SIMILARITY BASED IMPULSIVE NOISE REMOVAL IN COLOR IMAGES

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ABSTRACT

In this paper a novel approach to the problem of impulsive noise removal in color images based on the nonparametric density estimation is presented. The basic idea behind the new image filtering technique is the maximization of the similarities between pixels in a predefined filtering window. The new method is faster than the standard vector median filter and preserves better edges and fine image details. Simulation results show that the proposed method outperforms other standard algorithms of the reduction of impulsive noise in color images.

1. INTRODUCTION

A number of nonlinear, multichannel filters, which utilize correlation among multivariate vectors using various distance measures, has been proposed [1]-[13]. The most popular nonlinear, multichannel filters are based on the ordering of vectors in a predefined moving window. The output of these filters is defined as the lowest ranked vector according to a specific vector ordering technique.

Let $F(x)$ represents a multichannel image and let W be a window of finite size $n+1$, (filter length). The noisy image vectors inside the filtering window W are denoted as F_j, $j = 0, 1, ..., n$. If the distance between two vectors F_i, F_j is denoted as $\rho(F_i, F_j)$ then the scalar quantity $R_i = \sum_{j=0}^{n} \rho(F_i, F_j)$ is the distance associated with the vector F_i. The ordering of the R_i's: $R_{(0)} \leq R_{(1)} \leq ... \leq R_{(n)}$, implies the same ordering to the corresponding vectors F_i : $F_{(0)} \leq F_{(1)} \leq ... \leq F_{(n)}$. Nonlinear ranked type multichannel estimators define the vector $F_{(0)}$ as the filter output, (Vector Median Filter VMF). However, the concept of input ordering, initially applied to scalar quantities is not easily extended to multichannel data, since there is no universal way to define ordering in vector spaces.

To overcome this problem, distance functions are often utilized to order vectors. The orientation difference between two vectors can also be used as their distance measure. This so-called *vector angle criterion* is used by the *Vector Directional Filters* (VDF) to remove vectors with atypical directions [5],[8].

The *Basic Vector Directional Filter* (BVDF) is a ranked-order, nonlinear filter which parallelizes the VMF operation. However, a distance criterion, different from the L_p norm used in VMF, is utilized to rank the input vectors. The output of the BVDF is that vector from the input set, which minimizes the sum of the angles with other vectors. In other words, the BVDF chooses the vector most centrally located without considering the magnitudes of the input vectors.

To improve the efficiency of the directional filters, another method called *Directional - Distance Filter* (DDF) was proposed [5]. This filter retains the structure of the BVDF but utilizes a new distance criterion to order the vectors inside the processing window.

Another efficient rank-ordered a technique called Hybrid Directional Filter (HDF) was presented in [9]. This filter operates on the directional and the magnitude of the color vectors independently and then combines them to produce a unique final output.

All standard filters detect and replace well noisy pixels, but their ability of preserving pixels which were not corrupted by the noise process is not sufficient. In this paper we present the construction of a simple, efficient and fast filter which removes noisy pixels, but has the ability of preserving original image pixel values, [12, 13].

2. PROPOSED ALGORITHM

2.1. Gray-scale Images

Let us assume a filtering window W containing $n+1$ image pixels, $\{F_0, F_1, ..., F_n\}$, where n is the number of neighbors of the central pixel F_0 and let us define the similarity function $\mu : [0; \infty) \rightarrow R$ which is non-ascending in $[0; \infty)$, convex in $[0; \infty)$ and satisfies $\mu(0) = 1$ and $\mu(\infty) = 0$. The similarity between two pixels of the same intensity should be 1, and the similarity between pixels with far dis-

tant gray scale values should be very close to 0. The function $\mu(F_i, F_j)$ defined as $\mu(F_i, F_j) = \mu(|F_i - F_j|)$ satisfies the above conditions.

Let us now define the cumulated sum M of similarities between the pixel F_k and all its neighbors. For the central pixel we have M_0 and for the neighbors of F_0 we define M_k as

$$M_0 = \sum_{j=1}^{n} \mu(F_0, F_j), \quad M_k = \sum_{\substack{j=1 \\ j \neq k}}^{n} \mu(F_k, F_j), \quad (1)$$

which means that for F_k which are neighbors of F_0 we do not take into account the similarity between F_k and F_0 (summation from $j = 1$) when calculating M_k, which is the main concept of the new algorithm. The omission of the similarity measure $\mu(F_k, F_0)$ privileges the central pixel, as in the calculation of M_0 we have n similarities, $\mu(F_0, F_k)$, $k = 1, 2, \ldots, n$ and for M_k, $k > 0$ we have $n - 1$ similarity values, as the central pixel is excluded from the sum M_k.

In the construction of the new filter, the pixel F_0 in the window W is replaced by one of its neighbors if $M_0 < M_k$, $k = 1, \ldots, n$. If this is the case, then F_0 is replaced by that F_i for which $i = \arg \max M_i$, $i = 1, \ldots, n$. In other words F_0 is detected as being corrupted if $M_0 < M_k$, $k = 1, \ldots, n$ and is replaced by its neighbors F_i which maximizes the sum of similarities M between all its neighbors excluding the central pixel, (Figs. refcumukla, 3).

Our basic assumption is that a new pixel must be taken from the window W, (introducing pixels which do not occur in the image is prohibited like in the VMF and VDF). For this purpose μ must be convex, which means that in order to find a maximum of the sum of similarity functions M it is sufficient to calculate the values of M only in points $F_0, F_1, \ldots, F_n$.

2.2. Color Images

The presented approach can be applied in a straightforward way to color images. We use the similarity function defined by $\mu\{\mathbf{F}_i, \mathbf{F}_j\} = \mu(||\mathbf{F}_i - \mathbf{F}_j||)$ where $||\cdot||$ denotes the specific vector norm. Now, in exactly the same way we maximize the total similarity function M for the vector case.

We have checked several convex similarity functions in order to compare our approach with the standard filters used in color image processing listed in Tab. 1 and we have obtained the best results (Tab. 2, Fig. 2) when applying the following similarity functions, [12]:

$$\mu_0(x) = \exp\left\{-\left(\frac{x}{h}\right)^2\right\}, \mu_1(x) = \exp\left\{-\frac{x}{h}\right\}, \ h \in (0; \infty),$$

$$\mu_2(x) = \frac{1}{1 + x/h}, \ \mu_3(x) = \frac{1}{(1+x)^h}, \ h \in (0; \infty),$$

$$\mu_4(x) = 1 - \frac{2}{\pi} \arctan\left(\frac{x}{h}\right), \ h \in (0; \infty),$$

$$\mu_5(x) = \frac{2}{1 + \exp\left\{\frac{x}{h}\right\}}, \ \mu_6(x) = \frac{1}{1 + x^h}, \ h \in (0; \infty),$$

$$\mu_7(x) = \begin{cases} 1 - x/h & \text{if } x \leq h, \\ 0 & \text{if } x > h, \end{cases} \ h \in (0; \infty). \quad (2)$$

It is interesting to note, that the best results were achieved for the simplest similarity function $\mu_7(x)$, which allows to construct a fast noise reduction algorithm. In the multichannel case, we have

$$M_0 = \sum_{j=1}^{n} \mu(\mathbf{F}_0, \mathbf{F}_j), \quad M_k = \sum_{\substack{j=1 \\ j \neq k}}^{n} \mu(\mathbf{F}_k, \mathbf{F}_j) \quad (3)$$

where $\rho\{\mathbf{F}_i, \mathbf{F}_k\} = ||\mathbf{F}_k - \mathbf{F}_l)||$ and $||\cdot||$ is the L_2 vector norm, as it yields the best results. Applying the linear similarity function μ_7 we obtain

$$\mu(\mathbf{F}_i, \mathbf{F}_k) = \begin{cases} 1 - \rho(\mathbf{F}_i, \mathbf{F}_k)/h & \text{for } \rho(\mathbf{F}_i, \mathbf{F}_k) < h \\ 0 & \text{otherwise} \end{cases}$$

$$(4)$$

Then we have from (1)

$$M_0 = n - \frac{1}{h} \sum_{j=1}^{n} \rho(F_0, F_j) \quad (5)$$

$$M_k = \sum_{\substack{j=1, \\ j \neq k}}^{n} \left(1 - \frac{\rho(F_k, F_j)}{h}\right) = n - 1 - \frac{1}{h} \sum_{j=1}^{n} \rho(F_k, F_j)$$

$$(6)$$

The difference between $M_0 - M_k$ is

$$M_0 - M_k = n - \frac{1}{h} \sum_{j=1}^{n} \rho(F_0, F_j) - \left[n - 1 - \frac{1}{h} \sum_{j=1}^{n} \rho(F_k, F_j)\right]$$

$$= 1 - \frac{1}{h} \sum_{j=1}^{n} [\rho(F_0, F_j) - \rho(F_k, F_j)] \quad (7)$$

In this way we obtained a condition for retaining the central pixel, as it is considered by the algorithm to be noise-free:

$$M_0 - M_k > 0 \quad \text{if} \quad h > \sum_{j=1}^{n} [\rho(F_0, F_j) - \rho(F_k, F_j)] \quad (8)$$

If this condition is satisfied, then the central pixel is considered as not disturbed by the noise process, otherwise the pixel $\mathbf{F}_i$ for which the cumulative similarity value achieves maximum, replaces the central noisy pixel. Thus, the new filter replaces the central pixel only when it is really noisy and preserves the original undistorted image structures.

The parameter h can be set manually by the designer or can be determined adaptively, [12].

Notation	Filter	Ref.
AMF	Arithmetic Mean Filter	[2]
VMF	Vector Median Filter	[7]
ANNF	Adaptive Nearest Neighbor Filter	[11]
BVDF	Basic Vector Directional Filter	[8]
HDF	Hybrid Directional Filter	[9]
AHDF	Adaptive Hybrid Directional Filter	[9]
DDF	Directional-Distance Filter	[5]
FVDF	Fuzzy Vector Directional Filter	[10]

Table 1. Filters compared.

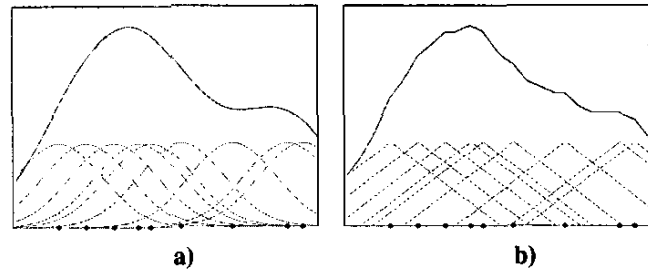| METHOD	NMSE [10^{-4}]	RMSE	PSNR [dB]
NONE	514.95	32.165	17.983
AMF	82.863	12.903	25.917
VMF	23.304	6.842	31.427
ANNF	31.271	7.926	30.149
BVDF	29,074	7.643	30.466
HDF	22.845	6.775	31.513
AHDF	22,603	6.739	31.559
DDF	24.003	6.944	31,288
FVDF	26.755	7.331	30.827
PROPOSED FILTERING SCHEME			
$\mu_0(x)$	5.056	3.163	38.137
$\mu_1(x)$	4.959	3.157	38.145
$\mu_2(x)$	5.398	3.294	37.776
$\mu_3(x)$	9.574	4.387	35.288
$\mu_4(x)$	5.064	3.190	38.054
$\mu_5(x)$	4.777	3.099	38.307
$\mu_6(x)$	11.024	4.707	34.675
$\mu_7(x)$	4.693	3.072	38.384

Table 2. Comparison of the new algorithm based on different kernel functions with the standard techniques (*LENA*) contaminated by 4% of impulsive noise.

3. CONCLUSIONS

The new presented algorithm is a modification and improvement of the VMF. The efficiency of the proposed filter has been compared with the standard algorithms of noise reduction, (Tab. 2, Figs. 2 and 4. The comparison shows that the new filter significantly outperforms the standard procedures used in color image processing, when the impulse noise should be eliminated.
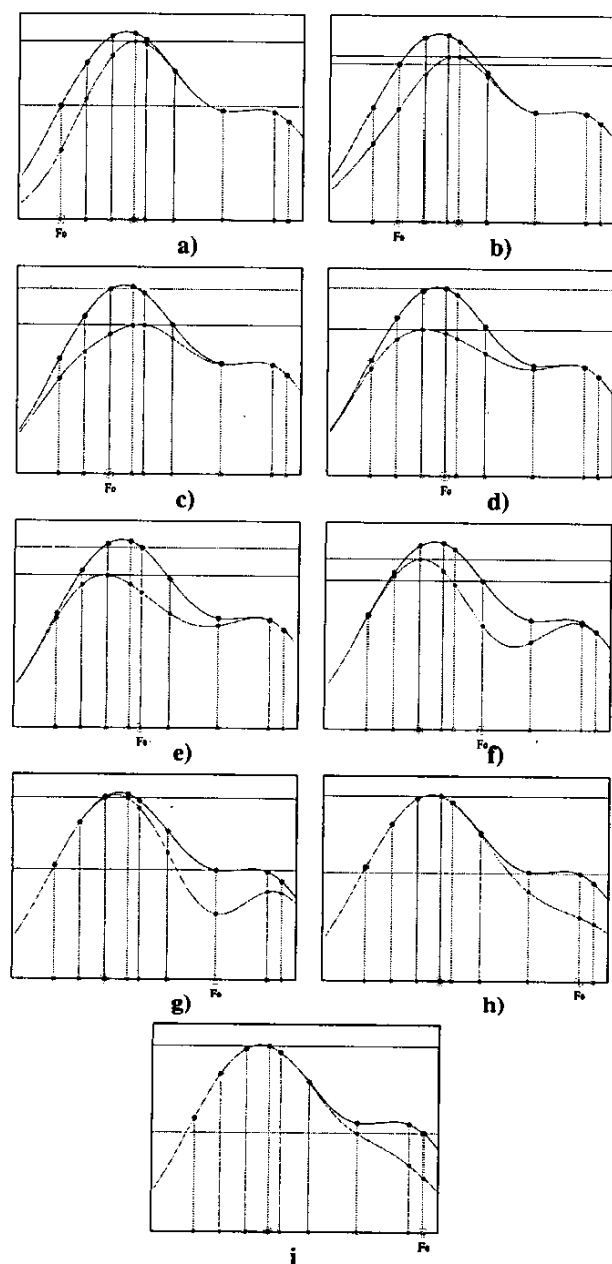
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Fig. 1. Cumulative similarity values dependence on the pixel gray scale value for a window containing a set of pixels with intensities $\{15, 24, 33, 41, 45, 55, 72, 90, 95\}$ using the μ_0 function (a) and μ_7 function (b).
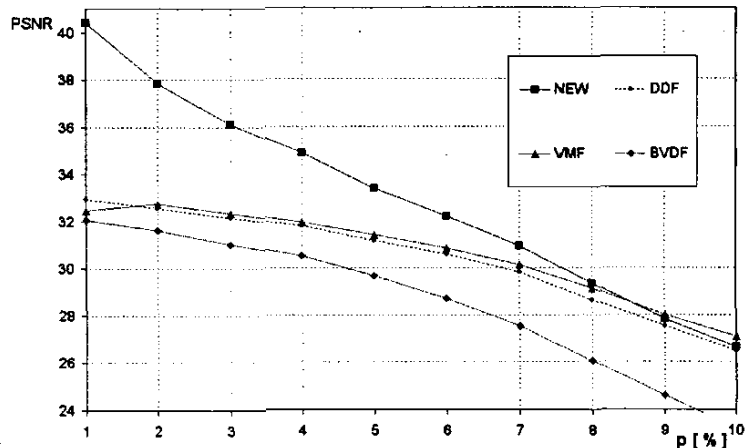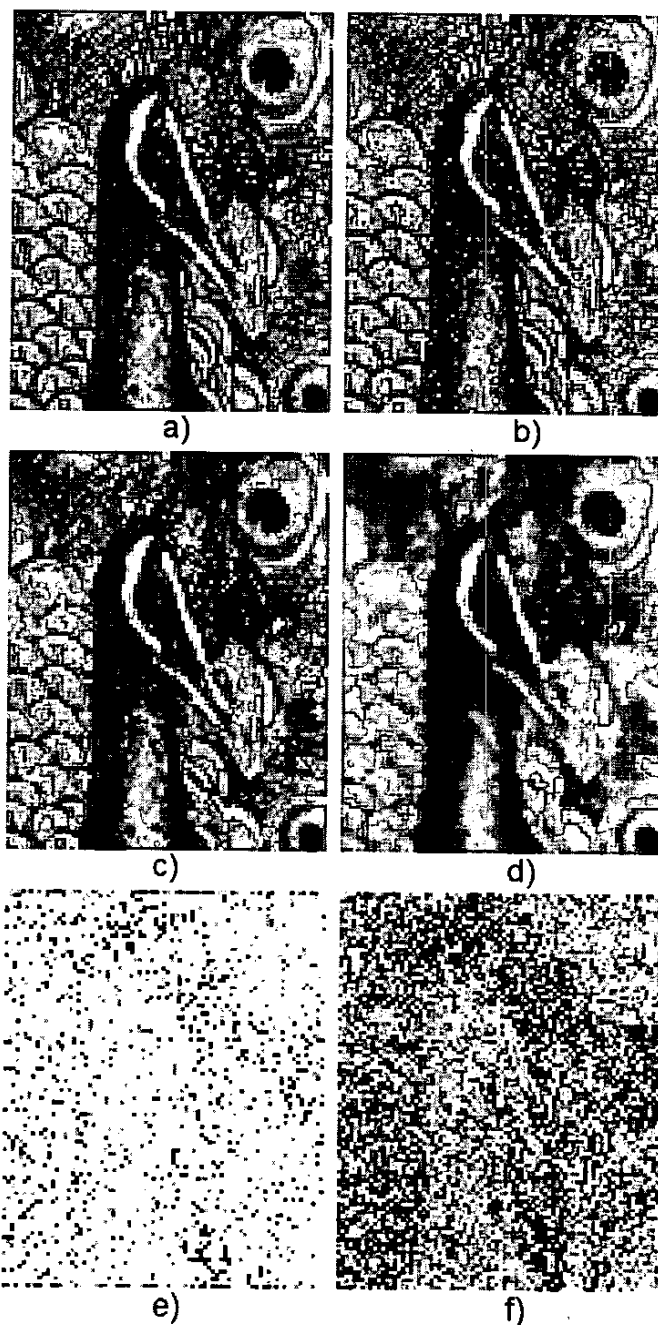

Fig. 2. Comparison of the efficiency of the new filter in terms of PSNR with VMF, BVDF and DDF using the *LENA* image corrupted by impulsive noise applied separately on each RGB channel ranging from 1 to 10%.

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Fig. 3. Illustration of the new filter construction. The supporting window W of size 3×3 contains 9 pixels of intensities $\{15, 24, 33, 41, 45, 55, 72, 90, 95\}$.

Each of the graphs from a) to i) shows the dependence of M_0 and $M_{/0}$, $(M_{/0} < M_0)$, where $M_{/0}$ denotes the cumulative similarity value with rejected central pixel F_0 on the pixel gray scale value. Graph a) shows the plot of M_0 and $M_{/0}$ for $F_0 = 15$, plot b) for $F_0 = 24$ and so on till plot plot i) which shows the graphs of M_0 and $M_{/0}$ for $F_0 = 95$. The central pixel will be replaced in cases: (a), (b), (f)- (i), as in those cases there exists a pixel F_i for which $M_0 < M_i$.

Fig. 4. Comparison of the efficiency of the vector median and the proposed filter: a) test image, b) image contaminated by 4% impulsive noise, c) image filtered using the proposed technique, d) the result of the filtering with the vector median, e) and f) depict the filtering error, (absolute value of the difference between the original and filtered image multiplied by 5).