Real-Time Integration of Intermittent Generation With Voltage Rise Considerations

Pirathayini Srikantha, *Student Member, IEEE*, and Deepa Kundur, *Fellow, IEEE*

***Abstract*—In the modern electric power grid, a commonly observable recent phenomenon is the increasing penetration of renewable generation sources especially at the distribution network (DN) level. The traditional DN is not designed for bidirectional power flow induced by these volatile sources and, therefore voltage rise is a major concern. In order to enable mass renewable integration into any type of existing radial DN without requiring expensive line/bus upgrades and avoiding adverse effects of voltage rise, these generation sources (with possible nonconvex discrete output levels) must be dispatched in real-time while taking into account nonconvex voltage constraints. Ubiquitous connectivity between power components is available in today's grid due to the cyber-physical nature of these devices. We leverage this to propose a distributed algorithm based on principles of population games for efficient dispatch that minimizes dependence of the DN on the main grid for sustainable system operation. Theoretical and simulation studies show that the proposed algorithm allows for the seamless coexistence of a large number of renewables that are highly responsive to fluctuations in demand and supply with strong convergence properties while successfully mitigating voltage rise issues.**

***Index Terms*—Distributed algorithms, optimization methods, power generation dispatch.**

I. Introduction

A COMMONLY observable recent phenomenon has been the broad deployment of small-scale sustainable generation systems such as roof-top solar panels, micro wind turbines and energy storage (e.g., electric vehicles) that connect to the power grid via a distribution network (DN). A traditional DN is not designed for two-way power flow. Back flow of power resulting from excess generation of renewables can lead to voltage rise across DN buses. As the penetration of micro-generation systems in the DN continues to increase, voltage rise across the buses can easily cross acceptable thresholds resulting in adverse equipment effects. Hence, voltage rise is one of the foremost concerns for Electric Power Utilities (EPUs) with heavy DG integration.

Manuscript received April 3, 2016; revised July 22, 2016, September 8, 2016, and November 13, 2016; accepted November 19, 2016. Date of publication November 23, 2016; date of current version June 17, 2017. This work was supported in part by grants from the National Science Foundation, in part by the Natural Sciences and Engineering Research Council of Canada, in part by the Natural Sciences and Engineering Research Council of Canada, and in part by the Hatch Graduate Scholarship Program for Sustainable Energy Research. Paper no. TSTE-00252-2016.

The authors are with the Department of Electrical and Computer Engineering, University of Toronto, ON M5S 3G4, Canada (e-mail: pirathayini.srikantha@ece.utoronto.ca; dkundur@ece.utoronto.ca).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSTE.2016.2632116

The steady state relationship between power flow and bus voltages based on line characteristics, local demand and supply is governed by the Kirchoff's voltage and current laws. Minimizing generation costs while satisfying constraints imposed on the underlying physical electrical attributes of the DN system can be formulated as an Optimal Power Flow (OPF) problem. One main challenge associated with directly solving the traditional OPF is in the non-convexity of the problem. There is an extensive body of work in the current literature that attempts to overcome this challenge by applying relaxations that convert the OPF into a convex problem (e.g., Semi-Definite (SD) and Second-Order Cone (SOC) relaxations) [1]–[5]. These references also show that the associated relaxations are exact only when specific conditions are met in a radial DN. One example of such a condition is load over-satisfaction (i.e. when loads can accept supplied real and reactive power that is greater than the actual demand [3]). However, if these conditions are not satisfied, then the resulting solution can be infeasible. In our paper, we consider a more generalized framework where we do not impose these conditions on the radial DN (i.e. take into account non-convex voltage constraints). Hence, we do not apply any relaxations to the OPF. Moreover, in order to include generation sources with discrete output levels, we consider a discrete optimization variable space. This further adds to the non-convexity of the OPF. The authors of [6] employ the optimal solution of the relaxed problem as an initial starting point of a heuristic algorithm for the non-convex OPF. However, theoretical convergence results are not available for heuristic approaches.

Another challenge with DG dispatch is the highly fluctuating nature of renewable generation capacities. Treating these as undispatchable sources can lead to bus voltages rising to unacceptable levels, which becomes especially the case when renewable penetration is high. Traditional central optimization techniques such as dynamic programming have been applied to compute DG dispatch in existing work (e.g., reference [7], [8]) which utilize forecast models to estimate generation capacities of DGs. However, these prediction models are associated with high error margins [9]. On the other hand, if DGs continuously send updates of current generation capacities to a central controller, excessive communication overhead can result. This can render centrally solving the OPF impractical for real-time integration of a large number of DGs, even if it is relaxed to a convex problem. Distributed methods have been proposed in the literature [10], [12]–[16]. Some of these proposals solve simplified economic dispatch that do not take voltage constraints into account. However, even with the omission of the non-convex constraints, these methods have been proven to converge asymptotically (i.e., not necessarily exponentially) to optimality and this convergence speed may not be suitable for real-time dispatch. References [10] and [11] apply decomposition techniques like our proposal but for relaxed physical network flow constraints and propose a distributed algorithm based on Alternating Direction Method of Multipliers (ADMM). Other proposals in the literature that are decentralized use only local bus voltage measurements to compute dispatch which may not fully utilize available generation potential due to incomplete information [17].

Thus, in order to effectively harness the potential of intermittent generation sources while suppressing the adverse physical issues that accompany mass integration, we propose a distributive two-tier dispatch algorithm that, by design, enables the seamless coexistence of a large number of distributed energy sources. The first tier consists of a decomposable convex optimization problem that aims to minimize dependence of the DN on the external grid which is typically composed of expensive and unsustainable power plants. The objective of this *master* problem is met in a distributed manner by intelligent agents that reside in the DGs. Evoking principles from population games, we show via a formal proof that the distributive dispatch strategy selections by DG agents result in exponentially fast convergence to optimality of the master problem when the overall generation capacity in the DN is adequate. Tier-two consists of a set of *secondary* problems in which every DG agent checks whether a dispatch strategy is locally feasible (i.e., meets local generation capacity and voltage rise constraints). The ubiquitous communication capability between physical nodes in the DN is leveraged to enable coordination amongst the agents to address the non-convex voltage rise constraints. We show that this feasibility check is constant in complexity via insights drawn from the underlying tree structure of the radial DN. We demonstrate via theoretical and empirical analysis that this two-tier distributed algorithm enables the system to rapidly adapt to fluctuations in demand and supply within the DN while maintaining voltages across the buses within acceptable limits.

Like our proposal, the existing literature consists of many works that also leverage game theory to allow for distributed dispatch and perform voltage control via reactive power support [17]–[20]. Our proposal significantly differs from these as we first demonstrate both practically and theoretically that the convergence of our algorithm at the first tier (i.e. economic dispatch) is exponentially fast especially in the presence of a large number of DGs. Also, voltage feasibility check is in the order of the height of the deepest feeder in the DN. These characteristics render our proposal highly suitable for real-time management of distributed energy sources. Moreover, our algorithm does not require participating agents or the EPU to be aware of the entire system topology as only local (for the agents) or aggregate (for the EPU) information is necessary. This enables plug-and-play integration of DGs along with enhanced privacy.

Thus, our contributions are three-fold in this paper: 1) We propose a novel distributed dispatch algorithm that takes into account non-convex constraints governing voltage rise issues; 2) We provide theoretical analysis and bounds to demonstrate how the proposed algorithm enables a large number of agents to converge to the optimal dispatch strategy in real-time; 3) We present simulation studies evaluating the efficacy of the proposed algorithm when applied to a low-voltage Danish DN consisting of 34 buses supplying power to 75 homes. The remainder of this paper is organized as follows. In Section II, an overview of the system settings and associated challenges with the traditional OPF problem is discussed. Then, the proposed dispatch algorithm along with theoretical analysis is provided in Section III. Section IV presents an implementation of the proposed algorithm via an empirical study with final remarks in Section V.

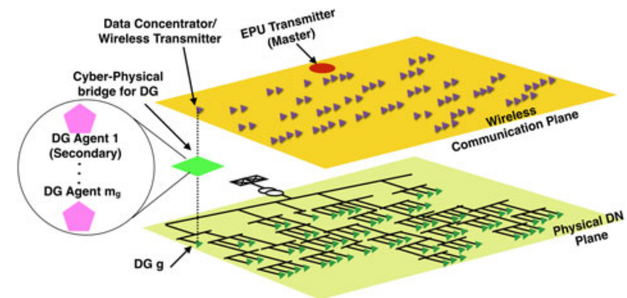


Fig. 1. System Model.

II. SYSTEM SETTINGS AND MODEL

In this paper, we consider a low-voltage DN governed by an EPU that consists primarily of residential consumers within a neighbourhood and a large number of micro-generation systems. Traditionally, DGs such as solar and wind energy sources are considered to be undispatchable as these have highly intermittent generation capacities influenced by external factors (e.g., cloud cover, wind speed, etc.). Computing economic dispatch for renewables in typical day-ahead markets by evoking prediction models is not common practise as forecast errors introduced by these models, especially across longer prediction horizons, are not negligible [9]. In this paper, we propose a distributed algorithm that solves, in real-time, an OPF problem that is implicitly reformulated every minute. Constraints in the OPF formulation consisting of generation capacity limits are updated every minute - thereby allowing us to accurately capture the intermittencies of these. This real-time approach allows us to generalize DGs and consider these to be dispatchable entities having generation potential that varies every minute. This extension that enables power dispatch capability to all DGs is supported by recent advances in inverter design. An inverter serves as an interface between a DG and the power grid. As noted by reference [21], these have virtually unnoticeable ramp up and down rates (in the order of milliseconds) and therefore is practical for our proposal.

Our set up is analogous to a grid-connected cyber-physical microgrid as illustrated in Fig. 1. Intelligent agents residing in every DG infer local generation capacity from decentralized measurements and utilize signals broadcast by the EPU every second to determine appropriate DG power output levels that heed system limits and also strive to balance overall demand with available supply. Excessive generation can be directed into storage systems, which are also considered to be DGs, for use during high demand periods that may occur later. These assumptions are formally presented in the following section.

A. Assumptions

We assume the following of our system model:

1) The EPU has access to data concentrators;
2) The EPU broadcasts signals every 1 second;
3) The real power of DGs is dispatchable at discrete levels;
4) DGs are equipped with real-time measurement devices;
5) DGs consist of intelligent agents that can communicate with one another via cellular transmitters and receivers;
6) Load and supply remain constant for 1 minute intervals;
7) Steady-state properties of the DN are considered;
8) The number of DGs deployed in the DN is large;
9) The DN is grid connected.

Assumption 1 is a commonplace practice within Advanced Metering Infrastructure (AMI). For Assumption 2, communication delays in broadcast wireless across a residential neighbourhood spanned by the DN is typically in the range of microseconds, thus making it feasible. In the third assumption, DGs adjust power dispatch at discrete levels to enable the modeling of storage devices such as EVs or batteries available in today that supply power at discrete levels [22]. As we consider significant penetration of DGs in the DN, we treat DGs as dispatchable entities in this paper so that excessive generation does not result in voltage rise issues. Excessive local generation can be redirected to storage systems such as that recently commercialized by reference [22] for later use during demand peaks. Discrete dispatch also allows us to apply population game theoretic constructs for solving the dispatch problem. Phasor Measurement Units (PMUs) support Assumption 4 by providing the local grid state information. Assumption 5 is in line with the cyber-physical vision of the smart grid. The sixth assumption supports our goal for real-time dispatch; existing dispatch research [23] is designed for time scales ranging from 10 minutes to hourly intervals. Forecast models that are highly accurate for small time horizons such as the one considered in this paper can be evoked to infer renewable generation capacities and demands over these one minute intervals [9]. In Assumption 7, we consider only the steady-state voltage behaviour in the system as transients are considered negligible due to non-simultaneous revisions of local strategies (please see Section III for further details). Assumption 8 is consistent with the notion of mass DG integration at the DN level. The final assumption allows for reactive power balance in the DN to be accounted for by the main grid.

B. Original Problem

The goal of DG dispatch is to minimize the dependence of DN consumers on the main grid for real power while accounting for physical grid constraints. We formulate this OPF problem as $\mathcal{P}_C$. Here, we consider two types of generation sources in the DN: the DGs that collectively form a set $\mathcal{G}$ and the main grid m such that p_g and p_m denote the power generated by a DG $g \in \mathcal{G}$ and the main grid m, respectively. The former may take values only from a discrete set. All power consumers in the DN form a set $\mathcal{D}$ and the real (reactive) demand from each $d \in \mathcal{D}$ is given by p_d (q_d). All lines and buses in the DN form the set $\mathcal{E}$ and $\mathcal{B}$, respectively such that $a \leftrightarrow b \in \mathcal{E}$ denotes a line between Bus a and Bus b. $Y_{a,b}$ is the corresponding component of the DN admittance matrix. V_a is the voltage at Bus a. H is a conjugate transpose operator. p_a^B (q_a^B) denotes the overall real (reactive) power injected into Bus a such that $p^l_{a,b}$ ($q^l_{a,b}$) is the real (reactive) power loss in line $a \leftrightarrow b$.

$$(\mathcal{P}_C) \ \min_{P,p_m} f_o(P, p_m)$$

$$\sum_{d\in D} p_d = p_m + \sum_{g\in\mathcal{G}} p_g - \sum_{a\leftrightarrow b\in\mathcal{E}} p^l_{a,b} \tag{C1a}$$

$$\sum_{d\in D} q_d = q_m - \sum_{a\leftrightarrow b\in\mathcal{E}} q^l_{a,b} \tag{C1b}$$

$$p_a^B + iq_a^B = \sum_{a\leftrightarrow b\in\mathcal{E}} Y^H_{a,b}(V_a^H - V_b^H)V_a \ \forall\, a, b \in \mathcal{B} \tag{C2}$$

$$\underline{V_a} \le |V_a| \le \overline{V_a} \quad \forall\, a \in \mathcal{B} \tag{C3}$$

$$0 \le p_g \le c_g \ \forall\, g \in \mathcal{G} \tag{C4}$$

The objective of $\mathcal{P}_C$ is to minimize the cost of DG power generation $P \in \mathbb{R}^{|\mathcal{G}|}$ and power commissioned from the main grid p_m. The gth element of P represents the real power generated by DG $g \in \mathcal{G}$. Since $f_o(.)$ is typically quadratic for economic dispatch, we adapt this consistent structure for this paper [23]. (C1a) and (C1b) are referred to as the coupling constraints and require that the aggregate real/reactive power generated matches the aggregate demand after accounting for power losses in the DN. (C2) specifies the overall power injected into every bus due to Kirchoff's voltage and current laws and is referred to as the bus injection model [1]. (C3) sets the upper and lower bounds ($\overline{V}_a$ and $\underline{V}_a$) on the voltage magnitude at each bus a. (C4) limits the real power generated by each DG g to its generation capacity c_g.

Based on Assumption 6, $\mathcal{P}_C$ represents a real-time optimization problem in which generation capacities and consumer demands vary every minute. Hence, the goal of this paper is to efficiently solve $\mathcal{P}_C$ every minute in order to take into account the intermittencies of generation and supply. Changes in renewable generation capacities and consumer demands are updated at a minute-by-minute basis. For notational convenience, we do not include an index to indicate the dispatch period in the formulation as updates of these parameters are implicitly accounted for in our proposed algorithm. Main challenges in directly solving this OPF are (C2) which is non-convex and the possible discrete nature of the optimization variables p_g. Applying convex relaxations to the OPF will lead to the exact solution under specific conditions on the DN. As we aim to propose a solution with general applicability to any radial DN with a tree configuration, we do not apply these relaxations.

III. Distributed Dispatch Strategy

Our system consists of a large number of participants (i.e. DGs) with a common set of discrete strategies (i.e. Assumption 3). The optimization problem that we consider (i.e. $\mathcal{P}_C$) consists of a coupling constraint that instills a balance between overall demand and aggregate supply and a set of separable constraints (i.e. generation capacities and voltage limits). We apply a decomposition and divide the problem into *master* and *secondary* tiers. The participants (i.e. DG agents) revise their strategies based on the costs of these strategies (i.e. cost signals transmitted by the EPU). These revisions induce a state trajectory that moves the system to an equilibrium exponentially fast and we show that this equilibrium is in fact the optimal solution of the master problem under certain conditions. The secondary tier consists of local constraint feasibility checks conducted by a revising DG agent and these serve to limit the set of strategies available to the agent as necessary. Thus, we are able to establish a bridge between optimization and population game theory for general problems such as that considered in this paper and show that desirable convergence traits can be achieved via a distributed approach.

More specifically, the master tier is akin to the economic dispatch problem in which local voltage constraints are not taken into consideration. With a change of variables, this is converted into a strictly convex optimization problem that is solved distributively by DG agents based on common cost signals broadcast every second by the EPU. These signals are computed by the EPU by capitalizing on the master problem structure and using real-time aggregate measurements (Assumptions 1 and 2). The second tier involves a local check whereby every time a DG agent makes a dispatch revision based on the cost signals, it ascertains whether this change is locally feasible in terms of local generation capacity and voltage rise in local buses. In our two-tier approach, EPU signals indirectly guide DG agents in selecting appropriate dispatch strategies that enable rapid convergence of the system to the optimal master problem solution while also ensuring that local physical constraints are heeded.

A. Decomposition of OPF Problem

The goal of decomposition is to decouple the EPU computational tasks from the local physical constraints in $\mathcal{P}_C$. This is achieved by moving the decision-making related to constraints (C2) to (C4) in $\mathcal{P}_C$ to the DG agents themselves. This decomposition significantly reduces communication and computational overhead for the EPU as highlighted later in this section.

Consider the gth DG. Every physical DG can consist of one or more DG agents as this allows for parallel processing. Specifically, each DG agent i of this DG will use a *revision protocol* to select one of n power levels $p_g^i \in y = [y_1 \dots y_n])$ based on the current cost signal transmitted by the EPU (Assumption 3) as long as this change heeds constraints (C2) to (C4) in $\mathcal{P}_C$. These power levels will be predefined and remain fixed for all DG agents in accordance to typical population game theory construction (i.e. all player have at their disposal the same number and type of strategies). Precision in dispatch enabled by these levels should match that of typical DG power rating. The number of DG agents simultaneously active in the DG, denoted m_g, depends on the maximum generation capacity c_g of the DG (i.e. $m_g = \lfloor c_g / y_n \rfloor$). This flexibility in the number of agents active in a physical DG enables the overall power dispatched by the physical DG via the simultaneous operation of multiple DG agents to reach various generation output values between 0 to the current generation capacity c_g of the DG. Hence, continuous dispatch is mimicked via the activation of multiple discrete agents making incremental strategy decisions. Thus, the overall power dispatched by the physical DG g is $p_g = \sum_{i=1}^{m_g} p_g^i$. Moreover, allowing for multiple DG agents to exist per physical DG tailors the dispatch problem to fit into the population game theory framework (i.e., requirement of a large number of players with the same set of strategies at their disposal). This also enables the distribution of computational overhead amongst multiple agents, prevents surges in voltages via incremental dispatch decisions made by agents and provides means for tractable theoretical analysis by evoking stochastic theory for a large number of independently acting entities (i.e. agents). The reader is referred to Section III-C for details on the revision protocol, local feasibility check and theoretical analysis.

The remaining objective $f_0(\cdot)$ and the coupling constraint form the master problem to be solved by the EPU. We select $f_0(\cdot)$ to be $\sum_{L=1}^{n} C_L.(\sum_{g=1}^{G} \sum_{i=1}^{m_g} p_g^i.\mathbb{I}_L(p_g^i))^2 + C_m.p_m^2$ where $C_L < C_{L+1}$ is the cost of power level $y_L \in y$, $\mathbb{I}_L(y_j)$ is an indicator function that returns 1 when $y_j = y_L$, $G = |\mathcal{G}|$ and the operator $a.b$ represents multiplication of the scalars a and b. This cost function is quadratic in terms of the sum of the overall power selected by DG agents for each level in y. Although this cost function is similar in structure to objectives in economic dispatch problems of conventional generation systems [23], we utilize this model to prevent concentrated generation that can otherwise cause saturation of voltage limits and prevent other DG agents at lower levels of the congested branch from supplementing available power at times of need. This also avoids unnecessary system losses. Moreover, this quadratic cost function allows for decomposability that we leverage in this paper to enable real-time convergence of the system to optimality. This cost structure penalizes a DG agent selecting a larger dispatch level as the quadratic term associated with the larger dispatch will be more dominant in the cost computation. Hence, this evenly distributes the dispatch responsibility to all DG agents in the DN thereby allowing resources to be directed towards local demands that occur currently or in the future (i.e. direct excessive generation to storage systems). C_m is the cost of power p_m obtained from the main grid. This is selected to be much larger than the cost of power supplied by the DGs (i.e. $C_m >> C_L \; \forall \; L = 1 \dots n$) so that our goal of minimizing dependence on the main grid for power supply can be implicitly incorporated into $\mathcal{P}_C$.

In our system model, we consider the DN to be a grid-connected microgrid system with *significant* penetration of DGs. As we are interested in how DGs can be actuated so that the DN is independent of the main grid for real power, we set $p_m = 0$ in the master problem. In case of a deficit in local supply, the DN will automatically draw power from the main grid so that uninterruptible services can be provided to local consumers. Hence, the power supply from the main grid is not directly controlled but serves as an ancillary service. Optimization of $f_0(\cdot)$ requires information about individual power dispatched by the DGs p_g^i.

As the EPU is concerned only with the aggregate system behaviour, we introduce a change of variables x where its ith component x_i represents the proportion of DG agents in the DN selecting power dispatch level i. As such x takes on values in the simplex $\triangle = \{x | x_i \geq 0 \, \forall \, i = 1 \dots n, \, \sum_{i=1}^{n} x_i = 1\}$ which reflects the fact that DG agents will necessarily select one of n dispatch levels and thereby making the sum of all x_i unity. As we are considering real power dispatch, coupling constraint (C1b) in $\mathcal{P}_C$ is not included (since reactive power q_m is supplied by the main grid). Moreover, as the DN is grid-connected, any line losses in the DN are supplemented by the main grid. The resulting master problem $\mathcal{P}_M$ becomes:

$$(\mathcal{P}_M)\ \min_{x \in \triangle} f_o(x) = \sum_{i=1}^{n} C_i.(m.y_i.x_i)^2$$

$$\text{s.t.} \sum_{i\,=\,1}^{n} m.y_i.x_i = \sum_{d \in D} p_d \qquad \text{(C1)}$$

where $m = \sum_{i=1}^{G} m_i$ is the total number of agents in the system. In this problem, the power dispatched by the DGs exactly match the overall demand in the DN. After this match is established, any power that is still drawn by the DN from the main grid will account for the line losses in the DN. The cumulative demand in constraint (C1) in $\mathcal{P}_m$ can be implicitly measured by the EPU (Assumption 1). The objective $f_o(x)$ and (C1) in $\mathcal{P}_m$ is equivalent to $f_o(P)$ and (C1a) in $\mathcal{P}_c$ when $m \to \infty$. Error resulting in the non-limiting case is $\pm 1/m$. Due to Assumption 8 and with the introduction of multiple DG agents per physical DG, m will be a large value and thus the error will be insignificant.

Given that the new optimization variables x_i are continuous, the objective function is strictly convex and constraints are linear, $\mathcal{P}_m$ is a strictly convex optimization problem with a globally unique optimal solution. If the optimal solution x^* of $\mathcal{P}_M$ is such that constraints (C2)–(C4) in $\mathcal{P}_C$ are feasible (i.e., there exists sufficient generation capacity and voltage rise is within acceptable limits in the DN) then the optimal solution x^* can be transformed to the original problem variables to give P^* which is the optimal solution of $\mathcal{P}_C$. A detailed characterization of this equivalence is detailed in the Appendix.

B. *Signals from EPU*

The two-tier solution structure enables the DG agents to iteratively approach the optimal solution x^*. To facilitate this, cost signals that guide DG agents with decision-making are broadcast by the EPU every second. The cost is derived by constructing the mathematical *dual* $\mathcal{P}_D$ of $\mathcal{P}_M$:

$$(\mathcal{P}_D)\ \max_{\nu}\ \min_{x \in \triangle} \sum_{i=1}^{n} C_i(m.y_i.x_i)^2 + \nu \left(\sum_{i\,=\,1}^{n} m.y_i.x_i + K \right)$$

where $K = -\sum_{d \in D} p_d$

Let the objective function of $\mathcal{P}_D$ be denoted $f(x,\nu)$. This represents the Lagrangian of $\mathcal{P}_M$ and is referred to as a potential function. We consider the mathematical dual as the potential function $f(x,\nu)$ incorporates penalty when overall demand is not met by aggregate sustainable generation. The cost $F_i(x)$ of dispatch level y_i is selected to be the gradient of this Lagrangian multiplied by a constant $\mathcal{K}$:

$$F_i(x) = \mathcal{K}.m.y_i(2.C_i m.y_i.x_i + \nu^*) \qquad (1)$$

As DG agents make dispatch decisions based on the gradient of the potential function, every strategy revision will reduce the potential of the system and thereby indirectly move the system to optimality. Hence, $F(x) = [F_1(x) \ldots F_n(x)]^T$ is part of the signal broadcast by the EPU to all participating DGs every second. In order to compute the current gradient $F_i(x)$, the EPU requires information about the current value taken by x in the DN and ν^*. The former is available to the EPU via Assumption 1. The EPU can directly compute the later due to the following reasons. The variable x is a vector of size n (i.e. the total number of strategies in y and $n << |\mathcal{G}|$) that represents the proportion of active strategies in use within the DN by all the agents and is independent of local constraints that are specific to individual agents. Hence, x is an aggregate measure. For this reason, the EPU can directly solve $\mathcal{P}_M$ to easily obtain x^*. However, the main challenge lies in how these dispatch levels can be distributed to all DGs in the DN to achieve x^* while maintaining feasibility. The EPU uses $F(x)$ to guide the DG agents to distributively select dispatch strategies that heed local constraints and achieve this x^* in an iterative manner. The EPU utilizes the x^* (which it directly computes as mentioned earlier) to solve $\mathcal{P}_D$ so that ν^* can be computed in closed-form. This can now be leveraged by the EPU to compute the cost signals that are then broadcast to the DG agents. As the dimensions of the optimization variables of $\mathcal{P}_M$ and $\mathcal{P}_D$ are n (in our implementations in Section IV, n is set to 3) and 1 respectively, the EPU can solve these convex problems very quickly to obtain x^* and ν^*. Moreover, as consumer demands and renewable generation are constant over every one minute interval due to Assumption 6, the EPU solves $\mathcal{P}_M$ and $\mathcal{P}_D$ only once at the beginning of these one minute dispatch intervals. Using these values, the EPU computes Eq. 1 every one second to obtain the cost signals for broadcast (Assumption 2) using the current state of dispatch as reported by the data concentrators (Assumption 1). Computing Eq. 1 is also straightforward as it consists of simple arithmetic operations. When DG agents revise strategies based on the cost signals, a state trajectory is induced overtime (i.e. we consider x to be the system state). We show in Section III-C and in the Appendix that the cost signal defined in Eq. 1 result in a system state trajectory that converges exponentially fast to an equilibrium which is in fact also the optimal solution x^* of $\mathcal{P}_M$ when there are a large number of DG agents and adequate voltage slack in the buses.

The actions of the DG agents resulting from the transmitted cost $F(x)$, system state x and strategies y can be modelled as a population game [24]. These games represent systems consisting of a large number of agents (i.e. continuum of players) which are anonymized (i.e. invariant to permutations). Hence, individual actions of agents are infinitesimal from a system-wide perspective. When local feasibility checks are not met by the DG agents during revisions, we consider this to be a minor reduction in the population size. Decisions of each agent are dependent on the current state of the system and the cumulative actions of other agents (as reflected by the cost signals). Thus, this game $\mathcal{G}$ is completely specified by the players (i.e. DG agents forming the population $\mathcal{P}$), strategies y of each player, cost $F_i(x)$ of each strategy y_i and system state x. Random revisions are made by the DG agents in order to achieve the desired aggregate system goal (i.e. meet overall consumer demands while ensuring that minimum dispatch levels are utilized). These independent and anonymous revisions allow us to accurately approximate the ensuing stochastic evolution of the aggregate system state x using a deterministic average process as outlined in Section III-C. Moreover, posing the optimization formulation in $\mathcal{P}_C$ as population game theoretic formulation [24] allows us to evoke tractable mathematical analysis to ascertain optimality characteristics of the system as demonstrated later.

Nash Equilibrium (NE) of a game represents the state at which players cannot deviate from their current action without incurring more cost [25]. The conditions for NE of $\mathcal{G}$ are exactly the optimality conditions of P_M (i.e. the Karush Kuhn Tucker (KKT)). Hence, the NE and the globally optimal solution of $\mathcal{P}_M$ are the same. Moreover, $\mathcal{G}$ is a potential game as the full externality symmetry is satisfied by the cost F (i.e. $\frac{\partial F_j}{\partial x_i} = \frac{\partial F_i}{\partial x_j}$) which leads to interesting intuitive properties relating cost signals computed by the EPU and the corresponding actions of DG agents as discussed next [24].

C. *Response of DG Agents to Cost Signals*

Every DG agent will select a random time to revise its current dispatch strategy based on the most recently received cost signal from the EPU using a *revision protocol*. What we want is for the iterative responses of DG agents to adapt to track the optimal dispatch cost while ensuring that the physical grid constraints are met within a real-time interval. To achieve this, the revision protocol should be designed so that the real power dispatched by all DGs in the DN converges to the optimal solution x^* at an extremely rapid rate (ideally exponentially fast). The dynamics of the system state x induced by strategy revisions of DG agents will dictate the convergence behaviour of the DGs to x^*.

When a DG agent decides to switch from one dispatch strategy to another, this change must lead to a reduction in the cost $f(x,\nu)$ (i.e. potential) of the system. Various values taken by the system state x due to these switches represent the state trajectory. The most rapid manner in which the system dispatch can descend to optimality is when the state trajectory moves in a direction opposite to the gradient of $f(x,\nu)$. Thus, ideally state dynamic should be governed by:

$$\dot{x}_i = \frac{1}{n}\sum_{j=1}^{n} F_j(x) - F_i(x) \tag{2}$$

which indicates that the evolution of system states (i.e. state dynamic) should be the negative of the projection of the gradient $F(x)$ onto the simplex $\triangle$ as x is limited to taking values in $\triangle$. This is referred to as the *projection dynamic* in Evolutionary Game Theory (EGT) [24]. What is important is that Eq. 2 has an equilibrium that coincides with the optimal solution of $\mathcal{P}_M$ and the NE of $\mathcal{G}$. As demonstrated in the Appendix, it can be show via Lyapunov arguments that convergence rate is exponentially fast which enables real-time performance. In order to achieve this particular system dynamic, every DG agent will make a switch from strategy y_i to y_j according to a revision protocol defined by the conditional switch probability [24]:

TABLE I
STRATEGY REVISIONS BY DG AGENTS

Strategy Selection Process by DG Agent i
• Initialize time: $t_{\text{next}} \leftarrow 0, t \leftarrow 0$.
• Initiate strategy: $s_c \leftarrow r \text{ and } (y)$.
Start Algorithm: (repeat the following):
1) Time for next revision τ_i compute via exponential probablity distribution with rate μ. Next revision will occur at $t_{next} \leftarrow t + \tau_i$.
2) When $t > t_{next}$,
• Utilizing the latest F and x broadcast by the EPU, select strategy s_i according to $\rho_{i,j}$ defined by: Eq. 3.
• Set: $s_c \leftarrow \{\max\{y\} \vert y \leq \min\{s_i, c_i\}\}$.
– Check if s_c is feasible according to Table II.
– If not, set $s_c \leftarrow y_{c-1}$ and repeat previous step.
– Else, go to Step (1).

$$\rho_{i,j}(F(x),x) = \frac{[F_i - F_j]_+}{n.x_i} \tag{3}$$

$\mathcal{K}$ in Eq. 1 is selected so that $\rho_{i,j}(.)$ satisfies the conditions of a probability measure. According to Eq. 3, the DG agent will switch to strategy j from i with a positive probability if the cost of the incumbent strategy is higher than the strategy under consideration. The state dynamic $\dot{x}_i$ is defined by the rate at which agents switch into and leave strategy y_i :

$$\dot{x}_i = -x_i.\sum_{i=1}^{n}\rho_{i,j} + \sum_{i=1}^{n}\rho_{j,i}.x_j \tag{4}$$

Due to Assumption 8 (many DG agents are present in the system), strong law of large numbers will take effect. Hence, stochastic effects will be eliminated. Substituting Eq. 3 into Eq. 4 will result in the projection dynamic listed in Eq. 2.

As mentioned earlier, local constraints (C2)-(C4) from $\mathcal{P}_{\mathcal{C}}$ are moved to the DG agents. Due to the structure of the cost function f_o which imposes no restrictions on the physical location of DGs, this decomposition preserves the optimality of $\mathcal{P}_{\mathcal{C}}$. Hence, as long as the dispatch by DGs in the system (irrespective of the physical location) heed local voltage and capacity constraints and meet overall system demand by engaging in the minimum possible power dispatch levels, the optimal solution is achieved. Before making a strategy switch, the DG agent checks whether the impending switch is feasible (i.e. does the switch satisfy the physical constraints). If this is not the case, then subsequent dispatch levels are considered until the switch is feasible as outlined in Table I. When the strategy under consideration is not feasible, this is due to either the voltage rise at the buses being too high or due to insufficient local generation capacity. The subsequent strategy in y is the next best strategy to be considered as this is a smaller dispatch level. This will result in a lower voltage rise when compared to the strategy that was considered earlier and has a higher possibility of being accommodated by current available generation capacity. This compromise is translated to a reduction in population size. Constraint (C4) in $\mathcal{P}_C$ which is the generation capacity constraint can be easily validated locally. However, constraints (C2)-(C3) in $\mathcal{P}_C$ are dependent on how the change in dispatch strategy selected by a DG agent

TABLE II
FEASIBILITY CHECK BY DG AGENTS

Feasibility Check by DG Agent i of Dispatch Change ΔP

- Backward sweep: Let $b \leftarrow i$, repeat until parent node is the substation:
  - Set $a \leftarrow \mathrm{parent}(b)$ and send Δp to node a.
  - Update $p_{a,b}$ according to Eq. 5. Set $b \leftarrow a$.
- Forward sweep: Let $a \leftarrow$ substation bus and $b \leftarrow \mathrm{child}(a)$ in the updating feeder branch. Repeat until descendent bus is a leaf or alarm activation:
  - If $p_{a,b} > 0$, then solve for V_b using Eq. 6 otherwise use Eq. 7.
  - If (C3) in $\mathcal{P}_C$ is violated activate alarm across the updating branch
  - Otherwise, set $a \leftarrow b$ and $b \leftarrow child(a)$ and send V_a to node b. Repeat for all children of a.

affects power flow across the branch in which the DG resides in. As a DG agent selects a random time using an exponential distribution that is continuous with an arrival rate of 1 second to revise its strategy, it can be assumed that only one DG agent performs revision at a particular time instant while dispatch by other DG agents remains fixed. This is a key simplification in EGT enabling a DG agent to use simple information exchange (as outlined in Table II) between buses within its branch to infer the violation of voltage constraints. Communication complexity of this information exchange is in the order of the height of the corresponding local feeder branch. As discussed later in this section, the time required for this feasibility check is in the range of microseconds. This is mainly due to the fact that the nodes are located at close physical proximity (i.e. belong to the same DN). Moreover, when dedicated communication channels are used, network congestion is non-existent.

In order to check whether a strategy revision meets physical constraints, the revising DG agent will need to infer whether the generation increase or decrease $\Delta p = y_i - y_j$ accompanying the incumbent strategy change from y_i to y_j will affect voltage rise limits in the local feeder branch. Suppose the bus that the revising agent is currently residing at is B and the parent Bus is a (i.e. $a \leftrightarrow b \in \mathcal{E}$ and Bus a is one line closer to the substation). Agents in Bus a can approximate the power flow $p_{a,b}^{\mathrm{new}}$ in line $a \leftrightarrow b$ due to this change in dispatch using current power flow denoted by $p_{a,b}^{\mathrm{old}}$ obtained from measurements (Assumption 4) as follows:

$$p_{a,b}^{new} = p_{a,b}^{old} + \Delta p \tag{5}$$

This is an approximation as power loss is not accounted for in this update. This is an acceptable omission as voltage rise can now be estimated conservatively allowing for sufficient margins for unexpected transients. Δp is propagated up the feeder branch until all ancestors of Bus b (up to the substation) update the power flow according to Eq. 5. We refer to this as the *backward sweep*. In the worst case, the number of information updates required is the height of the feeder branch.

Then, the bus voltages in the feeder branch in which the dispatch change is to occur are updated via a *forward sweep* process starting with the bus (labelled b) that is located in the updating branch and is also the immediate child of the substation bus (labelled a). Voltage is updated using the approximated power flow computed previously in the backward sweep step and $q_{a,b}$ obtained from local measurements. If $p_{a,b} > 0$, a voltage drop will occur across line $a \leftrightarrow b$ and V_b is updated by solving:

$$Y_{a,b}^{H} V_a (V_a - V_b)^H = p_{a,b} + jq_{a,b} \tag{6}$$

Otherwise, there is a voltage rise and V_b is updated by solving:

$$Y_{a,b}^{H} V_b (V_b - V_a)^H = -(p_{a,b} + jq_{a,b}) \tag{7}$$

As V_a is fixed at the substation, V_b is the only unknown variable. After updating the voltage, Bus b checks whether the local voltage constraint met (i.e., constraint (C3) in $\mathcal{P}_C$). When this fails, an alarm is broadcast through this feeder to alert the revising agent that the dispatch change is not feasible. Otherwise, the descendants of Bus b repeat this until either an alarm is evoked or the leafs of the updating feeder branch are reached. In the first case, the revising DG agent will infer that the feasibility check has failed and in the second case feasibility is confirmed. This algorithm capitalizes the tree structure of a radial DN and is summarized in Table II.

As information is propagating in parallel across the branch, communication complexity of the feasibility check in the worst case is $3.h.n$ where h is the height of the tree, the constant 3 denotes three sets of information flow across the updating branch (i.e. the forward, backward and alarm information exchanges) and n denotes the number of strategies available for each DG agent. For instance, we consider a low-voltage Danish DN consisting of 34 buses supplying power to 75 homes detailed in reference [26] in which, the height of the deepest feeder tree is 7 and the number of strategies available to agents is $n = 3$; as the latency of information exchange is in the range of microseconds, the time required to check voltage constraints will be much lesser than 1 second. Also, computational complexity at each agent is constant as the backward sweep involves one addition and the forward sweep requires the solving of a quadratic equation in the worst case. In general, the computations performed by each DG agent for the secondary tier involves determining $\rho_{i,j}$ using Eq. 3, estimating local power flows and comparing dispatch strategies (to ensure that generation capacity limits are heeded). All of these secondary tier computations entail simple arithmetic operations and thus are very straightforward.

D. Guarantees on Convergence to Optimality

Our proposal divides the original problem into master and secondary tiers. The master tier represents economic dispatch and ensures that the overall demand is met by the available sustainable generation capacity in the DN. Local generation capacity and voltage constraints are considered in the secondary tier. We transform the original problem into a strictly convex optimization problem that is used to construct cost signals by the EPU. These signals foster exponentially fast guaranteed convergence to optimal DG dispatch given that there exists adequate sustainable generation capacity and bus voltages. The former is proven in Section III-C. Bounds on the later are established in the following.

The secondary tier involves local feasibility checks in which DG agents check whether violations of generation capacity and voltage constraints are possible during every strategy revision.

A revising DG agent can easily check whether a new dispatch strategy will heed the local generation capacity constraint. One caveat is that the overall generation capacity in the system must be greater than or equal to the overall demand in the system (i.e. $\sum_{g\in\mathcal{G}} c_g \geq \sum_{d\in\mathcal{D}} p_d$) so that aggregate demand can be ensured to be met by all DGs in the DN. Next, voltage rises that accompany backflow of power in feeders due to DG power injections should be such that the resulting bus voltage magnitudes are within the prescribed limits (i.e. $|V_b| \leq \bar{V}_b \;\forall\, b \in \mathcal{B}$). Lower limits on bus voltage magnitudes are accounted for in the design of the DN to accommodate existing consumer demands. Moreover, adding DGs to the DN can only result in a breach of upper voltage limits.

Every time a DG agent makes a revision, backward and forward sweep methods are used to ensure voltage feasibility. These checks result in conservative strategy revisions as power losses across the lines are not incorporated into the decision-making. In the backward sweep step, the incumbent change in dispatch is propagated up the feeder branch. Change in power flow across the lines due to this revision is estimated in this step. Then, in the forward sweep step, the voltages at the buses are updated based on changes in power flow across the lines previously estimated. In the worst case, suppose that there is only backflow of power in a particular feeder branch. Increase in dispatch at any bus in this feeder branch will result in an increase in voltage across all buses in this feeder. In our backward sweep algorithm, we do not account for power loss resulting from backflow of power due to increase in dispatch. In certain cases, even when an increase in dispatch can be physically accommodated by the system without violating voltage constraints, our algorithm will prevent this from occurring due to overestimation of voltage rise across the buses as power loss is not factored into the computation. This can result in false positives.

By analyzing the worst case scenario that can lead to these false positives, we next establish a bound on bus voltage magnitudes within which no false positives will result from our feasibility check algorithm. As mentioned earlier, suppose that a particular feeder branch consists of only backflow of power in the lines. Any increase in dispatch will result in voltage rise across the buses in that feeder. Since only one DG agent will be making a strategy revision at a particular time, the largest possible increase in dispatch at any instant is $\Delta P = \max(y)$ which is 0.02 kW in our paper as we consider $y = [0.0000001,\ 0.01,\ 0.02]$ kW in simulations presented in the next section. Since our algorithm neglects power loss, we assume an increase of ΔP in power flow across lines formed by buses that are ancestors of the updating bus. From this, we can derive an upper bound on the magnitude of voltage increase, ΔV, that can occur in any bus associated with lines having power flowing in the opposite direction. This is deduced to be $\Delta V = \Delta P \frac{|z_{a,b}|}{|V_s|}$. A detailed proof is presented in the Appendix.

Next, we consider the circumstances leading to bus voltage magnitudes reaching the upper limit. The upper limit on voltage magnitude is typically the same across all the buses and is 10% more than substation voltage (i.e. $\bar{V}_b = 1.1|V_s| \;\forall\, b \in \mathcal{B}$ where V_s is the fixed substation voltage). Henceforth, this upper limit is referred to generically as $\bar{V}$. Suppose that after an increase in dispatch, the voltage magnitude at the revising bus b reaches the limit (i.e. $|V_b| = \bar{V}_b$). Since we do not incorporate power losses, the voltage increase estimated at each bus that is an ancestor of the revising bus is also bounded above by ΔV. Hence, the bus voltage magnitudes of ancestors of the revising bus should be at least $\bar{V} - (L+1)\Delta V$ prior to a revision where L is the number of lines separating the ancestor bus from the revising bus. This allows the compounded voltage increase estimated to be at least $\bar{V} - \Delta V$ at the revising bus prior to the revision. Voltages greater than these bounds can drive the estimated bus voltage magnitude above the limit after a revision and result in a false positive. Hence, in the worst case, the revising bus is the deepest leaf node in the feeder. In this case, the most restrictive bound on the voltage magnitude of a bus in the feeder will then be $|V_b| \leq \bar{V} - (h+1)\Delta V \;\forall\, b \in \mathcal{B}$ where h is the height of the feeder. If voltage magnitudes are within this bound then our algorithm will not produce any false positives. As the largest change in power dispatch ΔP is a small value (i.e. 0.02kW in our paper) and height of the DN is not a large value (i.e. $h = 7$ sample Danish low-voltage DN considered in this paper), the second term in the bound represents minor error. Hence, this indicates that false positives will occur in rare cases. These can be completely eliminated by sizing the DN for a smaller number of consumers or placing more DGs at buses that are closer to the substation.

E. Information Exchanges

For the computation of the cost signals, during every signalling iteration, the EPU solves $\mathcal{P}_M$ and $\mathcal{P}_D$ and computes $F(x)$ using the current distribution of strategies in the DN as reported by a data concentrator. The EPU broadcasts these signals to all the DG agents as these are common values. When a DG agent revises its current strategy at a randomly selected time, it will send its current dispatch p_g^i to the data concentrator. This information is used by the data concentrator to update x. For feasibility checks, DG agents measure current power flow in the lines via the PMUs (Assumption 4). In a backward sweep, these measurements are used to update the power flow along the lines formed by ancestors of the bus in which the revising agent is residing. Then, in the forward sweep, the voltages in the buses are approximated based on the updated power flows. An alarm is propagated when any estimated bus voltage violates limits. These information exchanges are illustrated in Fig. 2.

F. Summary of Our General Approach

Generally, as the OPF problem is non-convex, it is difficult to solve it directly and this is especially the case in our system model as we consider additional complexities such as the presence of a large number of dispatchable sources which are equipped with a discrete strategy set. In this paper, we do not claim to solve non-convexity. However, we attempt to overcome this difficulty by decomposing and translating the problem into a completely different domain whereby DG agents make distributed revisions based on cost signals broadcast by the EPU. The resulting distributed model fits well into a population game framework. We show in the limiting case when the number

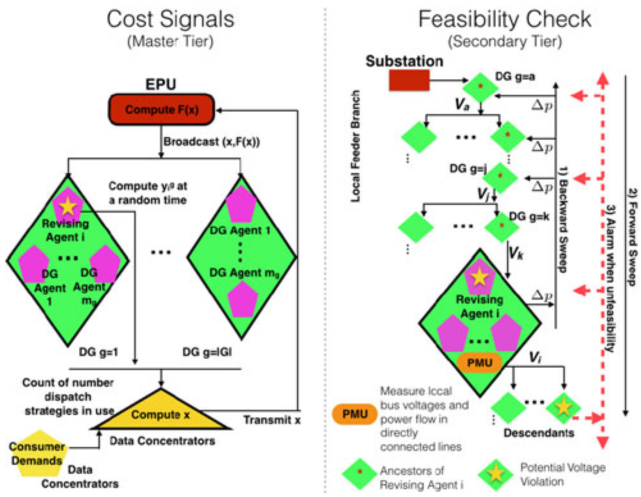


Fig. 2. Information Exchange Summary.

of DG agents $m \to \infty$, that the error $\pm 1/m$ in the optimal solution with respect to the master problem goes to zero. However, since it is not practically possible to introduce infinite agents, there will always be an error of $\pm 1/m$ which is negligible when m is sufficiently large. Hence, this population game theoretic approach allows us to work around the challenges associated with the size of the problem and discrete strategy set. The secondary tier consists of decision-making that is executed by each DG agent and this consists of voltage feasibility checks. The forward-backward sweep method conservatively approximates changes in bus voltages due to an incumbent strategy change. Hence, theoretically, when there is sufficient slack in the voltage constraints and in the limiting case of the number of agents, the optimal solution of the transformed problem will be the exact solution of original problem P_C.

G. Extensions of Proposed Algorithm

In this section, we present possible applicability and extensions of our proposed algorithm to other dispatch problems with diverse generation sources and network topologies.

Energy sources considered in this paper are renewable generation systems as these are common in DNs. Our proposal can be applied to also more traditional generators such as ones with ramping constraints that impose limits on changes in generation over two consecutive time intervals. As ramping constraints apply to specific generators, these can be decoupled from the original dispatch problem and integrated into the secondary tier. The feasibility check made by every revising agent will then consist of an additional step in which it is verified whether the impending revision combined with overall dispatch changes so far in the generator heed the ramping limits. We plan to take into consideration a wide range of heterogeneous energy sources for dispatch in future work.

Another extension of our proposal can be due to the underlying DN topology. The DN considered in this paper has a tree structure as the forward and backward sweep method for checking voltage feasibility is dependent on the underlying feeder topology having a tree structure. As DNs with tree topology are common occurrences in today's power grid, our proposal is in general widely applicable to most DNs. When voltage rise constraints are omitted, our proposal becomes a real-time economic dispatch solution. In existing literature, economic dispatch is formulated without accounting for physical voltage constraints (e.g. [12], [13], [18]). In our algorithm, the master problem remains unchanged and is used by the EPU to compute cost signals that depend on generation cost and supply and demand imbalance. When these signals are used by the DG agents to revise dispatch strategies without performing local voltage feasibility checks, real-time economic dispatch results and this is independent of the underlying system topology. Exponential convergence properties are guaranteed here as the OPF without voltage constraints can be transformed into a strictly convex optimization problem.

On the other hand, if the voltage constraint is an important consideration and the system consists of multiple feeders in which some have a tree topology and the remaining feeders have ring/mesh topologies, then it is still possible to apply our algorithm. The voltage feasibility check in our proposal requires only the local feeder in which the revising DG resides in to have a tree topology. Hence, all DGs residing in feeders with tree structure can operate as per our algorithm. DGs situated in feeders with ring/mesh topologies can serve local demands so that power is not directly injected into the buses. Excess generation can be directed into an energy storage system instead. In general, it is certainly desirable to impose no restrictions on the underlying system topology (e.g., tree, ring, and mesh) or balanced/unbalanced nature of the network. In the context of our work, applying an extension to the forward and backward sweep algorithm will allow for general applicability and we do plan to investigate this in future work.

IV. Implementation

In this section, the proposed DG dispatch algorithm is evaluated via MATLAB simulations conducted using practical models and parameters. These results highlight the desirable convergence characteristics of the proposed strategy and ensuing feasibility of the underlying physical DN constraints.

A. Models and Parameters

The proposed algorithm is implemented in the radial DN of [26] which models a low-voltage Danish distribution system consisting of 34 buses and 75 homes. This is a grid-connected distribution network that is supported by the main grid which absorbs surplus generation or supplements for deficiency in local generation in the system.We assert that evaluating the proposed dispatch algorithm in such a setting will enable more realistic insights on the challenges of renewable integration given the high DG penetration in Denmark DNs. Measurements of physical grid attributes such as voltage and power flow across lines facilitating Assumptions 1 and 4 are computed via Gauss-Seidel load flow analysis [27].

Power demands of urban homes are highly variable due to externalities such as diurnal effects and energy policies that differ across various regions. As such, we utilize appliance usage

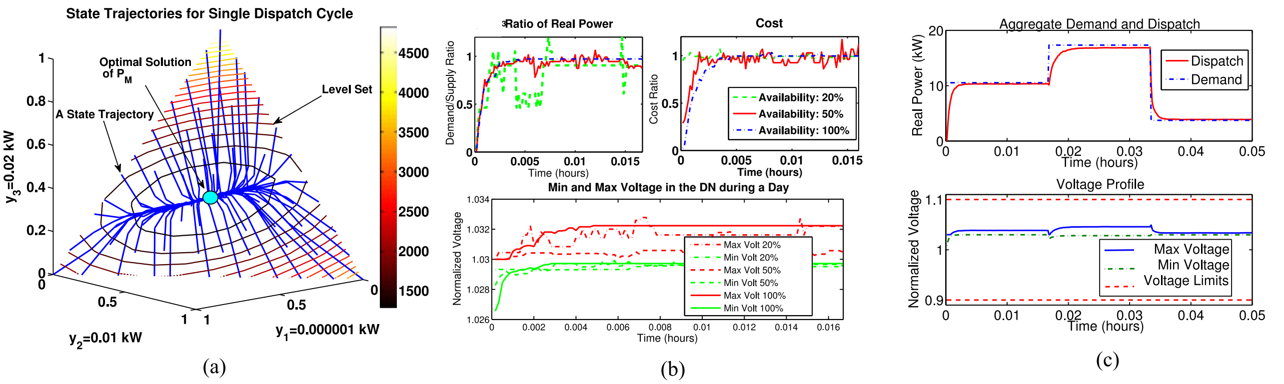


Fig. 3. Convergence Properties of Real Power and Voltage Profiles. (a) Projection Revisions. (b) Relaxation of Assumption. (c) Dispatch Cycle Transitions

profiles and patterns during the summer season for Europe provided in [29] for generating individual demand profiles of all 75 homes at a granularity of 1 minute (Assumption 6). Appliances considered in our demand model are air conditioner, dryer, washing machine, dish washer, freezer, fridge, electric stove, oven and water heater.

As this paper addresses mass DG integration, we consider the deployment of DGs such as solar panels and micro wind turbines in every home of the DN. Hourly solar generation data available in [30] is utilized to model generation from solar panels. Additional processing is applied to this data to smooth the hourly generation data and random noise is added to the data to model irregularities in solar irradiance due to differences in cloud cover in the region. For micro wind turbine generation, the power curve defined by $P_{\text{wind}} = \frac{1}{2}A\rho\theta v^3$ is utilized where A is the cross-sectional area of the turbine rotors, ρ is the air density, θ is the efficiency of the turbine and v is the wind speed. These specifications are obtained for a wind turbine rating of 1.9 kW as listed in reference [31]. Wind speed is modelled via the Weibull probability density function which has shape and scale factors of 1.94 and 4.48.

The EPU will broadcast cost signals computed according to Eq. 1 to DGs every 1 second (Assumption 2). Agents in a DG can select from one of $y = [0.0000001\ 0.01\ 0.02]$ kW dispatch levels. y_0 is selected to be a very small value so that the generation in the system reflects actual demands (i.e. there is no over-generation). Moreover, $y_0 \neq 0$ so that strict convexity of $\mathcal{P}_M$ is preserved. C_L is selected so that $C_L < C_{L+1}$. We have specified three strategies per agent (i.e. $n = 3$) as this will allow for easier visualization of state trajectories as will be discussed in Section IV-B. Moreover, the power dispatch levels defined in y will allow DG agents belonging to the same physical DG to realize a cumulative dispatch at a precision of 0.01kW which is also the precision of typical power ratings of DGs [22], [30], [31]. A DG g with generation capacity c_g kW can have $m_g = c_g/0.02$ number of agents. Every agent will select a random time based on Poisson distribution to re-evaluate its current dispatch. Every dispatch cycle is 1 minute in length and this is the period in which demand and supply remain constant (Assumption 6). Maximum and minimum voltage limits of the buses are set to be $\pm 10\%$ of the nominal 4 kV. Substation voltage is fixed to 1.03 p.u.

B. Convergence Characteristics of Distributed Revisions

We assess the ability of our two-tier optimization approach to achieve real-time dispatch, which is directly related to the convergence properties of x to the optimal solution of $\mathcal{P}_M$. To assess how the initial starting state affects convergence, the evolution of x (i.e. state trajectory) induced by distributed strategy revisions by DG agents for various randomly generated initial conditions is examined over a single dispatch cycle and the result is illustrated in Fig. 3(a). The simplex representing all possible values x can take is three-dimensional as there are three strategies (i.e. $n = 3$). Level sets are also included in this figure and each one of these curves represents all possible values of x that result in a particular system cost c (i.e. $f(x, \nu) = c$ where $f(.)$ is the objective function). These curves are differentiated from one another via the gradient bar. The optimal solution occurs when the cost is minimal (i.e. the level sets in the simplex converge to a central point). From Fig. 3(a), it is evident that all state evolutions, regardless of the initial starting point, converge to the optimal solution located at the middle of the level sets of the cost function $f(x, \nu)$ and thereby exhibiting global convergence. Moreover, as the trajectories taken by the states are orthogonal to the level sets (most rapid descent), this confirms that the revisions made by DG agents result in exponential convergence to optimality as suggested by our earlier theoretical analysis.

Next, the consequence of relaxing our assumption on the existence of a large number of agents is examined in Fig. 3(b). When there is insufficient local generation capacity or if feasibility checks fail, this results in a reduction of DG agent population size. Proportion of aggregate real power demand supplemented by the overall power dispatched by DGs when the percentage of agent availabilities take values in {20%, 50% and 100%} are depicted in this figure. Results indicate that the system is fairly stable after convergence to optimality for agent availabilities of 50% and 100%. However, marked oscillations are evident in the ratio between real power dispatched and current system demand over the one minute dispatch interval in Fig. 3(b) when only 20% of the agents are available. This is expected as the strong law of large numbers will not hold when the population size is small. Regardless of the availability of agents, revisions will always heed the voltage limits on the buses as every

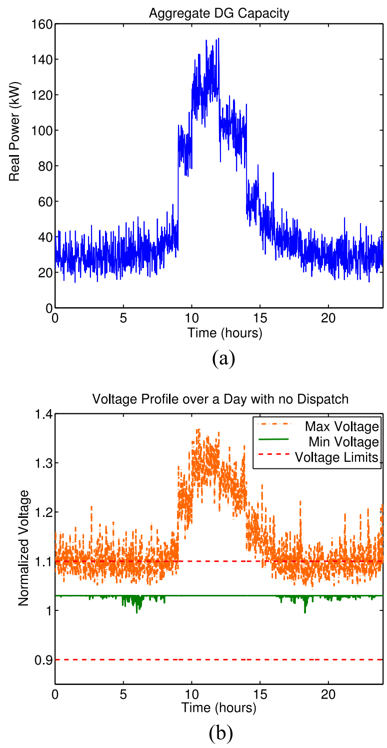


Fig. 4. Without Control. (a) DG Capacity. (b) Voltage Profile Over 34 Buses.

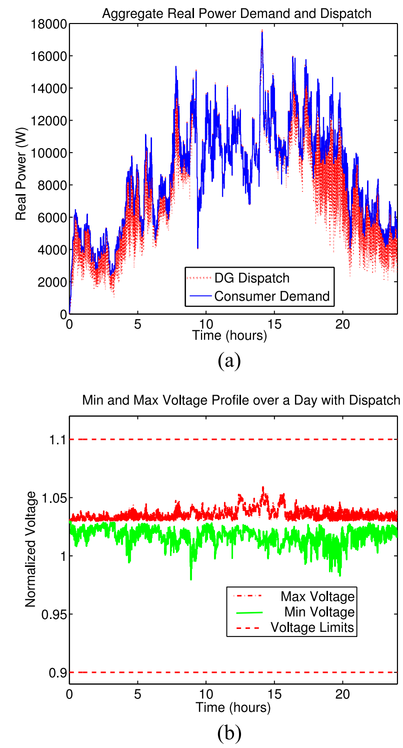


Fig. 5. Real-time Dispatch over a Day. (a) Real Power. (b) Voltage Profile Over 34 Buses.

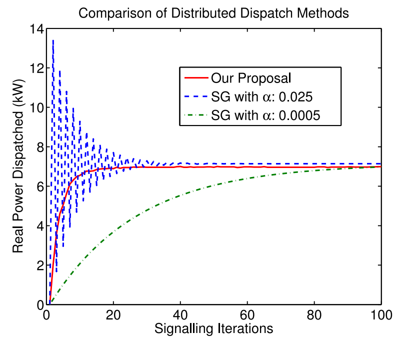
revising agent ascertains whether its strategy switch is feasible or not via conservation approximations. The impact of the number of DGs participating on the system cost is explored next. The second sub-figure in Fig. 3(b) illustrates the evolution of the ratio of system cost with respect to the optimal cost for a single dispatch cycle. As the objective is quadratic and is dependent on the magnitude of power dispatched, it is possible to observe that the fluctuations are not as pronounced as the previous result in this figure for the three modes of DG availabilities investigated. Next, the minimum and maximum voltage profile across all 34 buses for the three cases show that the system still remains within the acceptable $\pm 10\%$ limit. When agent availability is 100%, maximum and minimum voltage values are slightly higher than the other cases and this is expected as more active nodes are generating power in the DN causing more reverse power flows.

C. Overall Performance throughout Multiple Dispatch Cycles

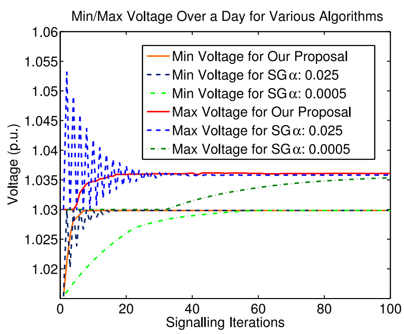
Here, we examine the transitional properties of aggregate DG dispatch between multiple constant demand and supply intervals when the proposed strategy is in effect. In Fig. 3(c), it is clear that overall DG dispatch is able to adapt to changes in supply and demand at every dispatch cycle. Bus voltages are well within the limits. Next, we extend dispatch over an entire day. Using the generation models of Section IV-A, and given that each home is fitted with DGs, the aggregate generation capacity of the system throughout a day is presented in Fig. 4(a). If these DGs are allowed to inject all power generated, then the resulting voltage profile in the system is illustrated in Fig. 4(b). Clearly, the maximum voltages of the buses are well above the acceptable limits (almost 30% above the maximum threshold) throughout the day. In contrast, when our proposed algorithm is activated, the resulting real power dispatch in Fig. 5(a) closely follows the aggregate system demand. Moreover, the maximum and minimum voltage profile of all 34 buses is well-within the $\pm 10\%$ limits. These results demonstrate the effective manner in which DG agents in our approach self-organize and rapidly adapt local dispatch using the overall information supplied by the EPU and information exchanged within their corresponding feeder branches. This illustrates the seamless coexistence of participating DG agents as changes in dispatch or demand in the system are implicitly inferred by the agents via the cost signals and the corresponding reactions lead to optimal system operating conditions.

D. Comparison with State-of-the-Art

Finally, we highlight the differences between our proposed strategy with the existing literature. We compare our approach to those of a similar flavour involving distributed dispatch via decomposition. We first implemented the iterative Sub-Gradient (SG) method [12]–[14] based on dual decomposition for a single dispatch cycle. Convergence of the SG method is dependent on the step-size used to update Lagrangian multipliers at each iteration. Hence, it is necessary for an operator to design the step-size based on system characteristics. Two step sizes $\alpha = 0.025$ and $\alpha = 0.0005$ are implemented and as illustrated in Figs. 6(a) and 6(b) these result in significant ringing and slow convergence, respectively, in both real power dispatch and the voltage profile. Ringing effects such as that observed in Fig. 6(b) are pronounced and can lead to the overshooting of bus voltages beyond the limiting thresholds. Slow convergence is not desirable for real-time dispatch. In contrast, the our solution



(a)



(b)

Fig. 6. Comparison of Dispatch Methods. (a) Real Power Dispatch. (b) Voltage Profile Over 34 Buses.

requires no custom parameter adjustment by the grid operator and exhibits rapid convergence to optimal dispatch while heeding voltage constraints. Hence, in our proposal DGs are able to implicitly infer the changes in conditions of the grid from the cost signals and react accordingly, this allows for the seamless coexistence amongst these DGs which thereby leads to the optimal solution.

In general, existing work on DG dispatch can be divided into three general classes and these consist of centralized, decentralized and distributed solutions. Solving a non-convex OPF in a centralized manner is not tractable as it is an NP-hard problem. In our proposal, on the other hand, the cost signals used by the DG agents for strategy revisions dictate the convergence attributes of the system. On average, our proposal requires 10 broadcasts of cost signals by the EPU prior to convergence to optimality. As each signalling iteration takes place every one second, the average time for convergence is 10 seconds. This real-time convergence is possible mainly due to the simplicity of the sub-problems solved by the EPU and the DG agents and the exponential convergence characteristics of the projection revisions. Decentralized solutions encompass a broad range of work in the existing literature. Some solutions take only local measurements (i.e. no communication is involved) and these result in sub-optimal dispatch [17].

More recently, fast techniques such as Alternating Direction Method of Multipliers (ADMM) have been applied to solve relaxed convex and non-convex OPFs in the literature. ADMM allows for a completely decentralized solution with performance comparable to our work. The following are some key differences. Reference [11] has applied ADMM to the relaxed OPF and has shown that the number of iterations required for convergence is linear with respect to the network size. This performance is comparable to our solution as we mathematically substantiate that the master problem converges exponentially fast and that the feasibility check for a revising agent is in the order of the height of the deepest feeder. In another instance, ADMM has been applied to non-convex OPF and the authors mention that their proposal will diverge unless there is zero duality gap [32]. This is the condition that results in the relaxed solution being equivalent to the exact solution. Hence, it will then be simpler to consider the relaxed OPF instead. Our solution does not result in divergence even when a solution for the original problem is infeasible. In cases such as this, our proposal results in a solution that yields maximum possible dispatch that conservatively heeds local voltage and capacity constraints.

Moreover with ADMM, each node exchanges local primal and dual variables with neighbouring nodes. This can result in significant communication overhead and privacy issues. In our solution, the EPU broadcasts generic cost signals periodically that expose no specific information about the network or agents. Moreover, for feasibility checks, revising agents exchange values indicating the *change* in dispatch that can result with the impending revision and not the actual dispatch. Other decentralized proposals such as those based on averaging consensus involve local information exchanges between neighbours and are dependent on the number of DGs participating in the system for convergence [33]. This will result in major latencies especially when DG penetration is high.

Like decentralized solutions, distributed strategies do not have a central entity imposing dispatch strategies on participating entities. However, in distributed proposals, a central entity can communicate signals that aid with individual decision-making by distributed agents. Distributed solutions in the literature are not suitable for real-time dispatch as these typically have asymptotic convergence properties (e.g. [15]). Although our solution is distributed, in which DG agents make their own dispatch decisions based on signals broadcast by the EPU, we show that our solution converges exponentially fast both in theory and simulations.

V. Final Remarks

In this paper, we have proposed a novel dispatch strategy for the integration of DGs into a low voltage distribution network that takes into account physical limitations of the system. As demonstrated via theoretical analysis and practical simulations, this strategy enables the seamless mass integration of intermittent generation into the DN while heeding voltage constraints. Our proposal requires minimal intervention from operators and indirectly controls DG agents via intelligent signals and communication exchanges so that these rapidly respond to fluctuations in supply and demand. These desirable attributes render the proposed DG dispatch strategy highly suitable for practical deployment and integration of a large number of highly variable generation into a low-voltage residential DN. As future work, we intend to study the generalization of our proposal to any underlying system topology and extend it to incorporate both demand response that provides consumer satisfaction guarantees and sustainable dispatch. Our long term vision is to derive strategies that will allow multiple DNs to cooperate with one another for sustainable grid operations.

Appendix

A. Game Characterization and Optimization Equivalence

A population game $\mathcal{G}$ is defined by individual active players selecting a strategy from the set y based on the allocated cost $F(x)$. x represents the proportion of players in the population selecting strategies in y. Suppose, there exists a function $f : \mathbb{R}_n \rightarrow \mathbb{R}$ such that $F(x) = \nabla f(x)\ \forall\ x \in \triangle$, then f is defined to be a full potential function for the game $\mathcal{G}$. Moreover, if the full symmetric condition $\frac{\partial F_i(x)}{\partial x_j} = \frac{\partial F_j(x)}{\partial x_i}\ \forall\ x \in \triangle,\ y_i \in y,\ y_j \in y$ is met, $\mathcal{G}$ is referred to a fully potential game. This translates to revisions being made by each player reducing the potential of the system and the Nash equilibrium x^*_{NE} of this system is the same as the optimal solution of the following problem when $m \rightarrow \infty$ [24]:

$$\mathcal{P}_p:\ \min f(x)$$

$$p^g_j \in y \qquad \forall\ j = 1 \ldots m$$

$$x_i = \frac{1}{m}\sum_{j=1}^{m} \mathbf{1}_{y_i}(p^g_j) \qquad \forall\ i = 1 \ldots n$$

Due to Assumption 9, reactive power and real power losses in the lines are considered to be supplemented by the main grid where $q_m = \sum_{d\in\mathcal{D}} q_d - \sum_{a\leftrightarrow\in\mathcal{E}} q^l_{a,b}$ and $p_m = \sum_{a\leftrightarrow b\in\mathcal{E}} p^l_{a,b}$ thereby simplifying $\mathcal{P}_C$ to:

$$\mathcal{P}'_C:\ \min\ f(P)$$

$$\sum_{d\in D} p_d - \sum_{g\in\mathcal{G}} p_g = 0 \tag{C1'}$$

$$p^B_a + iq^B_a = \sum_{a\leftrightarrow\in\mathcal{E}} Y^H_{a,b}\big(V^H_a - V^H_b\big)V_a \quad \forall\ a,b \in \mathcal{B} \tag{C2'}$$

$$\underline{V_a} \leq |V_a| \leq \bar{V_a} \qquad \forall\ a \in \mathcal{B} \tag{C3'}$$

$$0 \leq p_g \leq c_g \qquad \forall g \in \mathcal{G} \tag{C4'}$$

With the introduction of x and the notion of multiple agents per DG, the above problem can be equivalently posed as:

$$\mathcal{P}''_C:\ \min \sum_{i=1}^{n} C_i(m.y_i.x_i)^2 - \nu^*\left(\sum_{d\in\mathcal{D}} p_d - m\sum_{i=1}^{n} y_i.x_i\right)$$

$$m = \sum_{g\in\mathcal{G}} \lfloor c_g/\max(y)\rfloor \tag{C1''}$$

$$p^j_g \in \{y_i \in y | \mathbf{1}_{C_j}(y_i) = 1\} \qquad \forall\ j = 1\ldots m \tag{C2''}$$

$$x_i = \frac{1}{m}\sum_{j=1}^{m} \mathbf{1}_{y_i}(p^g_j) \qquad \forall\ i = 1\ldots n \tag{C3''}$$

where the coupling constraint (C1') from $\mathcal{P}'_C$ is moved to the objective and this term will be 0 at optimality as we assume that there exists sufficient generation capacity to meet overall demand. ν^* is a constant computed from $\mathcal{P}_D$. The full symmetric condition is satisfied by the objective of $\mathcal{P}''_C$. The indicator function $\mathbf{1}_{C_j}(y_i)$ determines the feasibility of strategy y_i based on constraints (C2') to (C4'). Problems $\mathcal{P}_p$ and $\mathcal{P}''_C$ are the same with the exception of the feasibility checks in $C2''$. m will depend on the availability of generation capacity during the current optimization interval. When $m \rightarrow \infty$, x can be considered to be continuous. It is important to note that $f(P)$ in $\mathcal{P}_C$ and $\mathcal{P}'_C$ result in non-unique optimal solutions with the same optimal value. To see this, consider $\sum_{i=1}^{n} C_i(m.y_i.x_i)^2$ which is equivalent to $f(P)$ in terms of x. x_i is computed by counting the total number of agents in the population using strategy i divided by m. Suppose, $x = \{0.2, 0.3, 0.5\}$ and $m = 1000$, then the total number of combinations of p^i_g that result in this x is $\binom{1000}{200}\binom{800}{300}$. A subset of these vast number of combinations result in the feasibility of $\mathbf{1}_{C_j}(y_i)$, given that there exists sufficient capacity in the system. Hence, there are many ways of arriving at x^*. Although x^* is unique, $\mathcal{P}'_C$ and $\mathcal{P}''_C$ can lead to the same optimal x^* with non-unique combinations of p^i_g. With the projection revision protocol and feasibility checks, the agents attempt to arrive at a configuration not necessarily unique that result in this x^*. Convergence characteristics and bounds to ensure that the feasibility checks are not overly conservative are provided in the following.

B. Exponential Convergence due to Projection Revisions

In order to show that the projection dynamic exponentially converges to the equilibrium x^* (i.e. $||x^t - x^*|| \leq e^{-\alpha t/2}||x^0 - x^*||$ where $\alpha > 0$) where x^t is the system state at time t and x^0 is the initial system state, it is necessary to show that there exists a Lyapunov function $L(x)$ such that $\dot{L}(x) \leq \alpha L(x)$ where $\alpha > 0$. We have selected the Lyapunov function to be $L(x) = (f(x) - f(x^*)) + \frac{1}{2}||x - x^*||^2$ where $f(x)$ is a short form of the potential function $f(x, v^*)$ and x^* is the optimal solution of $f(x)$. Prior to discussing the proof, it is important to note that the cost $F(x) = [F_1(x)\ldots F_n(x)]^T$ is strongly monotone as the following holds:

$$(F(x) - F(x^*))^T(x - x^*) \geq m * \min(C_i y_i)||x - x^*||^2$$

Let $\mu = m * min(C_i y_i)$ and it is evident that $\mu > 0$. Moreover, as the function $f(x)$ is convex, the following holds as well:

$$(F(x) - F(x^*))^T(x - x^*) \geq f(x) - f(x^*)$$

Equipped with these properties, the following is a proof of exponential convergence of the projection dynamic:

$$\begin{aligned}\frac{d}{dt}(L(x)) &= \nabla L(x)^T\dot{x} = (F(x) + (x - x^*))^T\dot{x}\\ &= (F(x) + (x - x^*))^T\left(\frac{1}{n}\left(\sum_{j=1}^{n} F_j(x)\right)\mathbf{1} - F(x)\right)\\ &= -||F(x) - \left(\frac{1}{n}\sum_{j=1}^{n} F_j(x)\right)\mathbf{1}||^2\\ &\quad -\ (F(x) - F(x^*))^T(x - x^*)\\ &\leq -(F(x) - F(x^*))^T(x - x^*)\end{aligned}$$

where $\mathbf{1}$ is a vector of ones. The first term in the third line of the above proof is ≤ 0 and removing it has resulted in the last inequality. We also use the fact that $F(x^*) = 0$ to obtain the third line. Combining the strong monotone condition of $F(x)$ and the convexity of $f(x)$, the following inequality results:

$$(F(x) - F(x^*))^T (x - x^*) \geq \frac{1}{2} (f(x) - f(x^*)) + \frac{\mu}{2} ||x - x^*||^2$$
$$\geq \min\left(\frac{1}{2}, \mu\right) \left((f(x) - f(x^*)) + \frac{1}{2}||x - x^*||^2\right)$$
$$= \min\left(\frac{1}{2}, \mu\right) L(x)$$

Hence substituting the above inequality into the last line of the initial proof, we are able to show that

$$\dot{L}(x) \leq \min\left(\frac{1}{2}, \mu\right) L(x)$$

where $\alpha = \min(\frac{1}{2}, \mu) > 0$ and thus the projection dynamic will exponentially converge to x^*. ■

C. Bound on Voltage Rise

Here, a proof is presented on the upper bound of voltage increase due to increase in power dispatch by ΔP at bus b. Suppose that bus a is the direct ancestor of bus b (i.e. $a \leftrightarrow b \in \mathcal{E}$). The relationship between bus voltages and power flow across the lines is as follows:

$$V_b(V_b - V_a) = (p_{b,a} + iq_{b,a} + \Delta P) z_{a,b}$$

V_b and V_a are the voltages resulting from the back flow of power in the line (i.e. $p_{b,a} + iq_{b,a} + \Delta P$) and ΔP is the increase in power dispatch across the line $b \leftrightarrow a$. Since the above relation consists of complex variables (i.e. $V_a, V_b, p_{b,a} + iq_{b,a}$), the magnitude of voltage difference across the line $a \leftrightarrow b$ is:

$$|V_b - V_a| = \frac{|(p_{b,a} + iq_{b,a} + \Delta P) z_{a,b}|}{|V_b|}$$

By evoking the triangular inequality, the above is bounded by:

$$|V_b - V_a| \leq \frac{|(p_{b,a} + iq_{b,a}) z_{a,b}| + \Delta P |z_{a,b}|}{|V_b|}$$

The first term on the right side of this relation is $|(p_{b,a} + iq_{b,a}) z_{a,b}| = |V_b'||V_b' - V_a'|$ where V_b' and V_a' voltages at buses a and b prior to the change in dispatch. Substituting this, the following inequality is obtained:

$$|V_b - V_a| \leq \frac{|V_b'||V_b' - V_a'|}{|V_b|} + \frac{\Delta P |z_{a,b}|}{|V_b|}$$

As the dispatch increase will result in a voltage rise, $\frac{|V_b'|}{|V_b|} \leq 1$. Moreover, since as the fixed substation voltage is the point of reference and $|V_b| \geq |V_s|$, it can be concluded that $\frac{\Delta P}{|V_b|} \leq \frac{\Delta P}{|V_s|}$. From these observations, the above bound is further generalized to:

$$|V_b - V_a| \leq |V_b' - V_a'| + \frac{\Delta P |z_{a,b}|}{|V_s|}$$

Hence, the first term in the above inequality represents the voltage rise across the bus prior to the increase in dispatch. The second term represents an upper bound on voltage rise contributed by the increase in dispatch.

References

[1] S. H. Low, "Convex relaxation of optimal power flow—Part I: Formulations and equivalence," *IEEE Trans. Control Netw. Syst.*, vol. 1, no. 1, pp. 15–27, Mar. 2014.

[2] S. H. Low, "Convex relaxation of optimal power flow—Part II: Exactness," *IEEE Trans. Control Netw. Syst.*, vol. 1, no. 2, pp. 177–189, Jun. 2014.

[3] J. Lavaei and S. H. Low, "Zero duality gap in optimal power flow problem," *IEEE Trans. Power Syst.*, vol. 27, no. 1, pp. 92–107, Feb. 2012.

[4] W. Shi, X. Xie, C. Chu, and R. Gadh, "Distributed optimal energy management in microgrids," *IEEE Trans. Smart Grid*, vol. 6, no. 3, pp. 1137–1146, May 2015.

[5] S. Bolognani, R. Carli, G. Cavraro, and S. Zampieri, "A distributed control strategy for optimal reactive power flow with power and voltage constraints," in *Proc. IEEE Int. Conf. Smart Grid Commun.*, 2013, pp. 115–120.

[6] S. Tomasin and T. Erseghe, "Power flow optimization for smart microgrids by SDP relaxation on linear networks," *IEEE Trans. Smart Grid*, vol. 4, no. 2, pp. 751–762, Jun. 2013.

[7] T. Niknam, M. Narimani, J. Aghaei, S. Tabatabaei, and M. Nayeripour, "Modified honey bee mating optimisation to solve dynamic optimal power flow considering generator constraints," *IET Gener., Transmiss. Distrib.*, vol. 5, no. 10, pp. 989–1002, Oct. 2011.

[8] Y. Kim, S. Ahn, P. Hwang, G. Pyo, and S. Moon, "Coordinated control of a DG and voltage control devices using a dynamic programming algorithm," *IEEE Trans. Power Syst.*, vol. 28, no. 1, pp. 42–51, Feb. 2013.

[9] J. Dowell and P. Pinson, "Very-short-term probabilistic wind power forecasts by sparse vector autoregression," *IEEE Trans. Smart Grid*, vol. 7, no. 2, pp. 763–770, 2016.

[10] Q. Peng and S. Low, "Distributed algorithm for optimal power flow on a radial network," in *Proc. IEEE Conf. Decision Control*, 2014, pp. 167–172.

[11] Q. Peng and S. H. Low, "Distributed optimal power flow algorithm for radial networks, I: Balanced single phase case," *IEEE Trans. Smart Grid*, pp. 1–11, 2016.

[12] W. Zhang, W. Liu, X. Wang, L. Liu, and F. Ferrese, "Online optimal generation control based on constrained distributed gradient algorithm," *IEEE Trans. Power Syst.*, vol. 30, no. 1, pp. 35–45, Jan. 2015.

[13] R. Mudumbai, S. Dasgupta, and B. Cho, "Distributed control for optimal economic dispatch of a network of heterogeneous power generators," *IEEE Trans. Power Syst.*, vol. 27, no. 4, pp. 1750–1760, Nov. 2012.

[14] J. Joo and M. D. Ilic, "Multi-layered optimization of demand resources using lagrange dual decomposition," *IEEE Trans. Smart Grid*, vol. 4, no. 4, pp. 2081–2088, Dec. 2013.

[15] P. Srikantha and D. Kundur, "Distributed optimization of dispatch in sustainable generation systems via dual decomposition," *IEEE Trans. Smart Grid*, vol. 6, no. 5, pp. 2501–2509, Sep. 2015.

[16] P. Srikantha and D. Kundur, "Distributed sustainable generation dispatch via evolutionary games," *IEEE Power Energy Soc. Innovative Smart Grid Technol. Conf.*, 2015, pp. 1–5.

[17] P. Vovos, A. Kiprakis, A. Wallace, and G. Harrison, "Centralized and distributed voltage control: Impact on distributed generation penetration," *IEEE Trans. Power Syst.*, vol. 22, no. 1, pp. 476–483, Feb. 2007.

[18] L. Du, S. Grijalva, and R. G. Harley, "Game-theoretic formulation of power dispatch with guaranteed convergence and prioritized best response," *IEEE Trans. Sustain. Energy*, vol. 6, no. 1, pp. 51–59, Jan. 2015.

[19] Y. Wang, X. Lin, and M. Pedram, "A Stackelberg game-based optimization framework of the smart grid with distributed PV power generations and data centers," *IEEE Trans. Energy Convers.*, vol. 29, no. 4, pp. 978–987, Dec. 2014.

[20] H. K. Nguyen, H. Mohsenian-Rad, A. Khodaei, and Z. Han, "Decentralized reactive power compensation using nash bargaining solution," *IEEE Trans. Smart Grid*, pp. 1–10, 2015.

[21] A. Cardenas, C. Guzman, and K. Agbossou, "Development of a FPGA based real-time power analysis and control for distributed generation interface," *IEEE Trans. Power Syst.*, vol. 27, no. 3, pp. 1343–1353, Aug. 2012.

[22] "Tesla Powerwall," *Powerwall | The Tesla Home Battery*, 2016. [Online]. Available: https://www.tesla.com/powerwall. [Accessed: 30-Nov-2016].

[23] J. Taylor, *Convex Optimization of Power Systems*. Cambridge, U.K.: Cambridge Univ. Press, 2015.

[24] W. H. Sandholm, *Population Games and Evolutionary Dynamics (Economic Learning and Social Evolution)*. Cambridge, MA, USA: MIT Press, 2010.

[25] L. Pavel, *Game Theory for Control of Optical Networks*. New York, NY, USA: Birkhauser-Springer, 2012.

[26] J. Pillai, P. Thøgersen, J. Møller, and B. Bak-Jensen, "Integration of electric vehicles in low voltage danish distribution grids," in *Proc. IEEE Power Energy Soc. Gen. Meeting*, 2012, pp. 1–8.

[27] T. Cutsem and C. Vournas, *Voltage Stability of Electric Power Systems*. New York, NY, USA: Springer, 2008.

[28] L. Zhang and T. Sidhu, "A new dynamic voltage and reactive power control method for distribution networks with DG integration," in *Proc. IEEE Elect. Power Energy Conf.*, 2014, pp. 190–195.

[29] P. Srikantha, C. Rosenberg, and S. Keshav, "An analysis of peak demand reductions due to elasticity of domestic appliances," in *Proc. ACM Int. Conf. Future Energy Syst.*, 2012, pp. 190–195.

[30] P. V. Watts, *AC energy and cost savings*, Oct. 2013. [Online]. Available: http://rredc.nrel.gov/solar/calculators/PVWATTS

[31] Canadian Wind Energy Association, "Small wind turbine purchasing guide: Off-grid, residential, farm and small business applications," 2015.

[32] T. Erseghe, "Distributed optimal power flow using ADMM," *IEEE Trans. Power Syst.*, vol. 29, no. 5, pp. 2370–2380, Sep. 2014.

[33] H. Liang, B. J. Choi, A. Abdrabou, W. Zhuang, and X. Sherman Shen, "Decentralized economic dispatch in microgrids via heterogeneous wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 6, pp. 1061–1074, Jul. 2012.

Pirathayini Srikantha received the B.A.Sc. degree in systems design engineering, Waterloo, ON, Canada, with Distinction - Dean's Honours List in 2009 and the M.A.Sc. degree in electrical and computer engineering both from the University of Waterloo, in 2013. She is currently working toward the Ph.D. degree in Edward S. Rogers Sr. Department of Electrical and Computer Engineering in the University of Toronto, ON, Canada.

Her research interests include investigating how effective solutions can be designed for current applications in the electric smart grid that include cyber-security, sustainable power dispatch, and demand response using convex optimization and game theoretic techniques. Her research has received the best paper award recognition at the 3rd IEEE International Conference on Smart Grid Communications (SmartGridComm 2012) for the symposium of "Demand Side Management, Demand Response, Dynamic Pricing."

Deepa Kundur (F'15) received the B.A.Sc., M.A.Sc., and Ph.D. degrees all in electrical and computer engineering from the University of Toronto, Toronto, ON, Canada, in 1993, 1995, and 1999, respectively. She is a Professor and the Director of the Centre for Power and Information in Edward S. Rogers Sr. Department of Electrical and Computer Engineering, University of Toronto, ON, Canada, where she is also an Associate Chair in the Division of Engineering Science. From January 2003 to December 2012, she was a faculty member in the Department of Electrical and Computer Engineering, Texas A&M University, and from September 1999 to December 2002, she was a faculty member in Electrical and Computer Engineering, University of Toronto. She is an author of more than 150 journal and conference papers. She has participated on several Editorial Boards and currently working on the Advisory Board of IEEE SPECTRUM. Her research interests include at the interface of cyber security, signal processing, and complex dynamical networks.

Dr. Kundur received the best paper recognitions at numerous venues including the 2015 IEEE Smart Grid Communications Conference, the 2015 IEEE Electrical Power and Energy Conference, the 2012 IEEE Canadian Conference on Electrical and Computer Engineering, the 2011 Cyber Security and Information Intelligence Research Workshop, and the 2008 IEEE INFOCOM Workshop on Mission Critical Networks. She has also received the teaching awards from both the University of Toronto and Texas A&M University. She is a Fellow of the Canadian Academy of Engineering. She also currently serves as General Chair for the Workshop on Communications, Computation, and Control for Resilient Smart Energy Systems at ACM e-Energy 2016, General Chair for the Workshop on Cyber-Physical Smart Grid Security and Resilience at Globecom 2016, General Chair for the Symposium on Signal and Information Processing for Smart Grid Infrastructures at GlobalSIP 2016, and Symposium Co-Chair for the Communications for the Smart Grid Track of ICC 2017.