Distributed Optimization of Dispatch in Sustainable Generation Systems via Dual Decomposition

Pirathayini Srikantha and Deepa Kundur

***Abstract*—Distributed generators (DGs) are being widely deployed in today's power grid. These energy sources are highly variable posing practical challenges for deployment and grid management. In this paper, a novel scalable distributed power dispatch strategy is proposed to effectively manage DGs at the distribution substation level, capitalizing on the recent push to cyber-enable power grid operations. We demonstrate how the inherent separability of the power dispatch problem allows the use of dual decomposition that enables every participating DG to locally compute its dispatch strategy based on simple broadcast data by the utility. Results and comparisons indicate that the DGs are able to rapidly converge to an optimal economical dispatch vector with significantly less concentrated computational effort and communication overhead, promoting security and privacy.**

***Index Terms*—Cyber-physical systems, distributed algorithms, power generation dispatch, power system security, renewable energy sources.**

I. Introduction

THE MODERN grid is rapidly evolving to meet a variety of end-user and regulatory demands and requirements. To cater to these needs, it is necessary to tap into a diverse set of generation sources such as solar panels and wind turbines that provide significant long-term benefits [18]. Such distributed generators (DGs) can be deployed in dedicated energy farms, at local points of electricity consumption such as individual residences, and can be routinely added or removed from the system in an *ad-hoc* manner. One technical challenge to DG wide-spread integration is the ability to accommodate their often highly fluctuating generation capacity in applications such as power dispatch. In this paper, we assert that an effective power dispatch strategy for DGs is scalable and supports plug-and-play DG deployment, maximizes utilization of available DG capacities, and promotes security and privacy amongst participants. Such extended functionality requires that the traditional more centralized physical power grid be cyber-enabled. The movement to this "smarter" cyber-physical power system promises overall improved efficiency, adaptability and consumer-centricity. In the context of dispatch, as we study, it facilitates a more optimal generation mix. Thus, in this paper, we propose a novel and practical strategy for real-time distributed power dispatch.

Existing power dispatch approaches can be categorized as offline or online. Offline techniques perform dispatch computations in advance (such as a day ahead) whereas online techniques are employed at shorter intervals (such as an hour). Centralized offline techniques [1], [10], [11] can conveniently offload the computation of optimal dispatch vector to appropriate resources as these strategies are not subject to strict timing constraints. A distributed offline approach (of similar Lagrangian decomposability flavor to this paper, but developed in parallel) is presented in [13]. Due to its slower convergence properties and communication overhead, the strategy is applied only for day ahead scheduling. Given that offline techniques rely extensively on prediction, one open challenge is addressing the accuracy of load and generation forecast models that tends to decrease as prediction horizon increases [12]. Moreover, if power dispatch is computed too conservatively, surplus generation capacity due to prediction error will not be utilized.

To address long-term modeling limitations, centralized online power dispatch techniques have been proposed [3], [22]. These approaches employ heuristic or soft computing techniques that speed up the dispatch vector derivation, but may often lead to sub- or near-optimal solutions due computational and time limitations [21]. The formulation of the online dispatch problem is based on either short term predictive models or generation capacities relayed directly by DGs to the central controller. The central controller will communicate the result to each DG at each dispatch cycle. Thus, in addition to powerful computational capabilities at the central controllers, centralized bi-directional communication infrastructure and protocols are needed.

To reduce computational and communication requirements, decentralized online power dispatch strategies [6], [14], [24] that rely on consensus-based distributed multiagent cooperation to iteratively compute aggregate load and generation capacity via short-range wireless links have been proposed. As the number and spatial distribution of DGs grow, however, scalability and communication complexity becomes a growing issue that we, in part, address in this paper.

Specifically, in this paper, we leverage the inherent separability and decomposability of the dispatch problem to propose a novel algorithm as below.

1) *Distributed:* Computational efforts are offloaded to local cyber-physical DG agents that compute their own dispatch via lightweight communications from the utility.

Manuscript received May 15, 2014; accepted August 14, 2014. Date of publication October 15, 2014; date of current version August 19, 2015. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada and in part by the Hatch Graduate Scholarship Program for Sustainable Energy Research. Paper no. TSG-00439-2014.

The authors are with the Edward S. Rogers Sr. Department of Electrical and Computer Engineering, University of Toronto, Toronto, ON M5S 3G4, Canada (e-mail: pirathayini.srikantha@ece.utoronto.ca; dkundur@ece.utoronto.ca).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSG.2014.2360586

2) *Real-time:* DGs use current local generation conditions for dispatch instead of forecast models.
3) *Scalable:* DG addition or removal has little impact on the complexity and convergence properties of the algorithm.
4) *Efficient:* Only unidirectional utility broadcast communication is required; signals transmitted by the utility enable rapid dispatch convergence.

Moreover, the mechanism does not require communication of individual DG state information, but merely an aggregate providing inherent privacy and resilience to denial-of-service attacks.

The contributions of this paper are five fold.

1) We propose a novel distributed dispatch strategy with three distinct implementations.
2) We provide an analytical and practical convergence analysis of the three techniques.
3) We evaluate the real-time performance of the proposed dispatch strategy for various generation capacities.
4) We perform a comparative study of existing and the proposed dispatch strategies against five performance metrics.
5) We study the impact of the proposed strategy on policy-making initiatives for various DG mix.

This paper is organized as follows. Section II contains an overview of system preliminaries and concepts. In Section III, the proposed distributed dispatch strategy along with detailed analysis is presented. Section IV outlines the models leveraged to implement a realistic system. Results from the implementation are provided in Section V and finally, the paper is concluded in Section VI.

II. Preliminaries and Setting

The dispatch problem is formulated at the distribution substation level for a residential community interspersed with a set of DGs (treated as cyber-physical sub-agents capable of making their own dispatch decisions). Moreover, the system can be considered a microgrid in interconnected mode.

A. Assumptions

We make the following assumptions.

1) The utility represents a master agent that employs wireless communications.
2) Each DG is a cyber-physical sub-agent comprised of a physical energy source as well as a cyber communications and control to compute and actuate local dispatch at the cyber-physical interface.
3) The utility monitors the power supplied or absorbed by the main grid.
4) The distribution system is connected to the main grid containing negative spinning reserves, synchronized generation resources and storage systems.
5) The cost of power dispatched by DGs is less than power purchased from the main grid.
6) Demand and generation capacities of DGs remain almost constant over a 10-min interval.
7) The cost function of each generation source is convex and the overall cost in the system is a linear combination of individual costs.
8) The system is equipped with a diesel generator (also considered a DG) sized for the distribution substation in case the system is islanded.

The first three assumptions support the cyber-enabled vision of the grid. The fourth assumption is necessary to ensure that the system remains stable during computation of the dispatch solution. The fifth assumption provides economic justification to tap into DGs. The sixth assumption is valid as this interval is less than the hourly dispatch performed in today's grid [16] and would therefore be less prone to predictive errors. The last two assumptions are necessary for the formulation of the distributed dispatch strategy.

B. Original Problem

The original economic dispatch problem is formulated as P_P

$$
\begin{aligned}
(\mathbf{P}_P)\quad & \min_x f_o(x) \\
& \text{subject to} \sum_{i=1}^{n} x_i - \sum_{j=1}^{p} L_j = 0 \\
& 0 \le x_i \le c_i \qquad \forall\ i = 1 \ldots n
\end{aligned} \tag{1}
$$

where $x = [x_1\ x_2\ \ldots\ x_n] \in R^n$ is the dispatch vector of n DGs, $f_o(x)$ is the overall generation cost function, $\sum_{j=1}^{p} L_j$ is the aggregate power demand in the distribution system from p loads and c_i is the generation capacity of each DG. The (physical) grid coupling constraint considered matches power supply with demand (i.e., $\sum_i^n x_i - \sum_j^p L_j = 0$) which favors tractability of the problem in lieu of incorporating detailed physical constraints (also adapted by several existing dispatch strategies such as [13]). Inequality constraints specify valid dispatch range for every DG. From seventh assumption and selecting $f_o(x)$ to be quadratic (without loss of generality), we let

$$
f_o(x) = 2x'Cx = \sum_{i=1}^{n} 2C_{ii}x_i^2 \tag{2}
$$

where C is a diagonal cost matrix with diagonal element $C_{ii} > 0$ that reflects generation cost (in \$/kW^2) of the ith generator. Since C is positive definite. We assert that the construction of P_P allows the optimization problem to satisfy Lemma 1 and therefore P_P is a centralized convex optimization problem [8].

Lemma 1 [8]: An optimization with convex cost function and linear constraints represents a convex optimization problem.

Lemma 2 [8]: Slater's condition, which enables viewing the dispatch problem P_P in terms of its dual P_D, holds if the intersection of the inequality and the equality constraints of the optimization problem are not empty.

Given that by design there exists sufficient capacity in all generators to support overall demand, not all generators (e.g., diesel) are functioning fully, and Lemma 2 and thus Slater's condition holds for P_P. Thus, in this paper, we consider the dual to the classical economic dispatch problem, which enables us to leverage the concepts of Lagrangian decomposability and sub-gradient search to formulate an effective novel online distributed dispatch strategy.

III. Distributed Dispatch

Our strategy architects the cyber-physical relationships for dispatch computation such that geographically dispersed physical information is employed by DG sub-agents for local computations only while broadcast cyber communications is used by the utility master agent to iteratively build cooperation amongst the sub-agents. First, the original problem P_P is converted to its Lagrangian dual P_D. Then the inherent decomposability of the problem is leveraged to construct simpler subproblems solved by individual sub-agent DGs and a master problem that is iteratively solved by the utility in order to compute and broadcast signals to guide the DGs in finding an optimal dispatch strategy. Every DG will compute its strategy based on this broadcast transmission. The master agent will iteratively update and rebroadcast this value each time implicitly inferring the dispatch vector computed by the individual generators until convergence. Convergence can be ϵ-sub-optimal or optimal depending on the update method chosen by the master agent.

A. Lagrangian Dual Function

The original (primal) problem P_P has one equality and $2n$ inequality constraints. These constraints can be eliminated by constructing the following Lagrangian dual function of P_P:

$$L(\lambda, v, x) = \sum_{i=1}^{n} 2C_{ii}x_i^2 + \sum_{i=1}^{n} \lambda_i(x_i - c_i) + v\left(\sum_{j=1}^{n} x_j - \sum_{k=1}^{p} L_k\right) \tag{3}$$

where v and $\lambda = [\lambda_1 \ldots \lambda_n]'$ are Lagrangian multipliers, $v \in \mathbb{R}$ and $\lambda_i \geq 0$. The resulting Lagrangian dual problem P_D has n inequality constraints $\lambda_i \geq 0$

$$(\mathbf{P}_D)\ \max_{\lambda_i \geq 0, v;}\ \min_{x \in X} L(\lambda, v, x). \tag{4}$$

Since, P_P satisfies Lemmas 1 and 2, Theorem 1 holds allowing us to focus on computing P_D for dispatch.

Theorem 1 [8]: Strong duality holds for the primal problem and the dual problem if the primal problem is a convex optimization problem and meets Slater's condition. The solution of the primal problem is therefore the same as the solution of the dual problem.

B. Sub-Agent Optimization

As the cost functions are separable, P_D can be decomposed into n sub-problems each solved by a sub-agent DG for fixed values of λ_i and v. Since the generation capacity constraints are local to each DG, the inequality constraint associated with the Lagrangian multiplier λ_i can be removed from the Lagrangian dual function and be treated as a local inequality constraint instead. For fixed v the sub-agent associated with DG i will need to solve the sub-problem P_S

$$(\mathbf{P}_S) \quad g_i(v) = \min_{x_i \in X} 2C_{ii}x_i^2 + x_i v$$
$$\text{subject to} \quad 0 \leq x_i \leq c_i \tag{5}$$

where the only optimization variable is x_i as v is fixed and broadcast. Thus, P_S is a single-dimensional constrained quadratic problem (QP) that is easily solved by each sub-agent.

C. Master Agent Optimization

The master agent will solve P_D for fixed values of x_i. Since the inequality constraints associated with λ_i are treated as local constraints addressed by the sub-agents, the inequality constrained P_D problem with $n + 1$ variables conveniently reduces to the following unconstrained maximization with single variable v:

$$(\mathbf{P}_M) \quad \max_{v} M(v) = \sum_{i=1}^{n} g_i(v) - v\sum_{j=1}^{p} L_j. \tag{6}$$

The master problem P_M is concave but not differentiable. Hence, it is not possible to use typical methods from calculus to compute the optimal value v^*. Iterative methods are employed to compute v^* by repeatedly solving P_M via the sub-gradient method. Given that v^k is the kth iteration of v, the sub-gradient method will compute v^{k+1} by employing implicitly inferred feedback from each sub-agent. The master agent will then broadcast its approximation of $v^k \approx v$. This continues until v_k converges to v^*. The following equation lists the sub-gradient method used to iteratively update v at iteration k:

$$v^{k+1} = v^k + \alpha^{k+1} q\left(v^k\right) \tag{7}$$

where $q(v^k)$ is the sub-gradient of $M(v^k)$ for step size α^k.

D. Sub-Gradient

The sub-gradient of a nondifferentiable function is not unique, but the sub-gradient $q(v^k)$ of a concave function $M(v^k)$ will need to satisfy [9]

$$M\left(v^{k+1}\right) \leq M\left(v^k\right) + q\left(v^k\right)\left(v^{k+1} - v^k\right). \tag{8}$$

Proposition 1: A valid sub-gradient for $M(v^k)$ is $q(v^k) = \sum_{i=1}^{n} x_i^*(v^k) - \sum_{j=1}^{p} L_j$ where $x_i^*(v^k)$ is the argument that results in $g_i(v^k)$.

Proof:

$$M\left(v^{k+1}\right) = \inf_{x \in X}\left(\sum_{i} 2C_{ii}x_i^2 + v^{k+1}\left(\sum_{i=1}^{n} x_i - \sum_{j=1}^{p} L_j\right)\right)$$
$$\leq \sum_{i}^{n} 2C_{ii}x_i^{*2}\left(v^k\right) + v^{k+1}\left(\sum_{i=1}^{n} x_i^*\left(v^k\right) - \sum_{j=1}^{p} L_j\right)$$
$$M\left(v^{k+1}\right) = M\left(v^k\right) + \left(v^{k+1} - v^k\right)\left(\sum_{i=1}^{n} x_i^*\left(v^k\right) - \sum_{j=1}^{p} L_j\right). \quad \blacksquare$$

Hence, the master agent can use Proposition 1 to update v using $q(v^k) = \sum_{i=1}^{n} x_i^*(v^k) - \sum_{j=1}^{p} L_j$ where $q(v^k)$ can be interpreted as the difference between the "aggregate dispatch" and "aggregate demand" in the system.

E. Computation of Step Size

The step-size α^k can be assigned as follows [4], [7].

1) Constant Step Size: Here, $\alpha^k = c$ for constant $c > 0$.

2) Square-Summable-but-not-Summable Step Size: α^k must satisfy: $\sum_{k=1}^{\infty}(\alpha^k)^2 < \infty$ and $\sum_{k=1}^{\infty}\alpha^k = \infty$. One possible assignment is $\alpha^k = c/(d+k)$ where $c > 0, d \geq 0$.

3) Dynamic Step Size: Here, v^{k+1} must be closer to v^* than v^k. Thus, substituting (7) into $||v^* - v^{k+1}||_2^2 < ||v^* - v^k||_2^2$ and rearranging gives

$$\alpha^k = \beta\left(M^k - M\left(v^k\right)\right)/||q\left(v^k\right)||_2^2 \tag{9}$$

where $0 < \beta < 2$ and M^k is an estimate of the optimal value of M. The dynamic step-size algorithm iteratively updates α^k to reach the target level M^k. In the literature, the target is set to $M^k = \max_{i \in 1\ldots k} M(v^i) + \epsilon_k$ [5] and ϵ_k is updated

$$\epsilon_{k+1} = \begin{cases} \rho\ \epsilon_k, & \text{if } M\left(v^{k+1}\right) \geq M^k \\ \max\{\gamma\epsilon_k, \epsilon\}, & \text{otherwise} \end{cases} \tag{10}$$

where $\epsilon_0 = \epsilon$, $\gamma < 1$, $\rho > 1$ are constants. The update algorithm at iteration k attempts to improve on each former value by ϵ_k. If the resulting value is greater than the current iteration, the value of ϵ_k is increased by a factor ρ. Otherwise, ϵ_k is decreased by factor γ as long as $\epsilon_k \geq \epsilon$.

An improvement to the above algorithm is the path-based incremental target level algorithm [15] that employs a different method to compute M^k by accounting for possible oscillations; we consider this approach in this paper where an additional variable ω_k is maintained to detect oscillations and $\omega_{k+1} = \omega_k + \alpha^k mQ$ where Q is an upper bound on $|q(v^k)|$ and m is a scaling constant. Equation (9) is modified to replace $q(v^k)$ with mQ. The target level value is set to $M^k = M_k^{\text{rec}} + \epsilon_k$ where M_k^{rec} and ϵ_k are updated as follows:

$$\begin{aligned} &\text{if } M\left(v^k\right) \geq M_k^{\text{rec}} + \frac{\epsilon_k}{2}; \begin{cases} \epsilon_k = \epsilon_{k-1} \\ M_k^{\text{rec}} = M\left(v^k\right) \\ \omega_k = \omega_{k-1} \end{cases} \\ &\text{otherwise if } \omega_k > b; \begin{cases} \epsilon_k = \frac{\epsilon_{k-1}}{2} \\ M_k^{\text{rec}} = \max_{i \in 1\ldots k} M\left(v^i\right) \\ \omega_k = 0 \end{cases} \\ &\text{else}; \begin{cases} \epsilon_k = \epsilon_{k-1} \\ M_k^{\text{rec}} = M_{k-1}^{\text{rec}} \\ \omega_k = \omega_{k-1} \end{cases} \end{aligned} \tag{11}$$

where b is path length bound ω_k, and M^k and ϵ^k are updated when the function value reaches the target level; here, M_k^{rec} can now be set to the latest value of the function or is updated when the path length exceeds the upper bound b. This means that ϵ_k is too high and therefore is reduced by half. When these conditions are not met, all variables remain unchanged.

F. Termination Criterion

During every dispatch cycle comprised of a 10-min interval, the iterative dispatch strategy terminates when a stopping criterion is met. One terminating criterion suitable in the context of this paper is $q(v^k) = 0$, where when the sub-gradient is 0, the aggregate dispatch of DGs will be equal to the overall demand in the system. Since the supply has met the aggregate demand in the system, the algorithm can terminate.

G. Convergence Analysis

The convergence of the step-size methods dictates the convergence to the optimal dispatch vector.

Proposition 2: The smallest difference d_k between the optimal value M^* of the cost function in P_M and the value of the cost function $M(v^k)$ in P_M evaluated up to iteration k can be bounded by $d_k = M^* - \max_{i \in 1\ldots k}\{M(v^i) \leq (R^2 + \sum_{i=1}^{k}(\alpha^i)^2 Q^2)/(2\sum_{i=1}^{k}\alpha^i)\}$ where $||v^1 - v^*||_2^2 \leq R$ and $||q(v^i)||_2^2 \leq Q$. If $R < \infty$ and $Q < \infty$, then d_k is an upper bound on the difference between the current and optimal value of the dual cost function.

Proof: The proof of Proposition 2 is possible through the following set of derivations [7]:

$$\begin{aligned} ||v^{k+1} - v^*||_2^2 &= ||v^k + \alpha^k q\left(v^k\right) - v^*||_2^2 \\ &= ||v^k - v^*||_2^2 + 2\alpha^k q\left(v^k\right)\left(v^k - v^*\right) \\ &\quad + \left(\alpha^k\right)^2 ||q\left(v^k\right)||_2^2 \\ &\leq ||v^k - v^*||_2^2 - 2\alpha^k\left(M^* - M\left(v^k\right)\right) \\ &\quad + \left(\alpha^k\right)^2 ||q\left(v^k\right)||_2^2 \end{aligned}$$

where the condition $M^* - M(v^k) \leq q(v^k)(v^* - v^k)$ which holds for concave functions is applied. Applying this relation recursively and using the facts that $||v^* - v^{k+1}||_2^2 \geq 0$ and $\sum_{i=1}^{k}\alpha^i(M^* - M(v^i)) \geq (M^* - \max_{i \in 1\ldots k} M(v^i))\sum_{i=1}^{k}\alpha^i$ will result in d_k. ■

For constant step size α^k, d_k reduces to $Q^2h/2$ as $k \to \infty$. Hence, the optimal solution lies within this threshold of the computed solution. Similarly, it can be shown that the square-summable-but-not-summable step size rule converges to the optimal solution. Since $\sum_{i=1}^{\infty}\alpha_i^2 < \infty$ and $\sum_{i=1}^{\infty}\alpha_i = \infty$, $d_k \to 0$ which implies that this step-size method converges to the optimal solution as $k \to \infty$.

A necessary condition for convergence of the dynamic step-size computation is $M^* < \infty$. The sub-gradient considered in this paper is bounded as $Q = \max\left\{\sum_{i=1}^{p} L_i, \sum_{i=1}^{n} c_i - \sum_{i=1}^{p} L_i\right\}$. Since Q is bounded, the sub-gradient is also bounded. Therefore, a necessary condition for dynamic step-size update convergence is met [23].

H. Summary

The proposed dispatch algorithm enables all participating DGs (sub-agents) to compute their own dispatch values in a distributed manner based on low bandwidth data periodically broadcast by the utility (master agent). DGs use their current generation capacities for computation which enables real-time dispatch that does not rely on forecast models. Moreover, as participating DGs form a parallel computational architecture, computational efforts are not concentrated at a single point as for centralized strategies. We assert that our approach is scalable and supports the *ad-hoc* addition or removal of

TABLE I
DISTRIBUTED DISPATCH ALGORITHM

Distributed Dispatch for a single cycle via Dual Decomposition
Initialization: Master agent sets $v^k = 0$ for $k = 1$ and broadcasts this to the sub-agents. 1) Sub-agent i computes dispatch strategy $x_i(v^k)$. 2) Master agent implicitly infers sub-gradient $g(v^k)$ by measuring the surplus or deficit power supplied by the utility to the system. 3) Master agent updates α^{k+1} and v^{k+1} according to a step size update and the sub-gradient method. 4) If termination criterion is met, the algorithm ends. Else, set $k \leftarrow k + 1$ and go to Step 1.

DGs. The utility computes signals based on implicitly inferred information about dispatch. Moreover, key strengths of the approach include that it does not require specific information about local conditions of each DG (i.e., generation capacity or dispatch values). The approach promotes DG privacy as the utility is effectively decoupled from the DG interworkings. Data computed by the utility is universal given that all DGs will react to the same information. No communication from the DGs to the central controller is necessary; the only communication that takes place is of downlink broadcast nature which can be easily implemented using existing cellular networks.

It is shown in Section V that the dispatch vector converges rapidly to optimality. For this reason, continuous communication from the utility is not needed throughout the dispatch interval. Table I outlines the approach. The dispatch procedure is repeated at every 10-min interval. We assert that the flexibility, scalability, and real-time nature of the proposed algorithm has potential for practical use.

IV. Implementation

To evaluate the performance of the proposed distributed dispatch algorithm, we implement the approach using the following models aimed at representing realistic scenarios.

A. Demand Model

Demand from the residential sector can differ due several factors such as seasonal changes, properties of households in the region, and penetration rate of electric appliances. We consider the fluctuating residential demand model from [2] that combines these factors to identify four classes of homes; here, demand is modeled as a continuous-time Markov process for which the state and rate matrices are constructed from actual measurement data obtained from a testbed of sample homes. Specifically, we consider homes belonging to “Class 1” with a peak load of 3902 W and base load of 142 W [2]. Given that a 4 MW distribution substation serves approximately 200 homes and a neighborhood consists of somewhat similar homes, the region considered in this paper is assumed to be comprised of 200 Class 1 homes. Moreover, since the demand model is based on measurements obtained during winter, only the winter season will be considered in this paper.

In addition to seasonal correlations, residential demands also reflect diurnal patterns. Hydro One has divided a winter weekday into on-peak (7 to 11 A.M. and 5 to 7 P.M.), mid-peak (11 A.M. to 5 P.M.), and off-peak time (7 P.M. to 7 A.M.) slots [16]. The demand model has assigned a set of states to each slot. These states are the representative loads of homes in a class during that particular slot. Class 1 homes are assigned six states for on-peak, six states for mid-peak, and five states for off-peak slots. Monte Carlo simulations based on the stationary Markov distributions derived from data provided in [2] are implemented in MATLAB to produce the residential load profiles. Fig. 2(a) and (b) illustrates a sample load profile of a Class 1 home and an aggregate load profile of 200 homes, respectively, during a 24 h period. Since it is assumed that the demand remains constant for 10 min, the demand from each home is computed at every 10-min interval.

B. Generation Capacity of Photovoltaics (PVs)

Photovoltaic (PV) generation capacity is related to the solar irradiance striking the solar panels. We employ hourly solar generation data by panels with a 3.08 kW ac rating in the region of Toronto on a randomly selected day from [19]. Power capacity of PVs is assumed to remain constant during an hour.

C. Generation Capacity of Wind Turbines

The power generation capacity of a wind turbine is characterized by four parameters: cut-in speed, rated speed, cut-off speed, and rated power output. For wind speed v in between the cut-in and rated speeds, power generation is given by $P_{\text{wind}} = (1/2)A\rho\theta v^3$ where A is the area of wind that passes through the rotors of the turbine with diameter D, ρ is air density and θ is the turbine power output efficiency. Wind turbines considered in this paper are assumed to have 50 kW power rating, 25 m height, 13 m rotor diameter, and 0.4 efficiency [20]. Wind speeds are modeled using the Weibull probability density model for 10 min forecasts [17] with a shape factor of 1.94 and a scale factor of 4.48.

D. Power Injecting and Absorbing Entities

Diesel generators are typically used to provision for peak loads or to supplement power to areas that can become isolated from the main grid. An active diesel generator in the 200 home system is assumed to be sized at 4 MW and is treated as another DG sub-agent by the dispatch scheme. Although it is assumed that the system is grid connected, diesel generators are included in the generation mix to reduce main grid dependency when aggregate demand exceeds the available generation capacity of DGs. It is assumed that a diesel generator has a fixed power generation capacity. To accommodate surplus power produced by the DGs during dispatch adjustments, flywheels are also included in the system.

E. Generation Costs

According to [18], the cost of wind, PV, and diesel power generation is 0.135, 0.802, and 2.08 \$/kWh, respectively. The cost of power supplied by the main grid is assumed to be the same as that of diesel generation.

V. Results

The models of Section IV that aim to represent practical environmental conditions and the proposed dispatch

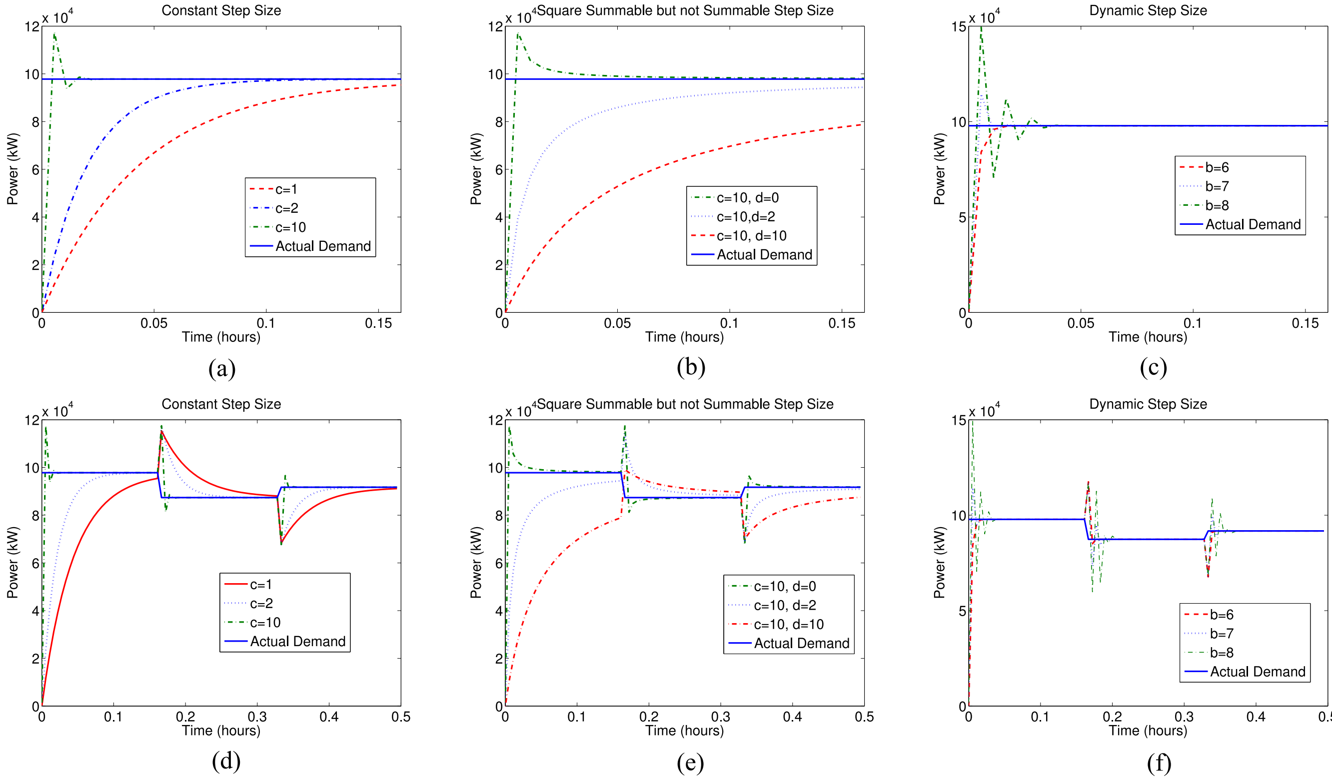


Fig. 1. Power dispatched by DGs for the three types of α computations. Power dispatched by DGs for (a)–(c) 1 interval and (d)–(f) 3 intervals.

strategy are implemented in MATLAB to perform four studies; comparisons with [13] are conducted.

A. Comparison of Sub-Gradient Methods

We compare the performance of our proposed algorithm for different parameter values and sub-gradient step size approaches of Section III-E. The system under consideration is composed of 200 homes, 2 wind turbines, 20 solar panels and one active diesel generator. There are 30 signaling iterations for every 10-min interval. At the beginning of each simulation, the master agent initializes v to 0 and for each subsequent interval, v is initialized to its last computed value of the previous interval. Aggregate generation and demand can differ at each interval. Fig. 1 presents the resulting demand and dispatch curves for a single 10-min interval and three consecutive 10-min intervals. The latter three plots demonstrate how the aggregate power dispatched by DGs transitions from one interval to another.

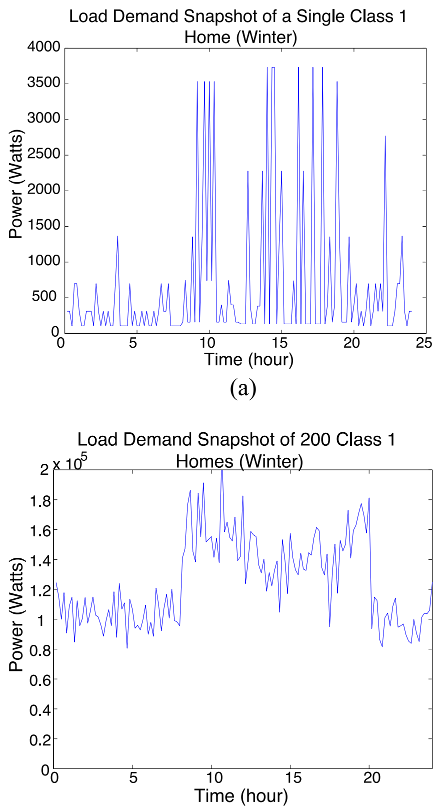
Fig. 1(a) and (d) corresponds to the master agent using a constant step-size c to compute v. For larger values of c, the power dispatched by DGs converges rapidly to the aggregate demand in the system with high initial overshoot. Extremely large values of c (not presented) are expected to exhibit oscillatory behavior with the overall power dispatched lying within the threshold $Q^2h/2$ based on the analysis of Section III-G. It is evident that Q can be as large as 100 kW and therefore the enclosing threshold is very large. Hence, care must be taken in selecting c so that it is high enough to allow quick convergence without inducing oscillatory response.

Fig. 1(b) and (e) corresponds to the square-summable-but-not-summable method used by [13] for day ahead scheduling. Parameter c is fixed while d is varied. When d is large, the step-size increment at the first iteration is small resulting in a larger number of iterations to reach optimality. Unlike the constant step size method, convergence to the demand curve is guaranteed as $t \to \infty$. However, this may not be suitable for real-time dispatch that requires rapid convergence within a short finite interval.

Fig. 1(c) and (f) illustrates results for the dynamic step-size sub-gradient method. The upper bound on the path length is varied by the factor b. It is clear that this method most rapidly converges within a small time interval with higher overshoot for larger values of b. Convergence is also guaranteed making this approach most appealing for real-time dispatch.

B. Real-Time Dispatch

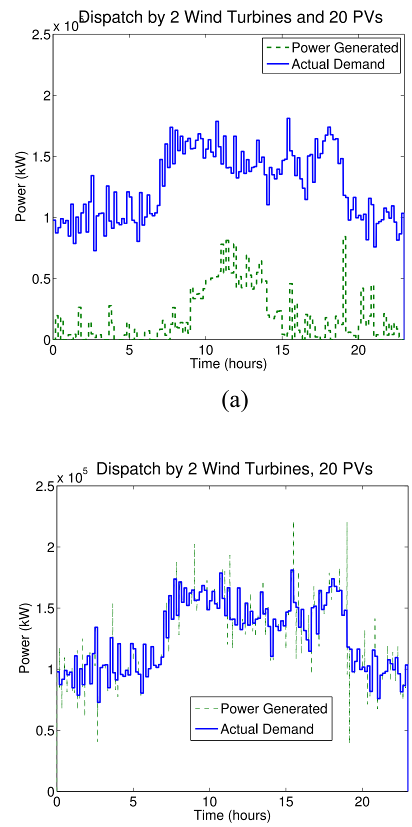
The distributed dispatch algorithm is applied to the system containing 2 wind turbines and 20 PVs over an entire day. The master agent uses the dynamic step-size sub-gradient method to compute v. When the diesel generator is inactive, the combined generation capacity of the wind turbines and PVs is not sufficient to meet the aggregate demand in the system, and hence Slater's condition will fail. It is evident from the dispatch curve in Fig. 3(a) that all active DGs are functioning at full capacity (although the equality constraint in P_P is not met) as expected. When the diesel generator is active, all DGs converge in a distributed manner without significant deviations from the highly fluctuating demand curve as illustrated in Fig. 3(b). These results indicate that each DG sub-agent is able to locally compute its dispatch based solely on the broadcasted



(a)

(b)

Fig. 2. Load profiles in the residential sector during winter. Load profile of (a) 1 sample home and (b) 200 homes.



(a)

(b)

Fig. 3. Real-time dispatch over a day. DG generation (a) without diesel generator and (b) with diesel generator.

v transmitted by the master and successfully match aggregate generation with demand in real-time.

C. Generation Mix Analysis

The cost of bulk (main grid) power especially at peak loads can be unsustainable. Thus, we explore the potential of establishing a DG mix to reduce the bulk power dependence. Fig. 4(a) illustrates the overall power supplied by DGs over an entire day for systems with a variety of generation mixes. Each generation mix is denoted by a tuple denoting the number of wind turbines and the number of PVs, respectively. As expected, when the number of DGs in the system is increased, the aggregate power saved increases. If the goal of the utility is to reduce overall power supplied by the main grid during a day, then analysis of Fig. 4(a) provides useful information for designing energy policy at the distribution substation level.

In some cases, information on overall power savings may not be as insightful as details on power savings during specific times in a day. Fig. 4(b) captures the achievable cumulative power savings from time 0 to t. Significant differences in power savings occur as the number of wind turbines in the generation mix are increased. Wind power is typically available throughout the day whereas solar power is available primarily during the morning and noon periods. The difference in the number of PVs deployed is noticeable after 10 A.M. During the summer, mid-peak and on-peak intervals occur between 7 A.M. and 7 P.M. Hence, if the goal is to offset power supplied by the grid during on-peak and mid-peak periods, the generation mix of the system should include more PVs. Thus, we observe that use of an efficient dispatch algorithm that harnesses the full potential of DGs enables formulation of more economical and sustainable energy policies.

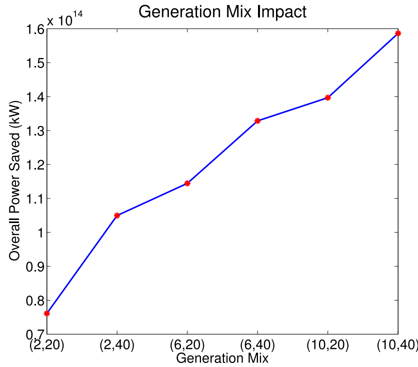
TABLE II
COMPARISON OF DISPATCH METHODS

	#DGs	Proposed Distributed	Centralized Online	Centralized/ Distributed Offline	Decentralized Online
Comm Costs	10	5	20	0	90
	20	5	40	0	380
	30	5	60	0	870
Info Overhead	10	5	20	0	180
	20	5	40	0	760
	30	5	60	0	1740
Dispatch Error from Forecast	10	0%	9.04%	28.23%	0%
	20	0%	9.04%	28.23%	0%
	30	0%	9.04%	28.23%	0%
Optimality of Solution	10	Yes	No	Yes	Yes
	20	Yes	No	Yes	Yes
	30	Yes	No	Yes	Yes

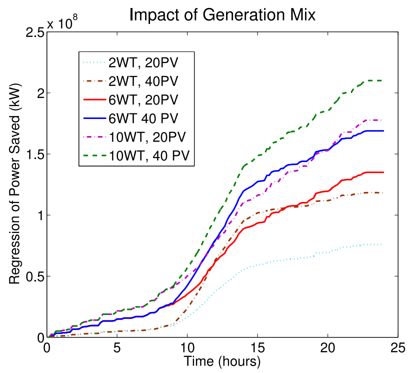
D. Comparison of Dispatch Strategies

In this section, characteristics of the proposed and existing dispatch strategies are evaluated against a variety of metrics, as summarized in Table II.

Communication cost refers to the number of connection links forged by nodes in a particular strategy at every dispatch cycle. For the proposed strategy, the utility does not make point to point connections with the DGs but broadcasts a single



(a)



(b)

Fig. 4. Power savings. (a) Generation mix impact on power savings. (b) Regression of power savings over a day.

transmission. The communication cost is the average number of iterations $E[T]$ required for convergence during a dispatch interval. Simulations indicate that $E[T]$ is approximately 5 for all combinations of generation mix considered earlier. For offline strategies, the entire dispatch schedule will be communicated in advance, so communication cost is negligible. For the centralized online strategy, all n DGs will need to communicate individual generation capacities to the utility that will then communicate n dispatch values to the DGs resulting in a communication cost of $2n$. For the decentralized algorithm, according to [14], communication cost is $m(m-1)$ where m is the number of relays assumed to be at least as large as the number of DGs.

Information overhead accounts for the total units of data exchanged between the nodes participating in dispatch. For the proposed approach, this metric is $E[T]$, the units of information broadcast during a dispatch cycle until convergence. The information overhead for centralized offline strategies is ignored since exchange occurs in advance. For centralized online strategies, DGs transmit current generation capacity and the central controller transmits the computed dispatch vector to all DGs. Given that there are n DGs in the system, this results in $2n$ units of information exchanged during a dispatch cycle. For decentralized strategies, nodes exchange load and generation information with one another resulting in $2m(m-1)$ data exchanges.

Forecast model inefficiencies can inject error into the computed dispatch vector due to generation prediction inaccuracies for example in wind models. Proposed distributed and decentralized strategies rely on real-time generation data for dispatch and do not use forecast models. Centralized online strategies can use short term (e.g., 3-h) generation models and offline strategies can use a-day-ahead prediction models. Errors introduced in these models are provided in [12].

Security and privacy are considerations as DGs may not choose to provide external access to their state information. Our approach whereby DGs do not communicate information regarding their state and simply rely on utility broadcast communication inherently promotes privacy. Moreover, the use of a broadcast channel for communications opposed to individual links makes the proposed system intrinsically more secure against denial of service or data corruption attacks.

Global optimality of the dispatch solution after convergence is another metric whereby a centralized online strategy may not guarantee an optimal solution due to the size of the problem and time constraints in contrast to the proposed strategy. Also distributed strategies such as that in [13] may not converge to optimality within a short interval when used for real-time dispatch. Thus, we conclude that the proposed strategy provides a good balance amongst a variety of metrics providing lower communication and information overhead and inherent security while enabling real-time dispatch.

VI. Conclusion

In this paper, we propose a scalable distributed dispatch strategy that converges rapidly to the optimal dispatch vector in real-time while entailing low communication and information overhead thus making it attractive for future systems comprised of a large and diverse number of DGs. We show how our distributed strategy reduces the dependence on bulk generation. Analysis of the proposed generation mix results can lead to insightful results that can aid in energy policy making initiatives. Future work will extend the dispatch strategy to work across multiple distribution substations and will include transmission level constraints and the incorporation of market considerations.

Acknowledgment

The authors would like to thank Prof. W. Yu for providing feedback on an initial development of this paper.

References

[1] A. M. Z. Alabedin, E. F. El-Saadany, and M. M. A. Salama, "Generation scheduling in microgrids under uncertainties in power generation," in *Proc. IEEE Elect. Power Energy Conf.*, London, U.K., Oct. 2012, pp. 133–138.

[2] O. Ardakanian, S. Keshav, and C. Rosenberg, "Markovian models for home electricity consumption," in *Proc. ACM Sigcomm Workshop Energy IT Green Netw. Smarter Syst.*, Toronto, ON, Canada, 2011, pp. 31–36.

[3] M. R. Avinaash, G. R. Kumar, K. A. Bhargav, T. S. Prabhu, and D. I. Reddy, "Simulated annealing approach to solution of multi-objective optimal economic dispatch," in *Proc. Int. Conf. Intell. Syst. Control*, Coimbatore, India, 2013, pp. 1–6.

[4] D. Bertsekas, *Nonlinear Programming. 6.252J Lecture Notes: Dual Computational Methods*. Cambridge, MA, USA: MIT, 2003.

[5] D. Bertsekas, A. Nedic, and A. E. Ozdaglar, *Convex Analysis and Optimization*. Belmont, MA, USA: Athena Scientific, 2003.

[6] G. Binetti *et al.*, "Distributed solution for the economic dispatch problem," in *Proc. 21st Mediterranean Conf. Control Autom.*, Chania, Greece, 2013, pp. 243–250.
[7] S. Boyd and A. Mutapcic, *Sub-gradient Methods. EE364b Lecture Notes*. Stanford, CA, USA: Stanford Univ. Press, 2008.
[8] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.
[9] S. Boyd, L. Xiao, and A. Mutapcic, *Subgradient Methods. EE393o Lecture Notes*. Stanford, CA, USA: Stanford Univ. Press, 2003.
[10] E. Constantinescu, V. Zavala, M. Rocklin, S. Lee, and M. Anitescu, "Unit commitment with wind power generation: Integrating wind forecast uncertainty and stochastic programming," Math. Comput. Sci. Div. Tech. Memorandum, Argonne Nat. Lab., Lemont, IL, USA, Tech. Rep. ANL/MCS-TM-309, 2009.
[11] M. Eghbal, T. K. Saha, and N. Mahmoudi-Kohan, "Utilizing demand response programs in day ahead generation scheduling for micro-grids with renewable sources," in *Proc. IEEE PES Innov. Smart Grid Technol. Asia*, Perth, WA, Australia, 2011, pp. 1–6.
[12] T. H. M. El-Fouly, E. F. El-Saadany, and M. M. A. Salama, "One day ahead prediction of wind speed using annual trends," in *Proc. IEEE Power Eng. Soc. Gen. Meeting*, Montreal, QC, Canada, 2006, pp. 7–12.
[13] J.-Y. Joo and M. D. Ilic, "Multi-layered optimization of demand resources using Lagrange dual decomposition," *IEEE Trans. Smart Grid*, vol. 4, no. 4, pp. 2081–2088, Dec. 2013.
[14] H. Liang, B. J. Choi, A. Abdrabou, W. Zhuang, and X. Shen, "Decentralized economic dispatch in microgrids via heterogeneous wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 6, pp. 1061–1074, Jul. 2012.
[15] A. Nedic, "Subgradient methods for convex optimization," Dept. Elect. Eng. Comput. Sci., Ph.D. dissertation, MIT, Cambridge, MA, USA, 2002, pp. 1–174.
[16] Hydro One, (2013, Oct.). *The Electricity Sector* Online. Available: http://www.hydroone.com/TOU/Pages/Default.aspx/
[17] K. Philippopoulos and D. Deligiorgi, "Statistical simulation of wind speed in Athens, Greece based on Weibull and ARMA models," *Int. J. Energy Environ.*, vol. 3, no. 4, pp. 151–158, 2009.
[18] Ontario Power Authority, (2013, Oct.). *The Electricity Sector* Online. Available: http://fit.powerauthority.on.ca/
[19] PVWatts, (2013, Oct.). *AC Energy and Cost Savings* Online. Available: http://rredc.nrel.gov/solar/calculators/PVWATTS/version1/International/pvwattsv1_hr_intl.cgi
[20] Wind Energy Resources, (2013, Oct.). *50 kw Wind Turbine Generator* Online. Available: http://www.wind-energy-resources.com/wer_50kw_wind_turbine.html
[21] A. K. Uchchkotiya, R. Singh, and A. S. Trivedi, "Evolutionary techniques of GAMS used in optimization of economic load dispatch in power system," *Int. J. Eng. Res. Technol.*, vol. 2, pp. 2359–2367, Sep. 2013.
[22] J. Xu, "Optimization of economic load dispatch for a microgrid using evolutionary computation," in *Proc. 37th Annu. Conf. IEEE Ind. Electron. Soc.*, Melbourne, VIC, Australia, 2011, pp. 3192–3197.
[23] Q. Ye *et al.*, "User association for load balancing in heterogeneous cellular networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 6, pp. 2706–2716, Jun. 2013.
[24] G. Zhabelova, V. Vyatkin, Z. Zhang, and M. Chow, "Agent based distributed consensus algorithm for decentralized economic dispatch in smart grid," in *Proc. 39th Annu. Conf. IEEE Ind. Electron. Soc.*, Vienna, Austria, 2013, pp. 1968–1973.

Pirathayini Srikantha received the B.A.Sc. degree in systems design engineering from the University of Waterloo, Waterloo, ON, Canada, with Distinction - Dean's Honours List, in 2009, where she received the M.A.Sc. degree in electrical and computer engineering in 2013. She is currently pursuing the Ph.D. degree from the Edward S. Rogers Sr. Department of Electrical and Computer Engineering, University of Toronto, Toronto, ON.

Her current research interests include investigating how effective solutions can be designed for current applications in the electric smart grid, including cyber-security, sustainable power dispatch, and demand response using convex optimization and game theoretic techniques.

Ms. Srikantha was the recipient of the Best Paper Award Recognition at the 3rd IEEE International Conference on Smart Grid Communications (SmartGridComm 2012) for the Symposium on Demand Side Management, Demand Response, and Dynamic Pricing.

Deepa Kundur received the B.A.Sc., M.A.Sc., and Ph.D. degrees in electrical and computer engineering from the University of Toronto, Toronto, ON, Canada, in 1993, 1995, and 1999, respectively.

From 1999 to 2002, she was an Assistant Professor with the Edward S. Rogers Sr. Department of Electrical and Computer Engineering, University of Toronto, and returned in 2012, where she became a Professor. She currently serves as an Associate Chair with the Division of Engineering Science. From 2003 to 2012, she was a faculty member in electrical and computer engineering with Texas A&M University, College Station, TX, USA. Her current research interests include cyber security of the electric smart grid, cyber-physical system theory, security and privacy of social and sensor networks, multimedia security, and computer forensics.

Dr. Kundur was the recipient of the Best Paper Recognitions at the 2008 IEEE INFOCOM Workshop on Mission Critical Networks, the 2011 Cyber Security and Information Intelligence Research Workshop, and the 2012 IEEE Canadian Conference on Electrical and Computer Engineering. She has been on several editorial boards and currently serves as an Associate Editor of the IEEE TRANSACTIONS ON INFORMATION FORENSICS AND SECURITY. She has served on the North American Reliability Corporation Smart Grid Task Force.