

# Higher Bit Rates for Quasi-Linear Optical Data Transmission Systems via Constrained Coding

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**Abstract:** Application of constrained coding to 40-Gb/s dispersion-managed optical communication systems limited by intrachannel four-wave mixing is considered. When transmitted sequences obey the so-called  $(2, \infty)$  constraint, a 50% increase in data rate is demonstrated.

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## 1. Introduction

In this paper, we propose the use of a low-complexity constrained coding scheme to achieve data rate improvements of about 50% in long-haul dispersion-managed fiber-optic communication systems operating in the quasi-linear regime. Such systems have attracted considerable attention in the last few years [1–6] and are known to be mostly limited by intrachannel nonlinear impairments. In particular, intrachannel four-wave mixing (IFWM) results in energy transfer between ‘1’s and creation of ghost (or shadow) pulses in spaces. Return-to-zero (RZ) pulses used for transmission of ‘1’s also experience timing jitter caused by intrachannel cross-phase modulation (IXPM).

The intensity of IFWM-induced perturbation depends on various system parameters. Most importantly for us, it is proportional to  $E_0^3 T_0 / T_B^4$ , where  $E_0$  is the pulse energy,  $T_0$  is the pulse width and  $T_B$  is the bit slot duration [1]. This means that shorter pulses are less affected by IFWM, and a low duty cycle is preferable. Certainly, such systems may have a lot of “white space” between ‘1’s and are less attractive from the spectral efficiency point of view. We will show how our coding scheme can turn this seemingly redundant white space into extra information.

## 2. System Model

In this work we consider long-haul optically amplified on-off keyed systems operating in the quasi-linear regime. Sometimes this regime of propagation is also called “highly dispersed pulse transmission” [1] and this term will be used below as well. We assume that the bit rate  $B$  is high enough to make IFWM a paramount concern. Gordon–Haus timing jitter and IXPM are also taken into account. At the same time,  $B$  must low enough so that effects like Raman-induced timing jitter could safely be neglected. We do not include the influence of third-order dispersion or polarization-mode dispersion either.

When optical amplifiers are inserted periodically to compensate for fiber losses, amplified spontaneous emission (ASE) generated by the amplifiers degrades optical signal-to-noise ratio (SNR). Therefore, certain pulse energy  $E_0$  is required to provide a satisfactory SNR at the receiver. On the other hand, the intensity of IFWM-induced perturbations is proportional to  $E_0^3$  if the pulse width is fixed. The key design tradeoff is to select the value of  $E_0$  in such a way that the contributions from ASE noise and IFWM are somehow balanced. To make a stringent test of our coding scheme, the probability of error at the receiver,  $p_e$ , is allowed to be as large as  $10^{-2}$ . We assume that an error-correcting code (ECC) may be applied to achieve the desired bit error rate.

A 40-Gb/s “benchmark” system is designed as follows. The dispersion map consists of a 40-km-long segment of a standard fiber with  $D_+ = 17$  ps/(nm·km) followed by a reverse dispersion fiber of the same length and exactly opposite dispersion  $D_- = -17$  ps/(nm·km). To reduce IXPM-induced timing jitter we use a precompensating fiber of length 17.5 km and dispersion  $D_-$  along with a postcompensating fiber of the same length and dispersion  $D_+$ . The average dispersion of the link is thus zero. All fibers have attenuation 0.25 dB/km and their nonlinear parameter is  $2.5$  W<sup>-1</sup>/km. Optical amplifiers are placed every 80 km at the beginning of the anomalous dispersion span, their spontaneous-emission factor being 1.4. The transmission distance is 4,800 km. Unchirped Gaussian pulses of width 5 ps (full width at half maximum) and peak power  $P_0$  in the range of 4–8 mW are transmitted. The timing jitter standard deviation is found to always be below 1 ps, which implies its negligible contribution to  $p_e$ . The receiver has a simple structure and consists of an ideal 280-GHz optical bandpass filter, a square-law detector and an integrate-and-dump electrical filter. If amplitude jitter dominates,  $p_e$  for the benchmark system can be computed as in [7].

Numerical simulations were carried out for 4096-bit De Bruijn pseudorandom input sequences using the split-step Fourier method [8]. The channel capacity was estimated based on the probability of error observed. We varied  $P_0$  to

find the best operating point. When  $P_0 < 4$  mW, the influence of IFWM is negligible and  $p_e$  was in a good agreement with [7]. For example,  $p_e = 0.018$  when  $P_0 = 4$  mW. However, the optimum  $P_0$  was found to be around 6–7 mW (Fig. 1). The minimum  $p_e$  was about 0.007, which corresponds to the channel capacity of approximately 37.6 Gb/s. In this region of operation ghost pulses make a significant contribution to the probability of error that could be 0.002 or less in a purely linear regime. In the next section we present the coding scheme along with the idea how to estimate the probability of error for a constrained-coded system.

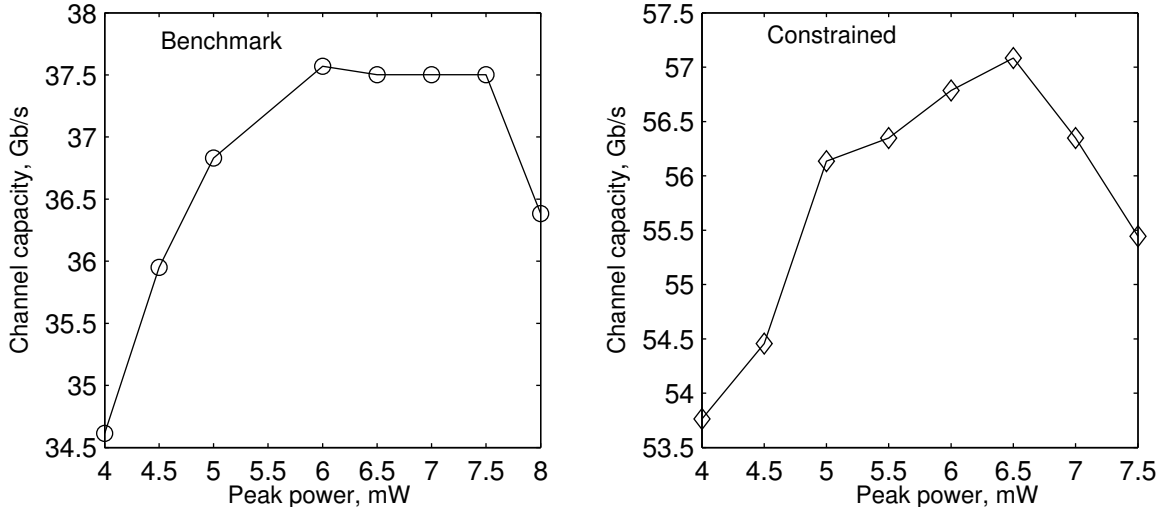


Fig. 1. Channel capacity for the benchmark system (left) and the constrained system (right).

### 3. Coding scheme

A binary channel is said to be runlength-limited (RLL), or  $(d, k)$ -constrained, if channel sequences are required to have at least  $d$  but at most  $k$  ‘0’s between any pair of adjacent ‘1’s. Information transmission through an RLL channel implies two necessary steps. First, an arbitrary source word must be mapped unambiguously to a sequence that satisfies the channel constraints. Second, some error control must be implemented to protect the data from noise. A popular approach is to utilize a concatenation of an inner modulation RLL (or constrained) code with an outer ECC. The received word is first demodulated by an RLL decoder, and an ECC decoder cleans up possible errors afterwards.

In this work we consider a  $(2, \infty)$ -constraint and use a rate  $1/2$  RLL code from [9]. A source word  $u = (u_1, \dots, u_p)$  of length  $p$  is mapped to a codeword  $x$  of length  $q = 2p$  with the help of Table 1. The concatenation of two codewords without a channel constraint violation is straightforward [9]. The decoder is implemented in a sliding-window fashion. A window of length 3 slides along the word  $x = (x_0, x_1, \dots, x_q)$  producing  $u_i = x_{2i-2} + x_{2i-1} + x_{2i}, i = 1, \dots, p$ , where the “plus” sign stands for logical “OR” operation.

Table 1. Encoding table for the RLL code.

User Data	0	10	110	111
Constrained Bits	00	1000	010000	100100

The output of the RLL decoder is an unconstrained binary sequence  $v = (v_1, \dots, v_p)$  with possible errors introduced by both the channel and the RLL decoder itself. Ignoring potential correlations between errors, we are interested in the crossover probabilities  $a = p(v_i = 1 | u_i = 0)$  and  $b = p(v_i = 0 | u_i = 1)$  that give us both the probability of error and a lower bound on the channel capacity.

In a purely linear regime of operation, when timing jitter has a known Gaussian distribution,  $a$  and  $b$  can be estimated by exploiting the fact that there are at least two ‘0’s between any pair of ‘1’s. In particular, if ‘000’ is sent, the central ‘0’ is not affected by pulses that occupy other bit slots. If, nevertheless, ‘010’ is received, we call this event a drop-in and denote the corresponding probability as  $p_{di}$ . Also, if ‘010’ is sent, we assume that only the central ‘1’ and no other ‘1’s can potentially influence either ‘0’ due to the timing jitter. Using this approximation we can compute the probabilities  $p(xyz|010)$  that ‘ $xyz$ ’ is received given that ‘010’ is sent. We introduce the following notation for the error events.

- $p_{do} = p(000|010)$  – a drop-out;
- $p_s = p(100|010) + p(001|010)$  – a bit shift;
- $p_{sdi} = p(110|010) + p(011|010)$  – a stimulated drop-in.

The remaining error events usually make negligible contribution to  $p_e$ . The crossovers  $a$  and  $b$  can then be estimated following the technique described in [9, 10]. The final result is

$$a = \frac{37}{112}(p_s + p_{sdi}) + \frac{131}{56}p_{di}, \quad b = \frac{2}{7}p_s + p_{do}. \quad (1)$$

If the probabilities of all error events are known, equations (1) are quite accurate as long as  $a < 0.1$  and  $b < 0.1$ , which was tested by Monte-Carlo simulations. When ghost pulses become an important factor, however, expressions (1) may only be used as a crude estimate. Nevertheless, we have found that constrained coding still provides a significant increase in achievable data rate, as will be discussed in the following section.

#### 4. Constrained coding for highly dispersed pulse transmission

Now we will apply the coding scheme from Section 3 to the benchmark system designed in Section 2. Due to the  $d = 2$  constraint we can potentially reduce  $T_B$  by a factor of 3 maintaining the same minimum pulse separation  $\Delta$  and, hence, approximately the same intensity of IFWM-induced perturbations. Actually, since the introduction of constrained coding makes channel sequences more irregular, ghost pulse formation may be suppressed even when  $\Delta$  remains the same, an observation made in [11].

If the channel were perfect, the application of our scheme would give a 50% increase in data rate. To estimate the gain for the real channel with ASE and IFWM, we simulated constrained systems with  $T_B = 8.35$  ps and variable  $P_0$ . Simulations were carried out using 12000-bit pseudorandom constrained sequences. A lower bound on the channel capacity was calculated based on the experimental crossovers  $a$  and  $b$  observed at the output of the RLL decoder. The maximum capacity of about 57.1 Gb/s is achieved at  $P_0 = 6.5$  mW (Fig. 1), a 52% increase in comparison with the benchmark system.

#### 5. Discussion and Conclusion

In this work we assumed an ideal ECC whose rate is equal to the channel capacity. Certainly, practical ECC's always have extra overhead. Nevertheless, since the probability of error at the decoder input is virtually identical for both systems, the ECC redundancy must be approximately the same in both cases. Therefore, our system comparison approach is fair even when the ECC overhead is taken into account. We also note that since both the encoding table and the decoding rule for the RLL code are very simple, their implementation implies very little extra hardware.

In conclusion, in this paper we have introduced constrained coding as a new approach to highly dispersed pulse transmission, and proposed a particular coding scheme. The results obtained in our example indicate that a gain in data rate of about 50% is quite achievable.

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