DG agent i performs distributed dispatch according to the steps summarized in Table I. The arrival times of strategy revisions for a DG agent i is a Poisson process. Suppose that the next revision obtained via this distribution occurs at time τ_i . The agent will switch from its current strategy y_i to another strategy y_j depending on the probability $\rho_{ij}(F(x))$ (computed using the latest $F(x)$ and x transmitted by the EPU) and the local generation capacity c_i . According to Assumption 6, the generation capacity c_i of DG i is updated every minute. Change in c_i is implicitly incorporated into the local revisions made by the DG agent as this alters the upper bound of $\min\{s_c, c_i\}$ in Step 2 of Table I which is used to select the revised dispatch strategy. If the selected power dispatch is greater than the available capacity, then the next largest dispatch setpoint that meets c_i is selected. Certain DGs such as the diesel generator may have significantly greater generation potential than other DGs in the system (i.e., PV). In these cases, the generation capacity of the DG is divided into $\lfloor c_i / \max(y) \rfloor$ units and these are each represented by a DG agent that operates based on the algorithm outlined in Table I.

In summary, we have proposed a distributed algorithm by first applying a series of transformations to the original

NP-hard problem formulation in \mathcal{P}_D to convert it into an equivalent problem that can be easily solved by the EPU. Due to the inherent decomposability of \mathcal{P}_D , generation capacity constraints are moved to the local dispatch revision process of DG agents. To eliminate the complexity introduced by integer variables, we apply a change of variables from z to x to transform it into a problem that is continuous up to a precision of $\pm 1/m$. As m becomes large (i.e., number of DG agents), the treatment of the transformed problem as one with a continuous domain becomes more realistic. This problem is solved by the EPU to obtain strategy costs and since $n \ll m$ this computation is not expensive. Based on these costs, every DG agent makes a dispatch decision using a revision protocol at a random time instance and this change causes an infinitesimal impact at the system-wide level. Moreover, these strategy revisions serve to reduce the potential of the system. We are able to derive the resulting state dynamic induced by these revisions in a closed form as the stochastic effects can be eliminated due to the strong law of large numbers which can be evoked only due to the presence of a large number of agents. This distributed approach prevents the need for concentrated computational efforts as these are offloaded to participating entities that solve simpler subproblems, which allow the system to iteratively arrive at the optimal solution. We have presented some very interesting theoretical insights into these state dynamics earlier in this section.

IV. IMPLEMENTATION

In this section, the proposed dispatch strategy is evaluated via comprehensive simulations implemented in MATLAB/Simulink using realistic demand and generation models [28], [29]. In addition to validating theoretical properties established in the previous section, we also compare the performance of our proposal with other techniques in the literature and investigate how the composition of renewable generation mix impacts policy-making initiatives. In general, we evaluate the proposed distributed dispatch strategy via simulations against various inputs (i.e., m DGs and k power consumers having highly varying generation capacities c_i and power demand patterns w_j over a day) and the resulting outputs (i.e., z_j which is the power dispatched by each DG according to the proposed strategy and the system state x).

A. Demand and Supply Models

The system considered in all our simulations is at the distribution substation level consisting of 1000 consumers whose power demands are primarily supplemented by DGs that include PVs, wind turbines, and diesel generators.

Consumer power demands are highly dependent on external factors that depend on location, lifestyle, and weather patterns. We adapt parameters such as power consumption, appliance usage probabilities, and appliance penetration rates during the summer season in Ontario, Canada for appliances (e.g., dishwashers, ovens, hobs, water heaters, fridges, freezers, dryers, washing machines, and air conditioners) as specified in [29].

Renewable power generation is also reliant on external environmental factors such as solar irradiance, cloud variability, and

wind speed. In our PV generation model, we utilize hourly data for 3.08 kW rated PVs in Toronto as provided in [27]. In order to introduce slight variability in generation that can arise due to the differences in the physical locations of the PVs, uniform random noise is added to this generation data. Next for wind generation, we consider wind turbines with a power rating of 11 kW, diameter of 13 m and efficiency of 0.4. The Weibull probability density model with a shape factor of 1.94 and scale factor of 4.48 is utilized to generate wind speeds [28] which are then applied to the wind power curve to obtain the power generated by wind DGs in a manner similar to [10].

B. Simulation Settings

The generation mix in the system for the initial set of simulations is fixed to 100 PVs and 100 wind turbines. The generation capacities of these DGs will vary according to the models presented above. We investigate the impact of varying the renewable generation mix in subsequent simulations. In order to balance the intermittent nature of these renewables, we also include a diesel generator with a capacity of 0.8 MW. Each DG can operate at one of three setpoints: $y = [0.001, 0.5, 1]^T$ kW. Hence, the diesel generator represents 800 DG agents. Demand and supply are assumed to be constant for every 1 min interval. Y_i is selected to be a positive value that increases with i (i.e., $0 < Y_i < Y_{i+1}$). The EPU transmits the cost of three dispatch strategies and x over the network at every signaling iteration (i.e., every 3 s).

C. Static and Dynamic Properties of Revision Protocols

In order to assess the convergence characteristics of the two revision protocols, state trajectories induced by these revisions starting from various initial conditions are presented in Fig. 3. As each DG agent can select from one of three strategies, the system state x at any time can take a value in the three-dimensional simplex (i.e., x can take a value only within the triangular region defined by $\{x \in \mathbb{R}^3 \mid \sum_{i=1}^3 x_i = 1, x_i \geq 0\}$). The level sets of the cost incurred by the EPU are also included in Fig. 3. Level sets represent contours-containing points for which $f(x, v^*) = c$ is satisfied and c is a constant. In order to differentiate every level set curve, a gradient bar is included at the side of Fig. 3(b). Larger values of c are associated with darker colors and smaller values with lighter colors. The point at which $f(x, v^*)$ is globally minimal is located at the center of these level sets.

From Fig. 3(a) it is evident that BR revisions result in state trajectories that cycle in and out of the global minimum. This is not desirable as the system state will not settle at an equilibrium. For PBR revisions, η is set to 0.1 and this adds some perturbation to the BR dynamic. Upon closely assessing the state trajectories in Fig. 3(b), it is evident that the system asymptotically converges to equilibrium with no limit cycles. Thus, results presented in Fig. 3(a) and (b) reaffirms the theoretical static and dynamic properties derived in Section III-B and the Appendix.

The smoothness in transitions of aggregate generation from one signaling period to another is investigated next in Fig. 3(c) for the two revision protocols. Three 1 min intervals are considered, where aggregate consumer demands are

[150, 250, 205] kW. It is clear that both revision protocols are able to adapt to changes in demand and local generation capacities across multiple dispatch intervals. In particular, dispatch resulting from PBR revisions is close to that resulting from BR revisions when $\eta = 0.01$. For larger values of η , the perturbations are pronounced. Hence, in the remainder of this section, all simulations will feature results obtained from PBR revisions, where $\eta = 0.01$ as this enables us to apply tractable analysis from the previous section to ascertain that the simulation results match our theoretical assertions.

D. Impact of Population Size

One major assumption made in this paper is that of the population size. Results presented so far are based on a system containing 1000 DG agents. When generation capacity becomes insufficient, certain DG agents may become unavailable to participate in the dispatch scheme. In Fig. 4(a), we investigate the impact on the convergence properties when Assumption 2 does not hold. Fig. 4(a) presents the proportion of demand met by DGs over a dispatch cycle using PBR revisions when there are 10, 100, and 1000 agents active in the system. When there are 100 and 1000 DG agents, the system converges smoothly to optimality within 3 to 4 signaling iterations (i.e., 9–12 s). A large number of participants in the system allows the population game theoretic constructs to take effect [24]. Moreover, as the incremental revisions by each DG agent are independent of one another, the more the DG agents there are, the more adaptive the system will be to changes. However, significant variabilities are evident when there are only 10 DG agents as stochastic effects are more pronounced. Convergence properties established earlier no longer hold as the strong law of large numbers no longer holds. However, this is not an issue in this paper as the dispatch problem formulation is based on the premise that a large number of distributed energy sources are present in the system.

E. Resilience Under Attack

The dispatch scheme proposed in this paper is highly dependent on cyber-physical interactions between the EPU and the DGs. Communication security issues can be exploited by an adversary to perpetrate attacks on the EPU via the DGs. One way an attacker can adversely affect the system is to force DGs to select the highest power dispatch level so that the EPU will incur unnecessary costs. The system is able to robustly self-adjust as long as the number DG agents c compromised remains below a threshold $T = D/\max(y)$ where D , is the current aggregate consumer demand. When this is not the case, excess generation $c * \max(y) - D$ cannot be offset by unattacked DR agents. On the other hand, an attacker may force the power dispatched by compromised DG agents to be 0. This can result in overall demand in the system not being met by the power dispatched by DGs. As long as the number of unattacked DG agents u is at least T , the system will be able to recover from perturbations such as these and meet the aggregate demand in the system. Suppose that all the compromised DG agents form the set \mathcal{C} , which consists of $|\mathcal{C}|$ number of agents. In general, when an attacker sets the power dispatched by compromised DG agent

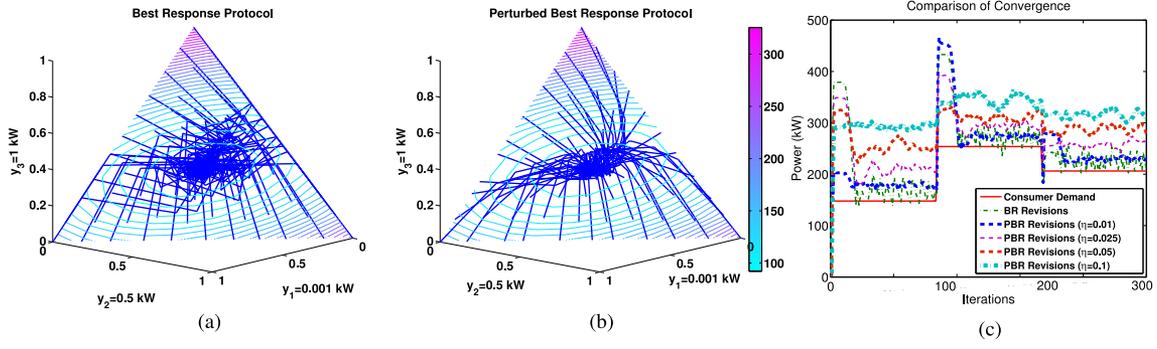


Fig. 3. Convergence properties of the revision protocols. (a) Best response revisions. (b) Perturbed BR revisions. (c) Comparison of convergence.

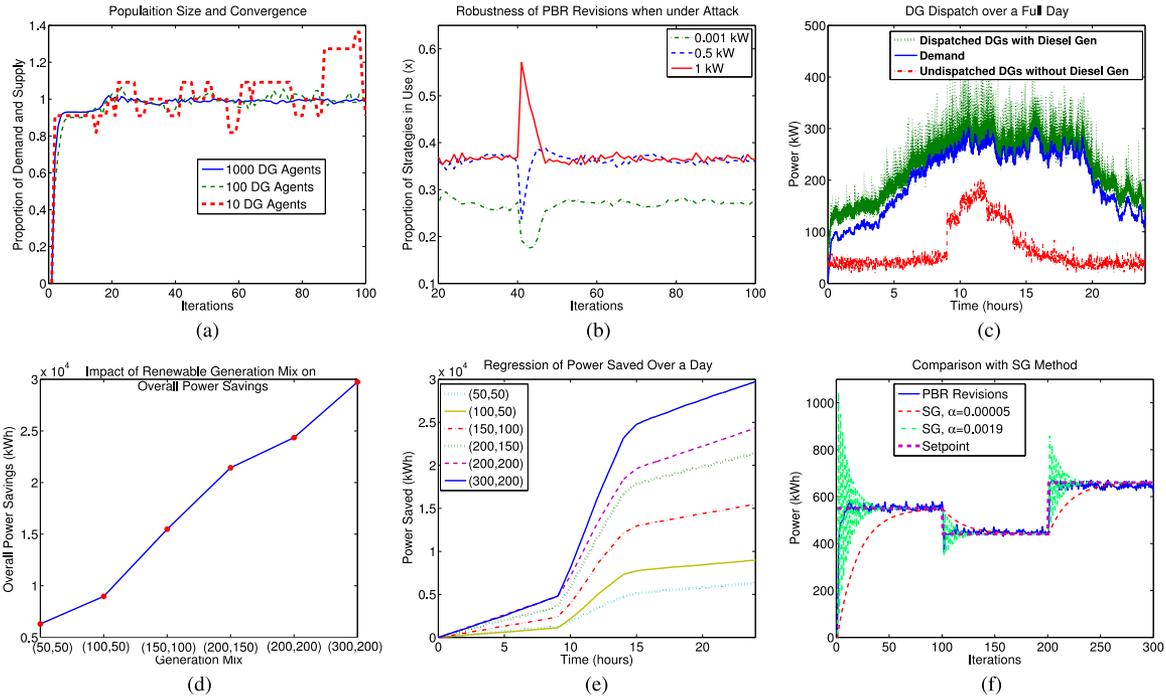


Fig. 4. Aggregate properties of PBR revisions. (a) Relaxing population size assumption. (b) Robustness to attacks. (c) Load following of DGs. (d) Overall power savings. (e) Power savings regression. (f) Comparison to subgradient method.

i to be a random value $a_i \in y$, the system will be able to recover from this perturbation as long as $C_p = \sum_{i \in \mathcal{C}} a_i \leq D$ and $m - |\mathcal{C}| \geq (D - C_p) / \max(y)$, where m is the total number of agents active in the system. The first condition requires that the overall power dispatched by the attacked agents to be less than or equal to the overall demand in the system so that there will not be any excess power dispatched. The second condition ensures that there exists adequate number of uncompromised agents to offset the perturbation introduced due to the attack. This is illustrated in Fig. 4(b) where 40% of DG agents are forced to select a random power dispatch level at the 40th signaling iteration. With PBR revisions, other DG agents are able to sense this discrepancy through the cost signals and are able to promptly revise their individual strategies to restore the system back to the optimal operating state as illustrated in Fig. 4(b). These attacks are, however, unlikely to occur in a large system

as compromising many DG agents is an onerous task for any attacker.

F. Load Following Over a Day

Next, the load following characteristics of DGs utilizing the proposed dispatch strategy is studied over the course of a day. When no diesel generation is available, as illustrated in Fig. 4(c), aggregate generation capacity of renewable energy sources is not sufficient to supplement all power demands in the system and therefore operate at maximum capacity. With the diesel generator functioning in tandem with the renewables, Fig. 4(c) indicates that our proposed dispatch strategy is able to very closely follow overall demand without significant surges and oscillations. This behavior corresponds to our theoretical analysis where we show that the system asymptotically converges

to the equilibrium for PBR revisions. Such fast convergence even in the presence of a large number of DG agents indicates that our algorithm is highly scalable and adaptive. In order to obtain quantitative results on the convergence speed while including computational and communication overhead, we have measured in our simulations the average time required for the convergence of the system to the optimal equilibrium over every 1 min dispatch interval. These intervals coincide with the 1 minute intervals in which the generation capacities and demands are updated due to Assumption 6. Our results indicate that on average convergence occurs within 12 s in every 60 s dispatch interval. As communication latencies incurred due to cost signal broadcasts by the EPU and information exchanges with the data concentrator are in the order of microseconds as outlined in Section III-A, these do not affect the convergence time. Hence, these simulation results reaffirm that the asymptotic convergence properties of the induced system dynamic from PBR revisions results in a highly scalable and responsive distributed dispatch system.

G. Renewable Generation Mix

The degree of sustainability in power generation depends on the penetration of renewable sources in the system. It is imperative for the EPU to strike a balance between the cost incurred by deploying these and promoting sustainable grid operations. As illustrated in Fig. 4(c), renewable generation capacity varies over an entire day due to external factors such as wind speed and solar irradiance. Moreover, consumer demands peak during certain periods of the day. This variability in renewable generation and consumer demand can be capitalized in energy policy initiatives as illustrated in Fig. 4(d) and (e). In these figures, each generation mix considered is denoted by a 2-tuple indicating the number of PVs and wind turbines deployed in the system. Fig. 4(d) illustrates the total consumer demand supplemented by each renewable generation mix combination over a day. Furthermore, in Fig. 4(e), the regression of cumulative demand supplemented by renewables over the course of the day provides vital information on how to allocate renewable investments based on the load composition in the system. For instance, if aggregate consumption significantly peaks during noon, then greater investments in solar generation sources rather than wind turbines will be ideal.

H. Comparison With Existing Literature

Finally, in this section, main characteristics of our proposal and strategies in the existing literature are evaluated. Comparable dispatch algorithms proposed in [10] and [21] based on the subgradient method are considered. Like our work, [10] and [21] decompose the dispatch problem into master (EPU) and slave (DG agent) subproblems. The main difference lies in the computation of the cost signals for which these references utilize the subgradient method. Convergence of the subgradient method to optimality is significantly reliant on the step-size α used for updating the cost signals at every iteration. Selecting an appropriate step size requires striking a balance between convergence speed and system oscillations. Aggregate dispatch resulting from the

subgradient method for $\alpha = 0.0019$ and $\alpha = 0.00005$ and our PBR revisions are depicted in Fig. 4(f) for three signaling iterations. The larger step size results in oscillations, whereas the smaller step size induces slower convergence to optimality. Fine tuning α at each signaling iteration in order to prevent these undesirable effects is an onerous task. Our algorithm, on the other hand, is self-adjusting and PBR revisions do not cause limit cycles in the system. In general, our proposal is distinct from existing work in DG dispatch as our solution guarantees optimality with fast convergence properties and low computational/communication overhead. We have theoretically shown in Section III-B that our distributed strategy results in asymptotic convergence to the equilibrium. In our simulations, we have observed that convergence to equilibrium occurs on average within 12 s and this result indicates that our proposal is a real-time solution. Moreover, the communication overhead entailed in our proposal is not excessive as illustrated in Fig. 2 and detailed in Section III-B. These features render our proposal ideal for real-time dispatch.

V. CONCLUDING REMARKS

In this paper, we propose a distributed dispatch strategy that is highly scalable and robust with strong static and dynamic properties as validated by theoretical and simulation analyses. Our strategy is capable of efficiently coordinating a large number of DGs that can rapidly respond to flux in system demand and local generation capacities optimally. We assert that our regression analysis establishes a foundation for policy makers to conduct cost-benefit analysis on a variety of generation compositions to facilitate the realization of more sustainable and reliable future smart grids.

APPENDIX

A. Asymptotic Convergence of PBR Dynamic

In order to prove that the PBR revisions made by DG agents induce a state dynamic that asymptotically converges to the equilibrium x_p^* , it is necessary to show that there exists a strict Lyapunov function that satisfies the following relation [24]:

$$\frac{d}{dt}L(x) = \nabla L(x)^T \dot{x} < 0.$$

We have already derived an analytical expression of the state dynamic \dot{x} that is induced by PBR revisions in (6) by evoking the strong law of large numbers. For further analysis, it is necessary to obtain a closed-form expression of (6).

Since $\rho^{\text{PBR}}(F(x), x)$ involves a minimization, it is indeed possible to obtain a closed-form solution for $\rho^{\text{PBR}}(F(x), x)$ by first constructing the first-order condition of the Lagrangian of (5) and using the primal feasibility condition as follows:

$$\nabla \mathcal{L}_{\text{PBR}}(y, \nu) = F(x) + \nabla v(y) + \nu = 0, \quad \sum_{i=1}^n y_i = 1$$

where ν is the Lagrangian multiplier associated with the simplex equality constraint. Combining these two equations will result

in the closed-form solution $y_i = \frac{e^{-\eta^{-1} F_i(x)}}{\sum_{i=1}^n e^{-\eta^{-1} F_i(x)}}$. Hence, this closed-form solution can be substituted into (6) to obtain the system state dynamics $\dot{x}_i = y_i - x_i$ and this is the state dynamic induced by PBR revisions.

Next, we show that there exists a strict Lyapunov function for PBR dynamic and this is $L(x) = (f(x) - f(x_p^*)) + v(x) - v(x_p^*)$, where $v(x) = -\eta \sum_{i=1}^n x_i \log(x_i)$. For the evolution of the dynamics to be asymptotically stable, the condition $\dot{L}(x) \leq 0$ must be satisfied by the strict Lyapunov function and we show that this is the case when we select $L(x)$ to be the aforementioned function as follows:

$$\begin{aligned} \frac{d}{dt} L(x) &= \nabla L(x)^T \dot{x} = (F(x) + \nabla v(x))^T (y - x) \\ &= -(\nabla v(y) - \nabla v(x))^T (y - x) - \nu \sum_{i=1}^n (y_i - x_i) \\ &= -(\nabla v(y) - \nabla v(x))^T (y - x) \leq 0. \end{aligned}$$

The second line is obtained by substituting the expression obtained from reorganizing the first-order condition of Lagrangian multiplier $\mathcal{L}_{\text{PBR}}(y, \nu)$ in terms of $F(x)$ into $F(x)$. The second term in the second line is 0 as $\sum_{i=1}^n y_i - \sum_{i=1}^n x_i = 0$. In the next line, as $v(x)$ is convex, it can be established that the gradient $\nabla v(x)$ is monotone and we apply this property to obtain the final inequality. This establishes that the PBR dynamic converges to x_p^* asymptotically. ■

B. Acronyms

Table II presents a list of all the acronyms used in this paper.

TABLE II
TABLE OF ACRONYMS

Notation	Description
DG	Distributed Generator
PV	Photo-Voltaic
EPU	Electric Power Utility
PMU	Phasor Measurement Unit
DO	Discrete Optimization
NP	Nondeterministic Polynomial
NE	Nash Equilibrium
KKT	Karush Kuhn Tucker
(P)BR	(Perturbed) Best Response

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