Resilient Distributed Real-Time Demand Response via Population Games

Pirathayini Srikantha, *Student Member, IEEE*, and Deepa Kundur, *Fellow, IEEE*

***Abstract*—The proliferation of high powered electric devices is a driving force in the rising of peak power demand from electric power utilities. One way to accommodate these rising consumption patterns involves the deployment of high capacity dispatchable, but largely unsustainable peak generation systems. To avert these extravagant costs and the likelihood of grid overload, demand response (DR) strategies can be employed to curtail overall consumption, thus reducing peak patterns. In this paper, we propose a distributed real-time DR approach. The proposed method fosters seamless cooperation between DR participants for rapid convergence to expected aggregate load curtailment, while accounting for individual consumer satisfaction needs. We assess this paper through theoretical analysis based on population game theory and simulations to demonstrate its inherent flexibility, scalability, and resilience making it attractive for practical widespread deployment.**

***Index Terms*—Demand response, distributed algorithms, cyber-physical systems.**

I. Introduction

As PEAK demands rise each year with the increasing penetration of high power-consuming end-devices (such as plug-in hybrid electric vehicles [1]), Electric Power Utilities (EPUs) are naturally identifying mechanisms to not just accommodate, but regulate demand peaks. Demand Response (DR) schemes are being employed to reduce peak demands through load shifting and energy conservation. Given that most consumers can tolerate a certain degree of adjustment in their energy requirements without affecting service satisfaction (through, for example, marginally varying thermostat setpoints, tumble drying, using cooler wash cycles, etc.), there exists an inherent demand flexibility that can be leveraged through monetary compensation to limit peak demand. A major technical challenge in DR is striking a balance between reducing costs and managing the fluctuating peak demands of a large number of consumers without negatively affecting consumer satisfaction.

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The authors are with the Department of Electrical and Computer Engineering, University of Toronto, Toronto, ON M5S 3G4, Canada (e-mail: pirathayini.srikantha@ece.utoronto.ca; dkundur@ece.utoronto.ca).

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Existing DR can be categorized as behavioural or Direct Load Control (DLC) schemes. Behavioural DR employs pricing to induce consumers to change individual power consumption patterns. On the other hand, DLC requires the EPU to directly actuate appliances belonging to participating consumers. Behavioural schemes such as Time-of-Use (TOU), Critical Peak Pricing (CPP) and Real-Time Pricing (RTP) compute prices for power over various time intervals in a day [2]. Given that these computations rely extensively on forecast models that are subject to inaccuracies that rise as the prediction horizon increases, prices are not representative of the actual demand-supply state in the grid [3].

In contrast, direct actuation of consumer appliances affords the EPU a higher degree of control. Smart appliances and controllable thermostats can be directly actuated via signals transmitted from the EPU [4]. However, existing DLC schemes are typically implemented in industrial or commercial settings and are not commonly applied to regulate residential demand. Since the residential sector is comprised of many end-users utilizing a diverse set of appliances with widely varying usage patterns, centrally computing an optimal real-time load shedding solution for all appliances (e.g., [5]–[7] focus only on thermostatically controllable loads for DR) is intractable for the EPU. Keeping track of the ever-changing properties of every single appliance along with user preferences for a large number of consumers in a central repository requires a vast amount of storage and communication resources. Moreover, centrally solving the typically non-convex demand response problem constrained by these highly varying local conditions to obtain the exact optimal load reduction becomes very computationally challenging especially as the number of participants increases in the system.

In order to overcome these issues, decentralized and distributed implementations of DLC that capitalize on the recent cyber-enablement of the power grid have been proposed in the literature. One class of these proposals incorporate multi-agent learning methods such as average consensus protocol, subgradient methods and machine learning heuristics [8]–[11]. Certain limitations that include slow convergence (convergence time increases in proportion to the number of participants for consensus based DR presented in [8]), excessive communication overhead (bi-directional communication flows are necessary between the EPU and all participants for the proposed DR algorithm in [10]) and radical changes to existing appliances (direct actuation of inductive and resistive components is required for the DR scheme proposed in [11]) are challenges that restrict practical implementation of these schemes for *real-time* DR.

Other proposals leverage game theoretic techniques for distributed/decentralized DR [12]–[17]. References [12]–[14] optimize appliance scheduling in participating homes via price signals. These proposals require scheduling to occur in advance (e.g., beginning of a day) and every participant must be aware of the appliances that he or she will be utilizing throughout the scheduling period. Not all appliances can participate as these schemes focus only on ‘shiftable’ appliances such as dishwashers, dryers, etc. Moreover, results presented in these papers are based on only a small number of DR participants. Reference [17] applies evolutionary game theory to accommodate a large number of DR participants. However, as the proposed algorithm in [17] results in non-unique solutions, the system will be subject to significant ringing and instability. Other applications of game theory are not directly used in DR but rather to assess whether there exist sufficient incentives for major adaptation of the proposed schemes [15], [16].

In this paper, we aim to address a significant challenge for DR design as observed earlier in the existing literature: managing a large number of consumers with highly variable demands in real-time. In particular, we propose a novel algorithm that is: 1) **distributed**: computational processing is offloaded to cyber-physical DR agents located at consumer premises; 2) **consumer-centric**: end-users configure load operation preferences on their respective DR agents based on comfort requirements; 3) **real-time**: the EPU broadcasts unidirectional communication signals employed by DR agents to elicit load shedding decisions based on current local conditions and these decisions rapidly converge to optimal aggregate peak reduction; 4) **resilient**: it is possible to rapidly recover from system perturbations when a subset of DR agents are corrupted; 5) **scalable**: better convergence is achieved with a larger number of participants. We capitalize on population games, an extension of Evolutionary Game Theory (EGT), to construct a theoretical DR framework that effectively defines strategic interactions amongst DR agents for achieving asymptotic convergence at minimal cost to the aggregate demand reduction goal set by the EPU.

Our contributions include: i) a novel distributed DR strategy with three distinct implementations and corresponding convergence analysis; ii) practical performance evaluation including a study of resilience; and iii) a comparative study of the proposed and existing DR strategies against four metrics.

This paper is organized as follows. Section II presents the system model and assumptions. Section III provides a detailed description of the strategy formulation along with theoretical system properties. In Section IV, simulation and comparative results obtained using realistic MATLAB/Simulink implementations are presented. Final remarks are drawn in Section V.

II. Preliminaries and Setting

We present a generalized framework for DR at the distribution substation level consisting of a diverse set of consumers.

A. Assumptions

We make the following assumptions about the DR system:

1) The EPU is equipped with a cellular transmitter that broadcasts cost signals in the downlink at 2 Hz;
2) The EPU can measure in real-time the proportion of power demand reductions resulting from the DR scheme at the distribution substation level;
3) Each DR participant is equipped with a DR agent supplied by the EPU at his/her premises that can be configured with individual load usage preferences;
4) The DR agent is a cyber-physical entity that is retrofitted with a wireless transmitter and a receiver and has the ability to directly actuate appliances in its local premise;
5) Each DR agent can commit to one of n power reduction levels and will automatically schedule local appliances according to this commitment;
6) There is a large number of DR participants;
7) Sufficient power reduction capacity exists in the system after factoring in consumer preferences to meet an aggregate load reduction of C kW;
8) DR costs are modelled as a strictly convex function;
9) Demand and supply remain constant for 1 minute.

The first, second and fourth assumptions correspond to the cyber-enabled vision of the smart grid. Standard PMUs typically transmit at 50 Hz which is a much higher rate than that required by the EPU in the first assumption [18]. The third and fifth assumptions abstract local operating conditions of consumer loads from the EPU. Behavioural uncertainties are quantified as occurring from a finite set of possibilities removing humans from the active control loop. A DR agent responds to signals transmitted by the EPU to make decisions on what local loads should be stalled or postponed according to the preference pre-configured by the consumer and the current appliances that are active. These preferences can be translated into a *comfort budget* which is the maximum overall energy reductions a consumer can tolerate over the course of a day. As loads consume power at discrete levels, these are generalized to n power reduction levels. Local wireless signals transmitted by the DR agent will either directly actuate smart appliances or indirectly actuate regular appliances via smart plugs using a smart home energy management system [4]. DR agents can also support the second assumption via local transmitters as these can be used to report local load reductions in a manner similar to smart meters and/or PMUs to the EPU for pricing purposes. The next three assumptions are necessary for the formulation of the distributed DR problem. The sixth assumption is practical as the residential sector, for instance, is composed of thousands of consumers. The eighth assumption, also adapted by many works that include [19], constructs cost of non-generation resources (i.e., demand response, energy storage, etc.) as a strictly convex function modelling after primary energy markets (e.g., power generation costs by plants such as thermal units are considered to be strictly convex and quadratic in [18] and [20]). A strictly convex cost function effectively captures greater utilization of these non-generation energy resources by consumers [19]. The final assumption is reasonable as the 1 minute interval of supply/demand constancy is much smaller than the prediction horizon typically used in DR to capture transitions in appliance power usage [18].

B. Demand Reduction via Conservation

In this paper, we consider consumer preferences for energy conservation (i.e., varying thermal setpoints, using cooler wash cycles, etc.). Although energy conservation is not the primary goal of many DR schemes, it is implicitly incorporated into schemes such as TOU. According to [21], TOU induces short-term behavioural changes in which many consumers use electric appliances more sparingly and carefully. One study has concluded that a 3-4% reduction in electricity usage is observable amongst these users [21]. In addition, programs such as PowerSaver in the U.S. promote energy efficiency by providing incentives to consumers who retrofit their homes with energy-saving equipment [22]. Energy conservation is now a natural option especially when there are many energy saving options in modern appliances. While energy conservation is one method to reduce grid overload, rescheduling appliance operations to flatten power consumption peaks can also be accommodated by our scheme. Comprehensive storage systems such as Powerwall by Tesla (home battery that charges via power generated by local solar panels) are now readily available in consumer markets [23]. When devices such as these function in tandem with demand response schemes such as ours, appliances can be rescheduled without causing rebounding or payback effects. Storage systems can be activated when demand reductions are required instead of conserving appliance usage. We consider these possibilities in future work.

C. Demand Response Problem Formulation

Consider an EPU that employs DR to shave aggregate demands by C kW during an impending energy strain such as grid overload or excessive generation costs. To encourage participation in the DR program, the EPU provides monetary incentives to consumers for demand reductions. Suppose all m participating DR agents form the population P. As discussed in the previous section, each DR agent in P must select amongst n power reduction strategies denoted by the set $S = \{s_1 \ldots s_n\}$ that correspond to n power commitment levels $y = [y_1 \ldots y_n]^T \in \mathbb{R}^{n\times 1}$ (such that $y_j \in \mathbb{R}$). Let $z = [z_1 \ldots z_m] \in \{0, 1\}^{nxm}$ (whereby $z_i \in \mathbb{R}^{n\times 1}$) represent the associated load shedding matrix such that if DR agent i selects to reduce power consumption by y_j kW within its precinct, the j^{th} component in z_i is set to 1 and all other components in z_i are set to 0. We can therefore denote the power committed by DR agent i by $z_i^T y$.

Let $f_o(z)$ represent the overall cost incurred by the EPU for imposing a load shedding of z. Thus the EPU is required to solve the following optimization problem P_C to achieve a desired aggregate demand reduction while minimizing compensation costs and preserving consumer preferences:

$$
\begin{aligned}
&(\mathbf{P}_C) \quad \min_z f_o(z) \\
&\text{subject to} \sum_{i=1}^{m} z_i^T y = C \qquad (1)\\
&\qquad 0 \le z_i^T y \le \min\{p_i, l_i\} \quad \forall\, i = 1 \ldots m
\end{aligned}
$$

where p_i and l_i denote the maximum power reduction possible given the remaining comfort budget and the power currently consumed, respectively, by consumer i. The equality (coupling) constraint requires aggregate power reductions by all DR agents to meet the targeted value of C. The inequality constraint ensures that the power committed by DR agent i meets the pre-configured consumer preference p_i given that the current consumer demand l_i is sufficient for the reduction. At the beginning of a day, p_i is set to the comfort budget the consumer has configured in his/her DR agent. As the day progresses, reductions made by the DR agents are taken into account by reducing the comfort budget accordingly. This ensures that a consumer does not experience more power reductions than what they prefer.

We observe that P_C is an integer programming problem, given the discrete nature of the variables for optimization. Integer programming problems fall under the NP complexity class [24]. Complexity of an NP problem increases exponentially with the problem size (i.e., number of participating DR agents in our case) [25]. Moreover, taking into account each consumer's current load conditions and highly fluctuating operation preferences imposes significant communication overhead. In the next section, we propose a distributed DR scheme that aims to address these challenges.

III. Distributed Demand Response

We assert that a distributed strategy in which participating DR agents respond to real-time cost signals from the EPU to compute their own load shedding commitment iteratively will eliminate pervasive practical challenges in DR schemes. DR agents are representatives of the EPU that reside at local premises of consumers. It is important to note that the compensation paid by the EPU to a consumer for reducing consumption by a particular level is reflected in $f_o(z)$ and this is not the same as the cost signals broadcasted by the EPU. Cost signals transmitted by the EPU foster cooperation amongst the DR agents so that the cumulative load shedding decisions of these agents ensure that minimal costs are incurred by the EPU while heeding local consumer preferences and appliance operating statuses.

In this section, to reformulate the new DR problem, we first shift the local inequality constraints in P_C to the corresponding DR agents and apply a change of variables to form P'_C that only consists of the coupling constraint. Then, the dual P_D of P'_C is constructed to form an optimization problem without the coupling constraint. We observe that a globally unique solution exists for P_D such that the EPU can achieve an aggregate demand reduction of C at minimal cost if there exists sufficient demand reduction capacity overall. If user preferences are overly restricting then P_C may become infeasible. In the proposed algorithm, each agent will select a strategy based on best-effort that heeds local constraints while attempting to achieve reductions that are close to C as discussed in Section III-C. Moreover, the objective function of P_D is used to compute strategy costs periodically to foster coordination amongst DR agents for selection of strategies that result in overall power reduction of C. As we see, such a distributive selection strategy by a large number of DR agents naturally defines a *population game* that is used for analysis.

A. Computation of Cost Signals

To maintain fairness, the EPU does not discriminate amongst DR agents and is therefore concerned with the cumulative effective of DR (in terms of overall cost and aggregate demand reduction) opposed to the individual strategies of each DR agent. Let x_i represent the fraction of DR agents in P that select strategy s_i (i.e., $x_i = \frac{\text{\# agents using } s_i}{m} = \sum_{j=1}^{m} \frac{z_j^T e_i}{m}$ where e_i is a unit vector in which the i^{th} element is 1). We introduce the vector $x = [x_1 \ldots x_n] \in \Delta$ that necessarily takes on values in the simplex $\Delta = \{x \mid \sum_{i=1}^{n} x_i = 1,\ 0 \le x_i \le 1\}$ since every DR agent in P will select one of n available strategies. We reformulate P_C of (1) in terms of x as follows:

$$\begin{aligned} (\mathbf{P}'_C) \quad & \min_{x \in \Delta} f_o(x) \\ & \text{subject to } m\ x^T y = C. \end{aligned} \tag{2}$$

The inequality constraints in P_C are removed and incorporated into DR agent cost signal responses discussed further in Section III-C. Complexity of the newly formulated P'_C is dependent only on the number of strategies n and does not change with the population size m. Without loss of generality, $f_o(z)$ is defined to be a quadratic function $\frac{1}{m}(\sum_{i=1}^{m} z_i)^T Y (\sum_{i=1}^{m} z_i)$. Quadratic cost functions are commonly used in DR works such as [19]. Y is a diagonal matrix with strictly positive diagonal entries. Diagonal entry Y_{ii} is the compensation paid by the EPU to the DR consumer for selecting s_i and committing to a y_i kW reduction. It is clear that the change of variable from z to x results in the transformation of the objective function to:

$$f_o(x) = m\frac{1}{2}x^T Y x = \frac{1}{2}m \sum_{i=1}^{n} x_i^2 Y_{ii} \tag{3}$$

Even if the decision variables are different, the objective values of $f_o(z)$ and $f_o(x)$ are the same when z is mapped to x. As Y is a diagonal matrix with strictly positive diagonal entries, Assumption 8 is satisfied (i.e., $f_o(x)$ is a strictly convex function).

We note that P'_C is a *strictly* convex optimization problem consisting of only continuous decision variables (i.e., x) and a strictly convex objective function with linear constraints (these define the simplex Δ that x can take values from). Thus, it is efficiently and *uniquely* [26] solved by the EPU to obtain x^*, the distribution of DR agents in P resulting in the lowest cost. It is straightforward for the EPU to compute x^*, but a practical challenge lies in assigning available strategies to specific DR agents to achieve distribution x^* while adhering to local operating constraints.

In our formulation, the EPU will use real-time cost signals to facilitate coordination amongst all DR agents so that their individual strategy selections iteratively converge to the optimal load shedding solution x^*. At this stage, the aggregate load would be reduced by at least C kW when sufficient shedding capacity exists or by the maximum commitment possible that is less than C kW when shedding capacity is insufficient.

To compute these cost signals, the Lagrangian dual P_D of P'_C is considered:

$$\begin{aligned} (\mathbf{P}_D) \quad & \max_{v} \ \min_{x \in \Delta} f_d(x) \\ & \text{where } f_d(x) = m\frac{1}{2}x^T Y x + v(C - m\ x^T y) \end{aligned} \tag{4}$$

where $v \in \mathbb{R}$ is the Lagrangian multiplier associated with constraint $m\ x^T y = C$. It is clear that $f_d(x)$ is strictly convex (thus representing a *potential function* [27]) for fixed v given that it is a linear combination of strictly convex $f_o(x)$ and the linear function $v(C - m\ x^T y)$ [26]. Moreover, P_D is a *simplistic problem* that can be easily solved by the EPU via standard solvers available to solve Quadratic Problems (QP) [18] to obtain the optimal value v^* by fixing x to the optimal x^* obtained by solving P'_C. Cost F_i assigned to strategy s_i by the EPU is dynamic and depends on the current distribution x as follows:

$$F_i(x) = m(Y_{ii}x_i - y_i v^*) \tag{5}$$

where $F_i(x)$ is strictly increasing and continuous in x and is precisely the gradient of $f_d(x)$ (i.e., $F_i(x) = \frac{\partial f_d(x)}{\partial x_i}$) thus defining the opposing direction in which the global minimum of the objective function resides allowing DR agents to select less costly strategies that reduce the potential of the system. The EPU computes $F_i(x)$ at every signalling iteration. The current value of x is available to the EPU according to our second assumption in Section II.

B. Potential Game Setup

DR agents are local EPU-coordinating entities that make strategic decisions based on the real-time cost signals broadcasted by the EPU at every signalling iteration. As DR agents will behave in a rational manner to achieve minimal individual costs, these entities may be considered to participate in a *population game* ϑ where the state of the population is adequately captured by x. The game ϑ is completely specified by x and $F_i\ \forall\ s_i \in S$. Theorem 1 shows that the unique structure of F_i allows ϑ to be classified as a *full potential game* [27] that has several desirable properties pertaining to the state dynamics resulting from the distributive strategy selections by the DR agents highlighted in the subsequent sections.

Theorem 1: The population game ϑ defined by $F : x \to \mathbb{R}^n$ where $F = [F_1, \ldots, F_n]$ is a full potential game as it satisfies the following full externality symmetry [27]:

$$\frac{\partial F_i}{\partial x_j} = \frac{\partial F_j}{\partial x_i},\ \ \forall s_i, s_j \in S$$

Proof: The partial derivatives of F_i defined for ϑ are

$$\frac{\partial F_i}{\partial x_j} = 0,\ \frac{\partial F_j}{\partial x_i} = 0\ \forall s_i \neq s_j$$

■

One can intuitively relate the potential function and population game as follows. Suppose a fraction of DR agents ϵ make a switch from a more costly strategy s_j to a less costly one s_i, then $\frac{\partial f}{\partial u} = F_i(x) - F_j(x) < 0$ where ∂u is the displacement vector representing the fraction of DR agents switching from s_i to s_j. From this observation, it is evident that when strategy switches are cheaper, the potential (i.e., the overall cost) of the game reduces. Hence, all "reasonable" strategy changes result in reducing the potential of the game and cost incurred by each DR agent. DR agents will repeatedly switch their strategies according to a *revision protocol* [28] until all DR agents reach a population state x^* where any further changes will result in increased cost. This point of stationarity is referred to as the *Nash Equilibrium* (NE), formally defined as:

$$NE = \{x \in \Delta | x_i > 0 \rightarrow F_j(x) \geq F_i(x); \forall s_i, s_j \in S\} \tag{6}$$

When $x^* \in NE$, strategies s_i that are in use (i.e., $x_i > 0$) incur the same minimal cost and all other unused strategies s_j (i.e., $x_j = 0$) have higher costs. There exists a relationship between the unique solution of P_D and the NE achieved through distributed revisions and this is stated in Theorem 2:

Theorem 2: The NE of the population game ϑ is identical to the global minimum of P_D for fixed v^* and is unique.

Proof: The optimal solution x^* of P_D is unique as the problem is strictly convex. This global minimum x^* must satisfy all of the Karush-Kuhn-Tucker (KKT) conditions [26]. These conditions reduce to exactly the necessary and sufficient conditions of NE in a game as listed in Eq. 6 (please refer to the Appendix for this proof). Hence, the global optimum x^* is indeed the NE of the game ϑ. ■

Local asymptotic stability properties of the unique NE provide insights to whether the distributed DR scheme will be robust. Suppose that after the DR agents reach the NE through "reasonable" distributed strategy revisions, a fraction ϵ of DR agents behave in an irrational manner and select strategies that are more costly. This situation is entirely possible if a subset of DR agents were compromised and forced to behave in a malicious manner. If the system is able to return to the original NE even after these perturbations, then the population game can be considered robust and the associated NE results in an Evolutionary Stable State (ESS) [27]. Theorem 3 indicates that a global ESS exists for the game ϑ and is precisely the NE.

Theorem 3: The unique NE of the potential game ϑ is the global ESS of the game.

Proof: As the game ϑ has a strictly convex potential function, it satisfies the relation $(y-x)'(F(y)-F(x)) > 0 \ \forall \ x \neq y$ and this is the condition necessary for a strictly stable game [29]. x^* is an ESS if it satisfies the following conditions: $(y-x^*)'F(x^*) \geq 0$ and if $(y-x^*)'F(x^*) = 0$ then $(y-x^*)'F(y) > 0$. The first condition dictates that x^* is an NE according to an alternative definition of NE in [30]. As ϑ is a strictly stable game, the second condition is naturally satisfied when $(y-x)'F(x) = 0$. ■

C. DR Agent Decisions and State Dynamics

We have established the static properties of existence and uniqueness of the NE and ESS in a population game. In this section, the dynamical evolution of the population state x when DR agents use various *revision protocols* to make distributive strategy selections is studied. We investigate the modified versions of revision protocols proposed in reference [27] as the objective in this paper is to minimize costs incurred by the agents whereas the objective in the reference is to maximize agent payoffs. In the distributed implementation, at a randomly selected time, each DR agent uses a *probability* proportional to a *conditional rate* $\rho_{i,j}(F(x), x)$ which is a function of the current strategy costs and system state to switch from s_i to s_j. As strategy revisions occur randomly according to a particular probability (and not all at the same time) and the population of DR agents is large, changes in the state and strategy costs occur in the overall system gradually due to these switches. The larger $\rho_{i,j}(F(x), x)$, the higher the rate that DR agents will switch from strategy s_i to s_j. It has been shown that the net change in the population state x_i when strategy revisions are made according to $\rho_{i,j}(F(x), x)$ is dictated by the mean dynamics [27]:

$$\dot{x}_i = \sum_{j=1}^{n} x_j \rho_{j,i}(F(x), x) - x_i \sum_{j=1}^{n} \rho_{i,j}(F(x), x). \tag{7}$$

The first and second terms of Eq. 7 represent the overall inflow rate of DR agents selecting s_i and the overall outflow rate of DR agents switching from s_i to other strategies, respectively. Although x_i is a random variable, since the population P is assumed to be large, the law of large number applies and we expect the population state will converge to the mean. The value taken by $\rho_{i,j}(F(x), x)$ depends on the particular revision protocol used by the DR agents. In this paper, three revision protocols with various degrees of information requirements and convergence characteristics are considered.

In the original problem formulation P_C, certain constraints were removed in order to decouple the EPU from consumer preferences and local appliance operating conditions. These local constraints are incorporated into the decision-making process of DR agents. If a DR agent i uses a particular revision protocol and selects s_j but local constraints are such that $y_j \geq min\{p_i, l_i\}$), then local power consumption will be reduced to $max\{y_k\} \leq min\{p_i, l_i\}$. Since it is assumed that the population is large and there exists sufficient capacity in the system, these deviations can be considered to be a minor reduction in the population size. Hence, the large scale of the DR problem is leveraged to accommodate limitations caused by local constraints. In Section IV, we investigate the impact of these restrictions on convergence and steady state behaviour.

The first type of revision protocol is *imitative* (I) [27]:

$$\rho^I_{i,j}(F(x), x) = x_j[F_i(x) - F_j(x)]_+ \tag{8}$$

A DR agent will randomly communicate with another *opponent* DR agent in the same population requesting the strategy the opponent is currently using. The probability that the DR agent will encounter an opponent using strategy s_j is x_j. The DR agent will then switch to the opponent's strategy with a probability that is proportional to the amount by which its current strategy cost exceeds the latter's strategy cost. The population state dynamic resulting from imitative revisions, derived by substituting (8) into (7), is:

$$\dot{x}_i = x_i(\bar{F}(x) - F_i(x)) \tag{9}$$

where $\bar{F}(x) = \sum_{i=1}^{n} x_i F_i(x)$ is the average cost incurred by all DR agents in the population. Eq. 9 is referred to as the *replicator dynamic*. When strategy s_i is completely unused (i.e., $x_i = 0$), then it becomes *extinct*. If the cost of the incumbent strategy is lower than the average cost of the population, then the rate at which that strategy is adapted is positive. Work in [14] proposes appliance scheduling via replicator dynamics and this is one of the closest works in the literature to that of this paper that uses EGT techniques for DR purposes. Hence, the replicator dynamic is our benchmark in this paper for comparison purposes.

The second type of revision protocol involves the *pairwise comparison* (PC) of strategies [27]:

$$\rho_{i,j}^{PC}(F(x), x) = [F_i(x) - F_j(x)]_+ \tag{10}$$

Evolutionary behaviour resulting from this pairwise comparison revision is called the *Smith dynamic* (Eq. 11) and is derived in a manner similar to the replicator dynamic.

$$\dot{x}_i = \sum_{j=1}^{n} x_j[F_j(x) - F_i(x)]_+ - x_i \sum_{j=1}^{n} [F_i(x) - F_j(x)]_+ \tag{11}$$

The conditional switch rate depends only on the cost of individual strategies. Unlike imitative revisions, even if strategies are not present in the system, the conditional switch rates to these will not be forced to 0.

The third revision protocol relies on *deficit costs* (DC):

$$\rho_{i,j}^{DC}(F(x), x) = [\bar{F}(x) - F_j(x)]_+ \tag{12}$$

A DR agent using s_i will switch to s_j at a higher rate if the cost of s_j is less than the average cost incurred by all DR agents in the system. This revision protocol results in *BNN dynamic* derived in a manner similar to the above:

$$\dot{x}_i = [\bar{F}(x) - F_i(x)]_+ - x_i \sum_{j=1}^{n} [\bar{F}(x) - F_j(x)]_+ \tag{13}$$

It is clear from the following table that the information requirement for the PC revision protocol is the least:

TABLE I
SUMMARY OF DISTRIBUTIVE DR SCHEME

Protocol	**Information Requirements**		
	Communicate with other DR agents	Cost of all strategies	Average cost of population
I	✓	✓	x
PC	x	✓	x
DC	x	✓	✓

Table I summarizes the distributed strategy selection process of each DR agent. In this algorithm, at the beginning of a day, the DR agent i initializes E_i (which represents the energy budget available to the consumer) with the total energy conservation c_i preferred by the consumer for a day and t_{cong} with the maximum time allocated by the consumer for conservation during a day. l_i is the sum of reducible power consumption by active appliances that have been given permission by the consumer to conserve (more details are provided in Section IV-B). Each DR agent randomly selects a strategy from y at the beginning of the first signalling. Then during congestion (as price signals are broadcast), a DR agent selects an exponentially distributed random time τ_i to revise its current strategy. When the revision time arrives, the DR agent updates $E_i,\ t_{cong},\ p_i$ accordingly and proceeds to revise its current strategy using the latest signal broadcast by the EPU. This is repeated until the current strategy cannot be further revised without incurring more cost and/or the exhaustion of comfort budget. The EPU stops broadcasting the cost signals when the optimal x^* is achieved by distributive strategy selections in the system. The EPU will be able to infer this via Assumption 2.

Distributed Algorithm for DR Agent i

Initialization:

- Current time: $t_{curr} \leftarrow 0$, $E_i \leftarrow c_i$, $t_{cong} \leftarrow t_i$
- Current strategy used: $s_c \leftarrow y(1)$

Algorithm (during congestion):

1) Compute τ_i using exponential distribution with rate 1. Set $t_{next} \leftarrow t_{curr} + \tau_i$ which is the next strategy revision time
2) While $t_{curr} < t_{next}$:
   - Set t_{curr} to the current internal clock time
3) Update:
   - $E_i \leftarrow E_i - s_c\tau_i$
   - $t_{cong} \leftarrow t_{cong} - \tau_i$
   - $p_i \leftarrow E_i/t_{cong}$

   Use the latest F and/or $\bar{F}$ broadcast by the EPU to select strategy s_c according to conditional switch rate defined by: Eq. 8, Eq. 10, or Eq. 12
4) $s_c \leftarrow \{max\{y\} | y \leq min\{s_c, p_i, l_i\}\}$
5) Go to Step 1

D. Convergence Properties

The state dynamic resulting from each revision protocol has various convergence characteristics. For instance, if the dynamic has limit cycle behaviour, the system may not converge to an equilibrium. On the other hand, when the system does converge, the equilibrium point may or may not be an NE. Two properties that are vital in identifying convergence characteristics for the three dynamics are *Negative Correlation* (NC) and *Nash Stationarity* (NS) defined as follows [27]:

$$\textbf{NC}: V_F(x) \neq 0 \text{ implies that} V_F(x)'F(x) < 0$$
$$\textbf{NS}: V_F(x) = 0 \text{ if and only if } x \in NE(F)$$

where V_F is the right hand side of the dynamical equation $\dot{x}$. NC indicates that the growth rate of the population state is negatively correlated with the corresponding costs and NS requires that all restpoints are precisely the NEs of the system. The NC and NS characteristics corresponding to the three dynamics considered in this work are listed in Lemma 1 and the proofs for these can be found in [27].

Lemma 1: For a full potential game, all dynamics resulting from the three revision protocols satisfy NC and the restpoints of the dynamics induced by the three revision protocols include the unique NE of the game ϑ.

Replicator dynamics resulting from imitative revisions will include restpoints that are not NE. For instance, suppose that one particular strategy is not used by any DR agent in the system. Since DR agents use imitation to switch strategies, the strategy not in use can never be imitated. Work in [27] shows that these equilibria are locally unstable and when the initial population state is an interior (i.e., $x \in \Delta$), the system always converges to the unique NE. Two other dynamics satisfy NS as the system equilibrium always coincides with the NE [27].

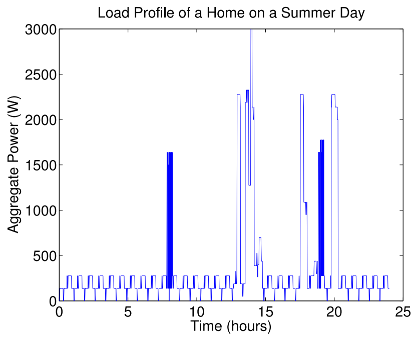


Fig. 1. Load profile in a home during a summer day.

Theorem 4: All three dynamics starting from any initial condition have unique trajectories that converge to the corresponding restpoints. Specifically, Smith and BNN dynamics converge to the global ESS.

Proof: The following holds for all three dynamics due to NC:

$$V_F(x)'F(x) \le 0 \Leftrightarrow V_F(x)'\nabla f(x) \le 0.$$

The above expression is equal to 0 only when x is one of the restpoints of $V_F(x)$. This is precisely the requirement for the existence of a strict Lyapunov function $L(x)$. From the above expression, it is clear that $L(x)$ is $f_d(x)$. Lyapunov theory can now be evoked to conclude that all three dynamics resulting from the potential game will always converge to its restpoints [31] with no divergent or limit cycle behaviours. Since the Smith and BNN dynamics satisfy NS, the trajectories of these systems will globally asymptotically converge to the unique NE which is also the global ESS. ∎

IV. Results

The theoretical properties introduced earlier on performance, convergence and robustness of the proposed DR strategy are evaluated in this section using MATLAB/Simulink for practical systems and models that mimic realistic scenarios.

A. System Modelling

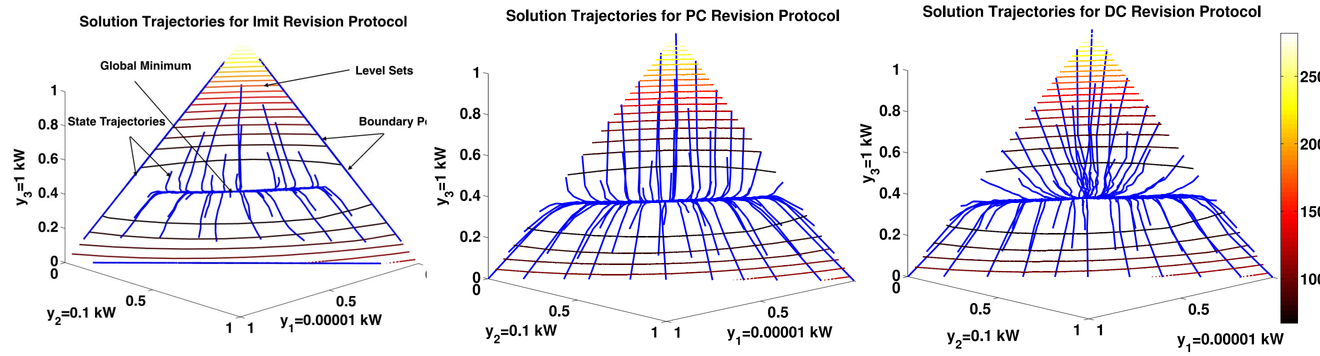
Household demands are influenced by many factors such as diurnal events, seasonal changes, penetration rates of appliances and appliance usage patterns. To capture the complexity encountered in a large system with many consumers and appliances, we consider DR for a residential neighbourhood consisting of 1000 homes. In [2] and [32], authors have summarized demand and appliance statistics for regions such as Ontario, France, India and Quebec to define penetration rates, power consumption patterns of common home appliances and probability of active appliance usage based on time of day and season. In our simulations, we use these parameters (specifically those provided in [2]) to generate demand profiles during the summer for homes located in a residential neighbourhood of Ontario consisting of washing machine, dryer, dishwasher, electric stove, oven, freezer, fridge, water heater and/or air conditioner. Fig. 1 illustrates the load profile of one such home over a day; reader is referred to [2] for more details.

B. Incorporating Consumer Preferences

Each participating home is fitted with a DR agent that can choose from one of three power commitment levels (i.e., $n = 3$) corresponding to $y = \{0.0001\ 0.1\ 1\}$ kW. As mentioned in Section II-B, we consider conservation of appliance operation for demand reductions. Consumers have the ability to incorporate their appliance operation preferences by configuring their comfort budget p_i and power conservation preferences into their DR agent. The comfort budget translates to the maximum energy reductions the consumer will tolerate per day. This ensures that the consumer will not be subjected to too much power reductions throughout the day. Consumers can also configure their preference on how their appliances should operate in conservation mode. In [2], authors have outlined that resistive heating elements in appliances typically consume the most power. Hence, in conservation mode, consumers can select options that reduce the activation of these heating elements. For instance, a washing machine can operate using cold water cycles. A dryer can operate in tumble drying mode. A dishwasher can operate using only cold water. An air conditioner can operate at a higher setpoint (on-cycles are less frequent) or not operate at all until the comfort budget is exhausted. The consumer may not want certain appliances to operate at reduced mode. In this case, the DR agent will update l_i (which represents the total active power currently available for reduction) at every signalling iteration by subtracting from the aggregate active power consumption in home i by the power consumption of all non-conservable local appliances that are active.

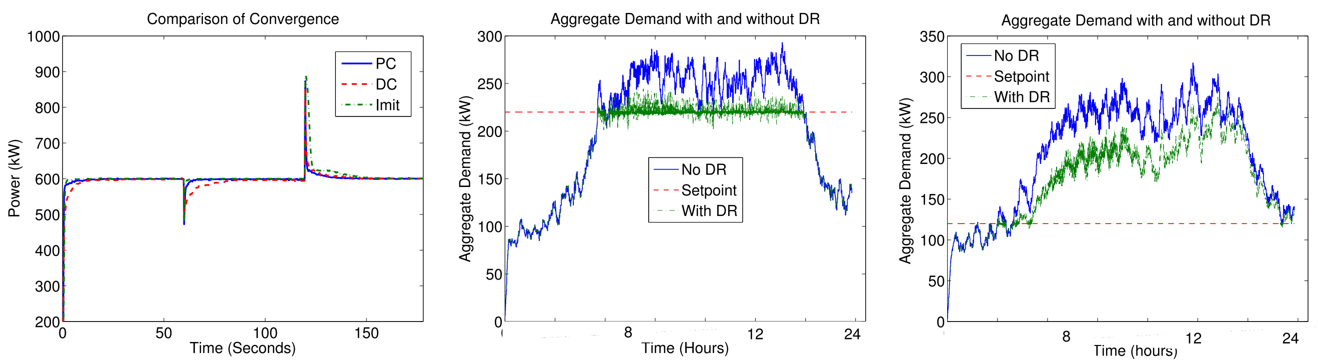
For simulations presented in this paper, the comfort budget allocated to every DR agent is 1 kW for a maximum of 1 hour per day. During conservation periods, appliances containing resistive components are assumed to operate according to the load profile information provided in [2] for the region Ontario with a slight modification. Resistive phases (i.e., time periods in the load profile in which a resistive element is activated) in these load profiles are configured in our simulations to randomly reduce power consumption by up to 90% during efficient operation modes. This captures various conservation operation modes manufacturers can equip appliances with. Air conditioners, fridges and freezers are appliances consisting of only inductive components. Fridges and freezers operate regularly (i.e., these do not reduce power consumption) and on the other hand air conditioners do not operate during congestion periods until comfort budgets are completely exhausted. These assumptions hold for most of our simulations with the exception of simulations conducted for obtaining results in Fig. 4b. For these simulations, we randomly vary the comfort budget, reduction of power drawn by resistive elements and frequency of on-cycles of inductive loads to assess the limitations of consumer preferences on aggregate demand reduction in the system.

EPU broadcasts cost signals every 0.6 seconds. A DR cycle lasts for 1 minute coinciding with the constant intervals of demand and supply. The cost function $f_o(x)$ represents the amount of compensation the EPU provides to a consumer for various levels of power reductions. This cost is completely



(a) Imitative Revisions (b) Pairwise Comparison Revisions (c) Deficit Cost Revisions

Fig. 2. Solution trajectories for the three revision protocols.



(a) Comparison of protocols for 3 cycles (b) Setpoint is 200 kW (c) Setpoint is 100 kW

Fig. 3. Ability of the proposed algorithm to maintain demands around a specific setpoint.

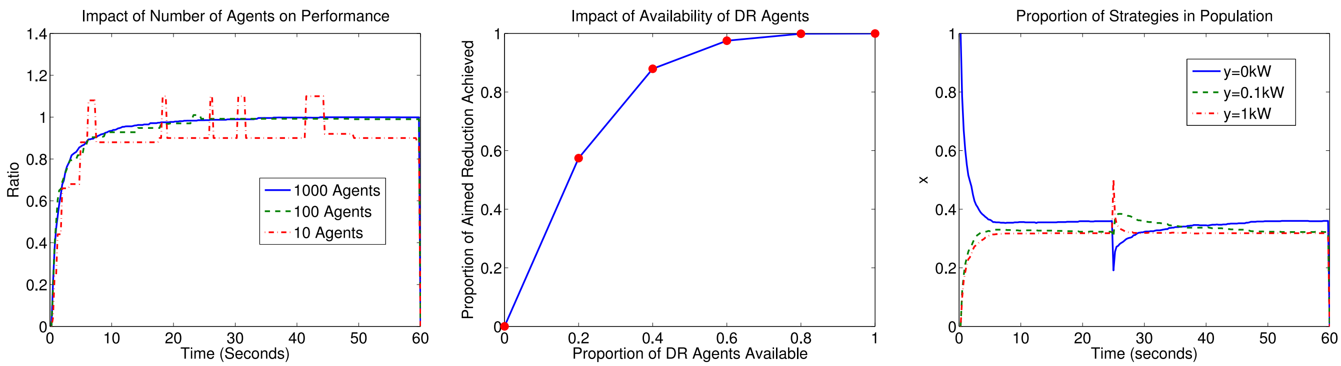
defined by the Y matrix according to the definition of $f_o(x)$ in Eq. 3. As Y is a diagonal matrix with strictly positive on-diagonal elements, strict convexity holds for $f_o(x)$. To preserve this structure, we select Y_{ii} to be a random strictly positive value that increases with i so that higher power reductions will lead to more penalty for the EPU.

C. Convergence Characteristics

First, we assess the impact of initial strategy distribution on the convergence behaviour of the three revision protocols. To explore all possible strategy distributions, it is assumed that all homes have been consuming more than 1 kW. Fig. 2a-2c, illustrate the resulting state trajectories for one DR cycle. State x remains within the simplex and exhibits no divergent or limit cycle behaviour. Trajectories generated by DC and PC revisions always converge to the optimal state $x = \{0.3636 \quad 0.3182 \quad 0.3182\}$ to achieve the expected aggregate reduction of 350 kW. Trajectories initialized at the boundary of the simplex for I revisions remain there and do not converge to the global optimum. This is not true for DC and PC revisions. This is expected as the replicator dynamic corresponding to I revisions does not satisfy NS whilst the Smith and BNN dynamics resulting from PC and DC revisions do satisfy NS as theoretically proven in Section III-D.

Next, the speed at which these systems descend to stationarity is investigated. Convergence speed is highest when system trajectories are orthogonal to the level sets of $f_d(x)$. From Fig. 2a-2c, it is apparent that PC trajectories are more orthogonal to the level sets of $f_d(x)$ than the other two systems. We consider aggregate demands during three consecutive DR cycles in Fig. 3a to gauge the adaptability of the DR system to changes in demands. As expected, PC revisions enable DR agents to rapidly respond to demand variations (a comparison with existing literature as discussed in Section IV-E illustrates this) and DC revisions result in the slowest convergence speed. In this paper, as mentioned earlier, the I protocol is our benchmark. It can be concluded that I revisions are not suitable for DR as it does not always converge to optimality. Also, the communication overhead for I protocol is high as DR agents are required to exchange information with one another to obtain the opponent's strategy whereby privacy can be compromised. For these reasons, only the PC protocol is considered in the remainder of this paper.

In our final convergence study, the DR system is examined for an entire day for two different aggregate load shedding goals. In Fig. 3b, the EPU is striving to maintain aggregate demands at 220 kW (i.e., peak-shaving attempt by the EPU to reduce power consumption when energy prices are high). Due to Assumption 9, the EPU will recompute C every minute so that even when there are fluctuations in demand and supply the overall demand in the system is maintained around the setpoint 220 kW. It is clear that all 1000 DR agents are able to distributively converge to the desired aggregate demand setpoint and consistently maintain this throughout the day. In Fig. 3c, the



(a) Setpoint is 600 kW (b) Availability of DR agents (c) System subjected to perturbation

Fig. 4. Performance of DR scheme under system limitations and perturbations.

demand setpoint is 120 kW. Excessive demand reduction by all DR agents is required to achieve this. As the comfort budget becomes exhausted, aggregate demand gradually increases until no more reduction is possible. As DR agents are still able to maintain the demand setpoint for a small fraction of the day, the EPU can use this DR strategy to buy time during contingency periods.

D. Robustness Under System Limitations and Perturbations

Figs. 3b and 3c illustrate that although our DR strategy is highly scalable, it is subject to limitations induced by the availability of comfort budgets and demands. In the following, various limitations and perturbations are applied to the system to evaluate the robustness of the proposed DR strategy.

First, we relax our assumption on the number of DR agents participating in the system and assess the impact of varying participants by one order of magnitude in Fig. 4a. This assessment is important as it allows us to gauge the impact of various reductions in population size that can occur when DR agents are unable to participate due to consumer preferences or inadequate local power demands. The ratio of actual and expected demand shed in the system remains at one when there are 100 or 1000 DR agents. However, when there are 10 DR agents, the system exhibits some oscillatory behaviour. This indicates that there is a tolerance of up to 90% reduction in population size for the convergence properties established in Section III-D to hold.

Next, we consider the effect of user preferences on the ability of the DR agents to meet EPU's load shedding goals. If a DR agent is unable to commit to a particular strategy without heeding comfort budget requirements, it will not participate. As outlined in Section IV-B, consumers may have various preferences for appliance operation in conservation mode. This can directly affect the cumulative effect of overall demand reductions. For instance, certain consumers may prefer increasing their thermal setpoint as a form of conservation and this translates to a decrease in the number of active cycles of air conditioners. Preferences such as these will directly impact the availability of DR agents for reduction in power consumption. In order to understand the limitations caused by consumer preferences, we have simulated the DR algorithm for various comfort budgets and appliance preference configurations that directly translate to the availability of DR agents. Supposing that the EPU has set a cumulative load shedding goal of 350 kW, results in Fig. 4b illustrate what fraction of this goal can be achieved for various availabilities of the DR agents. In this particular case, the EPU's goal can be achieved when almost 60% of the DR agents are able to participate. Hence, as consumer preferences can vary significantly, it is imperative that the EPU maintains a careful balance while setting demand reduction goals.

Our DR strategy relies extensively on cyber-physical interactions in the grid. A DR agent is a cyber-physical entity that has the ability to process communication signals to actuate loads under its control. Just like any other cyber device, DR agents are also prone to many well-documented vulnerabilities. A malicious adversary who successfully compromises a subset of DR agents through these vulnerabilities can perpetrate insidious attacks on the EPU. For instance, forcing these DR agents to choose expensive strategies incurs unnecessary costs for the EPU. Fig. 4c illustrates the evolution of states in a DR system where 20% of the DR agents (i.e., 200 DR agents) are forced to switch to the costliest strategy (i.e., 1kW) in the midst of a DR cycle. Unattacked DR agents are able to sense this discrepancy through cost signals transmitted by the EPU and react so that the system is able to recover immediately after the attack as indicated in Fig. 4c and return to the original optimal state. This example reinforces the validity of the theory introduced earlier which asserts that the global minimum is in fact an ESS for a system driven by the PC revision protocol. Massive system disruption is only possible if a large number of DR agents are compromised. More specifically, suppose that the number of DR agents compromised is c and these aggressively choose the most expensive strategy (i.e., $max(y)$). If $c > C/max(y)$, then it is not possible for other DR agents to re-adjust their strategies to offset the adverse effects of the attack as a surplus cost resulting from $c \times max(y) - C$ will always remain in the system. However, compromising DR agents to this extent is next to impossible as significant resources will be required to breach a vast number of geographically dispersed DR agents.

E. Performance Comparison With Other DR Schemes

Here, we compare the performance of the DR scheme proposed in this paper against other DR schemes.

We utilize population games in EGT to construct distributed DR. Work in [12] and [14] propose DR schemes that utilize EGT for distributed appliance re-scheduling over an extended period of time (i.e., not real-time). Reference [14] leverages a replicator dynamic based revision protocol for appliance scheduling throughout the day. We have implemented the imitative revision protocol that generates replicator dynamics used by authors of [14] for our real-time DR and this protocol serves as our benchmark. From the results presented in Section IV-C, we have concluded that PC revision results in better convergence and equilibrium properties which are suitable for real-time DR in comparison to I revisions. Work in [12], on the other hand, proposes day-ahead appliances scheduling via best-response dynamic. According to [27], there are two main issues with this dynamic. First, its convergence characteristics cannot be established using traditional analysis (similar to that presented in Section III-C) as the trajectories consist of differential inclusions. Secondly, as the state trajectories are not unique, the system state can cycle in and out of Nash Equilibria. As we are concerned with robustness and stability in our system, best-response dynamic is also not suited for real-time DR.

In order to directly compare the performance of our DR proposal with another distributed real-time DR scheme that attempts to meet goals similar to ours in the literature, we have implemented the DR technique outlined in the work authored by Deng *et al.* in [9]. This DR scheme utilizes a combination of dual decomposition, sub-gradient method and binary search to enable distributive and real-time DR. Like our proposal, authors of [9] have decoupled the main DR problem into master and slave problems which are solved by the EPU and DR agents respectively. However, unlike our proposal, the EPU iteratively improves cost signals based on the sub-gradient (SG) method in [9]. The convergence speed to optimality of distributed decisions made by DR agents depends on the step-size selected for the SG method. In order to directly compare the convergence properties, we have implemented this DR algorithm and our PC algorithm for the same system consisting of 1000 DR agents. Results are illustrated in Fig. 5. The original aggregate demand in the system is 800 kW and suppose that the setpoint decided upon by the EPU is 500 kW. Our algorithm enables convergence within a small number of iterations. However, the SG method exhibits very slow convergence for a step size of $\alpha = 0.0001$ and oscillatory behaviour for $\alpha = 0.0024$. Significant oscillations such as these may cause system and load damages. We have shown via extensive simulations and theoretical derivations that our PC algorithm always asymptotically converges to optimality with no cycling behaviour no matter what the initial conditions are and is therefore ideally suited for real-time and distributed DR.

For more general conclusions on our proposed DR strategy, we make comparisons with other existing DR schemes against four performance metrics in Table II. We compare our proposed strategy with three general classes of DR strategies which are centralized real-time, offline and decentralized real-time. In these classes, the main differences are the time horizons over which computations occur (i.e., hourly or day ahead intervals) and whether computations are conducted by a central entity or in a distributed manner by all participating entities.

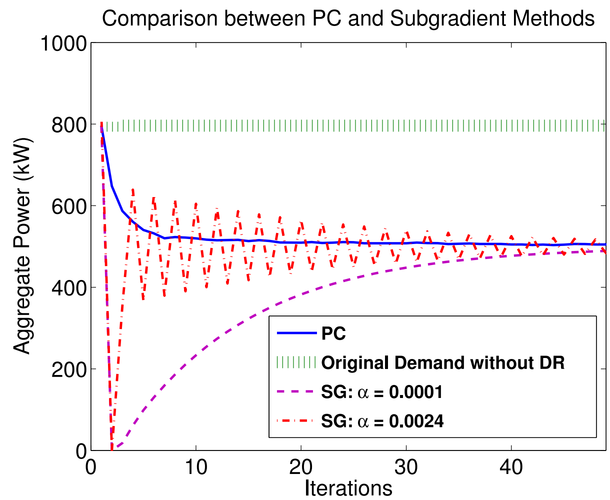


Fig. 5. Comparison between PC protocol and SG method.

TABLE II
COMPARISON OF DISPATCH METHODS

	Proposed Distributed	Centralized Real-time	Offline	Decentralized Real-time
Comm Cost (n DR Agents)	O(1)	O(n)	O(1)	O(n)
Forecast Error	0%	9.04%	28.23%	0%
Solution Optimality	Yes	No	Yes	Yes
Resilience	Yes	No	Yes	No

First, we consider the *communication cost* of DR schemes falling under these classes. The recent cyber-physical integration in the grid (supported by Assumptions 3 and 4 in Section II-A) enables various entities such as the EPU and DR agents to exchange information in order to facilitate DR. However, the more the information is exchanged, the greater will be the resulting communication overhead. In Table II, we use the O notation as this aptly describes the limiting communication cost in terms of n which is the number of DR agents in the system [33]. In our proposal, the communication overhead is constant (i.e., $O(1)$) and is independent of n for the following reasons. The EPU broadcasts the cost of strategies to all DR agents. As these costs $F(x)$ and/or $\bar{F}(x)$ are the same for all DR agents, these can be broadcast to all DR agents. Since communication occurs over the wireless channel for our proposal (according to Assumptions 3 and 4) and this is a broadcast medium, there is no need for the EPU to form individual downlink connection with every DR agent [34]. A single general broadcast of the cost to all DR agents is sufficient. If the communication network is not dedicated to only the DR participants, then additional mechanisms such as encryption will ensure that this broadcast can be deciphered only by authorized entities (i.e., DR agents). On the other hand, DR agents do not initiate communications with the EPU as these use the latest cost information transmitted by the EPU to make DR decisions based on local constraints. Moreover, as we consider only PC revisions after Section IV-C, no information exchange is required between the DR agents.

This is only required for I revisions which we do not consider due to undesirable properties as extinction identified in Sections III-D and IV-C for this revision protocol. Hence, communication is uni-directional and only occurs in the downlink for our strategy. For this reason, the communication cost is constant $O(1)$.

Centralized real-time schemes such as that proposed in reference [7] depend on updates from participating DR agents about local state (e.g., temperature, appliance operation, etc.) for computations. Also, as the central coordinating entity transmits signals containing information about how to adjust various local configurations of appliances (e.g., temperature setpoint) for every DR participant, these are specifically tailored for every DR agent and therefore cannot be broadcast to all participants. As communication occurs between the EPU and the DR agents in both the uplink and downlink directions, at least $2n$ information exchanges are necessary resulting in the communication cost being $O(n)$. Offline strategies, such as the day-ahead pricing schemes referenced in [11], use only forecast models (i.e., no need for information from consumers) to compute price of electricity. This price is common to all participating consumers and can be broadcast. Hence, the communication cost is constant $O(1)$. Finally, decentralized real-time schemes such as the consensus-based strategy proposed in reference [8] require information exchanges between participating agents. As presented by the authors of reference [8] themselves, this cost is $O(n)$.

Dispatch error from forecast is the second performance metric that we consider. This error is non-existent in the proposed and decentralized real-time schemes as prediction models are not employed. Centralized real-time schemes are typically conducted over hourly intervals and offline schemes are based on day-ahead forecasts. Errors resulting from these prediction horizons are calculated based on [3].

Solution Optimality is the third performance metric considered. This is always guaranteed for the proposed DR strategy when PC protocol is in effect as shown earlier via theoretical proofs and simulations. This is also the case with decentralized real-time solutions as proven in reference [8]. Centralized real-time solutions may not be optimal as overcoming computational overhead which increases exponentially with the number of participants can be difficult with time limitations. On the other hand, as offline schemes typically use facilities equipped with powerful computational resources with no time pressure, optimality is feasible.

Resilience is inherent in our DR strategy as shown via theory and simulations. Decentralized systems are vulnerable as false information can be propagated by compromised agents to other agents which can result in incorrect decisions. Centralized schemes that depend on bi-directional information transfer are subject to single point of failure attacks (attack on the central coordinating entity) or false information attacks on measurement data sent by consumers. Offline schemes, on the other hand, depend on forecast models for the computation of pricing signals and therefore are not dependent on feedback from participants. As pricing information can be made available from multiple sources, issues with single point of failure can be averted as well.

V. Conclusion

In this paper, we have proposed a distributed real-time DR strategy that harnesses the cyber-enabled vision of the grid to facilitate a highly scalable, flexible and versatile DR program that efficaciously meets EPU goals while encompassing a large range of consumer preferences. Resilience is integrated into the core of the strategy design rendering it an inherently robust and secure solution. EGT is leveraged to model distributive interactions between participants by constructing an elegant dynamical systems framework from which we have established important convergence and optimality properties. Comparative analysis shows that our scheme outperforms state-of-the-art DR solutions as it incurs minimal communication and computational overhead while efficiently utilizing system resources due to reduced prediction errors. We therefore assert that the proposed scheme has potential for practical implementation to minimize grid overload and encourage grid sustainability while also preserving consumer satisfaction. As future work, we intend to analyze our proposed strategy for smart home energy management systems that allow both energy conservation and appliance rescheduling.

Appendix

In this Appendix, a proof of Theorem 2 is provided. The Lagrangian of P_D and the associated first order Karush-Kuhn-Tucker (KKT) conditions necessary for the minimization of $f_D(x)$ are as follows:

$$L(x, \mu, \lambda) = f_D(x) + \mu\left(1 - \sum_{i=1}^{n} x_i\right) - \sum_{i=1}^{n} \lambda_i x_i$$

(1) Stationarity: $\frac{\partial L}{\partial x_i} = 0 \rightarrow F_i = \mu + \lambda_i \ \forall\ i \in S$

(2) Primal Feasibility: $\sum_{i=1}^{n} x_i = 1,\ x_i \geq 0 \ \forall\ i \in S$

(3) Dual Feasibility: $\lambda_i \geq 0 \ \forall\ i \in S$

(4) Complementary Slackness: $\lambda_i x_i = 0 \ \forall\ i \in S$

x^* satisfying the KKT conditions is the global minimum of P_D as this problem is a strictly convex optimization problem. It can be shown that any x^* satisfying the KKT conditions also satisfies the necessary and sufficient conditions needed for an NE. For all strategies in use (i.e., $x_i > 0$), condition 4 requires that the corresponding $\lambda_i = 0$. This results in condition 1 of all strategies i that are in use reducing to $F_i = \mu$. This implies that the cost of all of these strategies in use are the same and is μ. For all other strategies j not in use, from condition 1 $F_j = \mu + \lambda_j$ and it is clear that $\mu + \lambda_i \geq \mu$ as $\lambda_i \geq 0$ due to condition 3. As the cost of these strategies are greater, these are not in use. This shows that the cost incurred by the incumbent strategies are indeed minimal and the same. Hence the unique x^* satisfying the KKT conditions is also the NE of the game ϑ.

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Pirathayini Srikantha (GSS'14) received the B.A.Sc. degree in systems design engineering with distinction—Dean's Honors List–and the M.A.Sc. degree in electrical and computer engineering from the University of Waterloo, Waterloo, ON, Canada, in 2009 and 2013, respectively. She is currently pursuing the Ph.D. degree with the Edward S. Rogers Sr. Department of Electrical and Computer Engineering, University of Toronto, Toronto, ON, Canada.

Her current research interests include investigating how effective solutions can be designed for current applications in the electric smart grid that such as cyber-security, sustainable power dispatch, demand response using convex optimization, and game theoretic techniques.

Ms. Srikantha was a recipient of the Best Paper Award Recognition at the Third IEEE International Conference on Smart Grid Communications in 2012 for the Symposium of Demand-Side Management, Demand Response, and Dynamic Pricing.

Deepa Kundur (S'91–M'99–SM'03–F'15) received the B.A.Sc., M.A.Sc., and Ph.D. degrees in electrical and computer engineering from the University of Toronto, Toronto, ON, Canada, in 1993, 1995, and 1999, respectively.

From 1999 to 2002, she was a Faculty Member at the Edward S. Rogers Sr. Department of Electrical and Computer Engineering, University of Toronto, where she is currently a Professor and the Director of the Centre for Power and Information. She also currently serves as the Associate Chair of the Division of Engineering Science. From 2003 to 2012, she was a Faculty Member of Electrical and Computer Engineering at Texas A&M University, College Station, TX, USA. She is also a recognized authority on cyber security issues and has appeared as an expert in popular television, radio, and print media. She has authored over 150 journal and conference papers. Her current research interests include cyber security, signal processing, and complex dynamical networks.

Prof. Kundur was a recipient of the Best Paper/Poster Recognitions at various venues, including the 2015 IEEE Smart Grid Communications Conference, the 2015 IEEE Electrical Power and Energy Conference, the 2012 IEEE Canadian Conference on Electrical and Computer Engineering, the 2011 Cyber Security and Information Intelligence Research Workshop, and the 2008 IEEE INFOCOM Workshop on Mission Critical Networks, and the Teaching Awards at both the University of Toronto and Texas A&M University. She has participated on several editorial boards and is currently on the Advisory Board of IEEE SPECTRUM. She currently serves as an Associate Editor of the IEEE TRANSACTIONS ON INFORMATION FORENSICS AND SECURITY. She serves as the General Chair for Workshop on Communications, Computation, and Control for Resilient Smart Energy Systems at ACM e-Energy 2016. She served as the General Chair for the IEEE GlobalSIP 2015 Symposium on Signal and Information Processing for Optimizing Future Energy Systems, the 2015 International Conference on Smart Grids for Smart Cities, and the 2015 Smart Grid Resilience Workshop at IEEE GLOBECOM 2015.