Reduced Dimension MAP Turbo-BLAST detection

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Abstract—**The Bell-Labs Layered Space-time (BLAST) architecture is a simple and efficient multi-antenna coding structure that can achieve high-spectral efficiency [1]. Many BLAST detectors require more receiver antennas than transmitter antennas. We propose a novel turbo-processing BLAST detector based on a group detection strategy that can operate in systems with fewer receiver antennas than transmitter antennas. A maximum *a posteriori* (MAP) decision is made using a group of transmitted symbols and the remaining signal contribution is treated as interference. The interference is characterized as non-zero mean colored noise source that is whitened before a decision is made. The proposed detector, the Reduced Dimension MAP (RDMAP) detector, is a generalization of both the MAP detector and the turbo-processing Minimum Mean Squared Error (MMSE) detector in [2], [3]. Simulation is used to compare the GMAP detector with the MAP detector and MMSE detector.**

I. Introduction

The *Bell-Labs Layered Space-time (BLAST)* [1] architecture is a simple and efficient coding structure that can take advantage of the multiple-input multiple output (MIMO) channel capacity. The original detector proposed in [1] uses an Interference Cancellation and Nulling Algorithm (ICNA). An ICNA detector cannot however be applied to systems that have more transmitter antennas than receiver antennas. Such systems can exist in the downlink of a cellular systems where it is often infeasible to have a mobile station with many antenna's due to size limitations. A similar scenario can exist when there are more than one transmitters and a single receiver, thus the total number of transmitter antennas can easily exceed the number of receiver antennas.

There are several detection strategies that can be applied to systems that have an excess number on transmitter antennas. An optimal solution is the maximum likelihood (ML) detector, which unfortunately has exponential complexity. Suboptimal ML detectors have been applied to BLAST systems using tree-search algorithms [4] and group detection strategies [5], [6].

Turbo processing receivers have also been applied to systems with an excess number of transmitter antennas. The optimal turbo-BLAST detector is the maximum *a posteriori* (MAP) detector that has exponential complexity. A more computational feasible detector is the minimum mean squared error (MMSE) BLAST detector [2], [3] that is based on an CDMA MMSE detector proposed by Wang [7]. The detector in [2], [3] uses *a prior* information to partially cancel interference and an instantaneous MMSE filter to suppress residual interference. With successive iterations, the performance of the MMSE detector improves as more interference is cancelled, but falls short of the MAP detector performance.

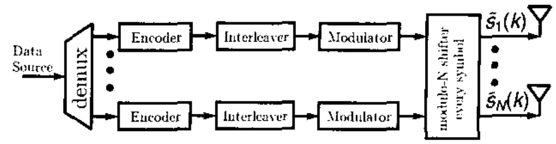

Fig. 1. Layered Space-Time Transmitter

In this paper, we a propose a novel BLAST detector, termed the Reduced Dimension MAP (RDMAP) detector, based on a group detection strategy. This detector bridges the performance gap between MAP and MMSE detectors in systems with an excess number of transmitter antenna. The RDMAP detector divides the set of transmitted symbols into two groups: an MAP group and an interfering group. The symbols in the interfering group are treated as an interfering noise source that is whitened by applying an appropriate filter. The prior probabilities for the interfering symbols are used to determine the mean of the interfering noise source. The size of the MAP group $|G|$ is an adjustable parameter that determines the complexity of RDMAP detector. Through the choice of this parameter, the RDMAP detector is a generalization of both MAP detector and MMSE detector in [2], [3]. Our group detection strategy is different from that in [5] as the solution in [5] does not use a noise whitening filter and different from the solution in [6] because we incorporate *prior* in the whitening filter.

The remainder of this paper is organized as follows. Section II provides a system model that includes the transmitter, channel model and turbo-processing receiver structure. Section III describes the RDMAP detector design. A complexity analysis and BER comparison is contained in Section IV, followed by a summary and concluding remarks in Section V.

II. System Model

Consider the transmitter structure in Figure 1 for a layered space-time architecture [2] having N transmitter antennas. Binary data is demultiplexed into N layers that are independently encoded, interleaved and modulated, then passed through a modulo-N shifter. We only consider QPSK modulation such that the modulated output for the n^{th} layer is given by $x_n(k) = \{2b_n(2k) - 1\} + \sqrt{-1}\{2b_n(2k+1) - 1\}$, where $\{b_n(l)\}$ is the coded binary $\{0, 1\}$ bitstream. The transmitted symbol on antenna n is given by $\tilde{s}_n(k) = \tilde{x}_\alpha(k)$, $\alpha = (n-k) \bmod N$. Assuming a flat fading channel model, the vector

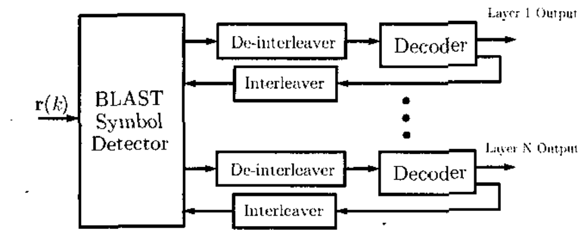

Fig. 2. Turbo Processing BLAST Receiver

channel output can be expressed as

$$\tilde{\mathbf{r}}(k) = \tilde{\mathbf{H}}\tilde{\mathbf{s}}(k) + \tilde{\mathbf{v}}(k) \tag{1}$$

where $\tilde{\mathbf{H}}$ is an $M \times N$ complex channel matrix, $\tilde{\mathbf{r}}(k)$ is the channel output, $\tilde{\mathbf{v}}(k)$ is a Gaussian noise source of variance σ^2, and M is the number of receiver antennas. It is convenient to transform the complex channel equation in (1) into real matrix equation

$$\mathbf{r}(k) = \mathbf{H}\mathbf{s}(k) + \mathbf{v}(k) \tag{2}$$

where $\mathbf{r}(k) = [\ \Re\{\tilde{\mathbf{r}}^{\mathbf{T}}(k)\}\ \ \Im\{\tilde{\mathbf{r}}^{\mathbf{T}}(k)\}\ ]$, $\mathbf{s}(k) = [\ \Re\{\tilde{\mathbf{s}}^{\mathbf{T}}(k)\}\ \ \Im\{\tilde{\mathbf{s}}^{\mathbf{T}}(k)\}\ ]$, $\mathbf{v}(k) = [\ \Re\{\tilde{\mathbf{v}}^{\mathbf{T}}(k)\}\ \ \Im\{\tilde{\mathbf{v}}^{\mathbf{T}}(k)\}\ ]$ and

$$\mathbf{H} = \begin{bmatrix} \Re\{\tilde{\mathbf{H}}\} & -\Im\{\tilde{\mathbf{H}}\} \\ \Im\{\tilde{\mathbf{H}}\} & \Re\{\tilde{\mathbf{H}}\} \end{bmatrix} \tag{3}$$

is the $M' \times N'$ real channel matrix with $M' = 2M$ and $N' = 2N$. The use of the real valued channel in (2) allows us to consider the in phase and quadrature phase components of each layer separately in the equalization process.

The block diagram for the turbo processing BLAST receiver is shown in Figure 2. The receiver consists of a BLAST symbol detector, a set of N channel decoders, and an interleaver and deinterleaver between each decoder and the detector. There are modulo-N shifters at the input and output of the detector that have been omitted from Figure 2 for clarity. In each iteration, the detector produces an *a posteriori* probability (APP) for each coded bit in the form of an log likelihood ratio (LLR) that is given by

$$\Lambda_1[b_n(l)] = \log\frac{P(b_n(l)=1|\mathbf{r}(k))}{P(b_n(l)=0|\mathbf{r}(k))} \tag{4}$$

$$\equiv \lambda_1[b_n(l)] + \lambda_2^p[b_n(l)] \tag{5}$$

where $\lambda_1[b_n(l)]$ is the *extrinsic* information that is fed to the channel decoder for the n^{th} layer and $\lambda_2^p[b_n(l)]$ is the *a priori* information provided by the n^{th} channel decoder. The channel decoders produce an LLR for each coded bit as

$$\Lambda_2[b_n(j)] = \log\frac{P(b_n(j)=1|\lambda_1[b_n(i)], i \neq j)}{P(b_n(j)=0|\lambda_1[b_n(i)], i \neq j)} \tag{6}$$

$$\equiv \lambda_2[b_n(j)] + \lambda_1^p[b_n(j)] \tag{7}$$

where $\lambda_2[b_n(j)]$ is the extrinsic information fed back to the symbol detector, $\lambda_1^p[b_n(j)]$ is the *a priori* information provided by symbol detector. After a sufficient number of iterations, estimates for the uncoded bits can be obtained as

$$\hat{c}_n(i) = \mathrm{sgn}(\Lambda_2[c_n(i)]) \tag{8}$$

where

$$\Lambda_2[c_n(i)] = \log\frac{P(c_n(i)=1|\lambda_1[b_n(j)])}{P(c_n(i)=0|\lambda_1[b_n(j)])} \tag{9}$$

The channel decoders can be efficient implemented using SISO APP module [8].

III. Detector Design

We perform an MAP decision using a subset of the elements of the symbol vector $\mathbf{s}(k)$ and treat the contribution of the remaining signal contribution as interference. For a signal vector $\mathbf{s} = \mathbf{s}(k)$, let $G = \{\alpha_1, \ldots, \alpha_{|G|}\}$ be the set of numbers corresponding to the indices of the elements of $\mathbf{s}$ used in the MAP decision and let $\bar{G} = \{\beta_1, \ldots, \beta_{|\bar{G}|}\}$ be the set of numbers corresponding to the indices of the elements not in the MAP decision group. For a particular choice of G, the channel output can be expressed as

$$\mathbf{r} = \mathbf{H}_G\mathbf{s}_G + \mathbf{H}_{\bar{G}}\mathbf{s}_{\bar{G}} + \mathbf{v} \tag{10}$$

where $\mathbf{s}_G = [s_{\alpha_1}, \ldots, s_{\alpha_{|G|}}]^T$ is the reduced dimension signal vector, $\mathbf{s}_{\bar{G}} = [s_{\beta_1}, \ldots, s_{\beta_{|G|}}]^T$ is the interference vector, $\mathbf{H}_G = [\mathbf{h}_{\alpha_1}, \ldots, \mathbf{h}_{\alpha_{|G|}}]$, $\mathbf{H}_{\bar{G}} = [\mathbf{h}_{\beta_1}, \ldots, \mathbf{h}_{\beta_{|\bar{G}|}}]$, and $\mathbf{h}_i$ is the i^{th} column of $\mathbf{H}$. The time index k has been omitted for clarity. The contribution of the interference and Gaussian noise can be treated as a colored noise source. Let $\mathbf{w} = \mathbf{H}_{\bar{G}}\mathbf{s}_{\bar{G}} + \mathbf{v}$ be the colored noise source whose mean is $\bar{\mathbf{w}} = \mathrm{E}[\mathbf{w}] = \mathbf{H}_{\bar{G}}\hat{\mathbf{s}}_{\bar{G}}$, where $\hat{\mathbf{s}}_{\bar{G}} = [\hat{s}_{\bar{G}1}, \ldots, \hat{s}_{\bar{G}|\bar{G}|}]$ and $\hat{s}_{\bar{G}i}$ is evaluated using the prior probabilities from the channel decoders as

$$\hat{s}_{\bar{G}i} = \sum_{s_{\bar{G}i} \in \{+1,-1\}} s_{\bar{G}i} P(s_{\bar{G}i}) = \tanh\left(\frac{\lambda_2^p[b]}{2}\right) \tag{11}$$

where b is the bit that determines the symbol $s_{\bar{G}i} = 2b - 1$. The covariance of $\mathbf{w}$ is given by

$$\mathbf{R}_w = \mathrm{E}[(\mathbf{w} - \bar{\mathbf{w}})(\mathbf{w} - \bar{\mathbf{w}})^T] = \mathbf{H}_{\bar{G}}\Omega\mathbf{H}_{\bar{G}}^H + \mathbf{I}\sigma^2 \tag{12}$$

where $\Omega = \mathrm{diag}(\omega_1, \ldots, \omega_{|\bar{G}|})$ and $\omega_i = \mathrm{E}[|s_{\bar{G}i} - \hat{s}_{\bar{G}i}|^2] = 1 - \hat{s}_{\bar{G}i}^2$. The noise $\mathbf{w}$ can be whitened by first removing the mean $\bar{\mathbf{w}}$ and then applying an appropriate noise whitening filter $F = \mathbf{\Sigma}^{-1/2}\mathbf{Q}^H$, where $\mathbf{\Sigma}$ is a diagonal matrix and $\mathbf{Q}$ is an orthogonal matrix, both obtain from the eigenvalue decomposition of $\mathbf{R}_w = \mathbf{Q}\mathbf{\Sigma}\mathbf{Q}^H, \mathbf{Q}\mathbf{Q}^H = \mathbf{I}$. The whitened channel observation is given by

$$\mathbf{y} = \mathbf{F}(\mathbf{r} - \bar{\mathbf{w}}) \tag{13}$$

The APP for the bit b_i corresponding to the i^{th} symbol in $\mathbf{s}_G$ can be evaluated as

$$\Lambda_1[b_i] = \log\frac{P(b_i=1|\mathbf{y})}{P(b_i=0|\mathbf{y})} \tag{14}$$

$$= \log\frac{\sum_{s_{Gi}=+1} P(\mathbf{s}_G)\exp\left(\frac{-\|\mathbf{y}-\mathbf{F}\mathbf{H}_G\mathbf{s}_G\|^2}{2}\right)}{\sum_{s_{Gi}=-1} P(\mathbf{s}_G)\exp\left(\frac{-\|\mathbf{y}-\mathbf{F}\mathbf{H}_G\mathbf{s}_G\|^2}{2}\right)} \tag{15}$$

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where S_G is the set of possible $\mathbf{s}_G$ and s_{Gi} is the i^{th} element in $\mathbf{s}_G$.

The choice of the groups G and $\bar{G}$ is critical to the performance of the RDMAP detector. A direct approach to find $\{G, \bar{G}\}$ is to examine all possible choices of G and $\bar{G}$, and look at the minimum distances between constellation points in the noise-whitened channel observation space. The enumeration of the possible G and $\bar{G}$ is however computationally infeasible. A simpler approach is use a matched filter (MF) detector and examine how the mean squared error (MSE) at the filter output is affected by each interfering symbol. If an interfering symbol has a high MSE, then it will likely have a significant impact on the detector output and should be included in the MAP group G. To choose G for a bit decision b_i, we treat the interference from a symbol s_j as a single colored noise source, match filter with $\mathbf{h}_i$ to produce an MSE given by

$$\varepsilon_j^2 = |\mathbf{h}_i^T \mathbf{h}_j|^2 \omega_j^2 \quad (16)$$

G is formed by the indices of the N_G largest ε_j^2's.

The RDMAP detector is equivalent to the MAP in the limiting case of $N_G = N'$ and equivalent to the MMSE detector [2], [3] detector in the limiting case of $N_G = 1$. For $N_G = N'$, $\mathbf{H}_G = \mathbf{H}\mathbf{P}^T$, where $\mathbf{P}$ is a permutation matrix. Since $\mathbf{P}$ does not affect the MAP decision, the RDMAP detector in this case is equivalent to the MAP for $N_G = N'$. Looking to the $N_G = 1$ case, the MMSE detector in [2], [3] forms a decision according to

$$\lambda_1[b_i] = \frac{2\mathbf{h}_i^T \mathbf{R}_i^{-1}(\mathbf{r} - \bar{\mathbf{w}})}{1 - \mathbf{h}_i^T \mathbf{R}_i^{-1} \mathbf{h}_i} \quad (17)$$

where $2\mathbf{h}_i^T \mathbf{R}_i^{-1}$ is the MMSE filter, $1 - \mathbf{h}_i^T \mathbf{R}_i^{-1} \mathbf{h}_i$ is an estimate of the noise variance at the filter output, $\mathbf{R}_i = [\mathbf{H}\mathbf{\Omega}_i \mathbf{H}^T + \sigma^2 \mathbf{I}]$, $\mathbf{\Omega}_i = \mathrm{diag}(\omega_1, \ldots, \omega_{i-1}, 1, \omega_{i+1}, \omega_N)$ and $\omega_j = \mathrm{E}[|s_i - \hat{s}_i|^2] = 1 - \hat{s}_i^2$. For $N_G = 1$, the extrinsic component $\lambda_1[b_i]$ of the RDMAP decision in (15) can be simplified as

$$\lambda_1[b_i] = 2\mathbf{h}_i \mathbf{R}_w^{-1}(\mathbf{r} - \bar{\mathbf{w}}) \quad (18)$$

Substituting $\mathbf{R}_w = \mathbf{R}_i + \mathbf{h}_i \mathbf{h}_i^T$ in (18) and using the matrix inversion lemma [1] with $\mathbf{A} = \mathbf{R}_i$, $\mathbf{B} = -1$ and $\mathbf{X} = \mathbf{h}_i$ gives the MMSE decision in (17) after some simplification. Thus the RDMAP detector is equivalent to the MMSE detection in [2], [3] for $N_G = 1$.

IV. Simulated Results

This section analyzes the BER performance and complexity of the RDMAP detector. The MAP detector and MMSE detector in [2], [3] are used for comparison in terms of both complexity and performance. Simulations were preformed using bursts of 100 symbols and each layer was encoded using a rate 1/2 convolutional code with generating polynomial $(7, 5)$. A random interleaver and deinterleaver was used. Estimates for uncoded bits were produced after 10 turbo iterations. An independent Rayleigh fading model was used to determine the channel matrix $\mathbf{H}$ and perfect channel knowledge was assumed at the receiver.

A complexity analysis for the RDMAP detector is shown in Table I along with the complexity of the MAP and MMSE [2], [3] detectors for reference. All operations are shown for a single coded bit decision. The number of multiplications (mult) and additions (add) is approximate since only the highest polynomial term of M', N', N_G etc. is shown for clarity and lower power terms are omitted. The number of elementary operations involve in a matrix inverse (inv) and eigenvalue decomposition (eig) is difficult to evaluate, thus the complexity is expressed in $O(n)$ notation. It was assume the noise free channel outputs $\mathbf{r} = \mathbf{Hs}$ were precomputed for the MAP detector, but produced online for the RDMAP detectors. If the group size N_G is chosen to be moderately small, the complexity of RDMAP detector is polynomial with respect to N', M', and only moderately higher than that of the MMSE detector.

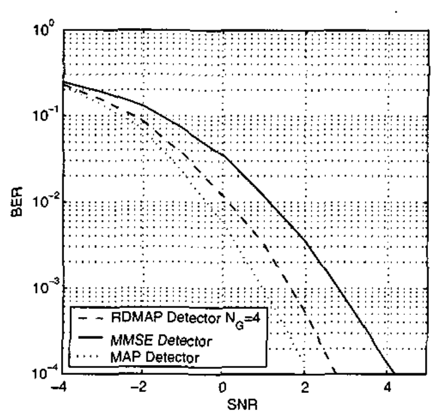

Fig. 3. BER Comparison $N = 6, M = 3$ for RDMAP, MMSE, and MAP detectors

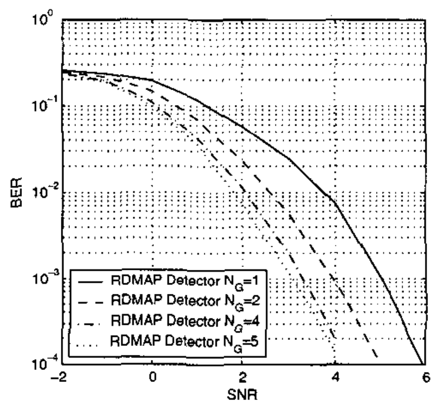

Fig. 4. BER for $N = 10, M = 4$ system using RDMAP detector for $N_G = 1, 2, 4, 5$

[1] $(\mathbf{A} + \mathbf{X}\mathbf{B}\mathbf{X}^T)^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{X}(\mathbf{B}^{-1} + \mathbf{X}^T \mathbf{A}^{-1} \mathbf{X})^{-1} \mathbf{X}^T \mathbf{A}^{-1}$

	MAP	MMSE	RDMAP
mult	$2^{N'}(M+N)$	$M^2 N$	$M_{\bar{G}}^{N} + 2^{N_G} M N_G$
add	$2^{N'} M$	$M^2 N$	$M^2 N_{\bar{G}} + 2^{N_G} M N_G$
inv		$O(M^3)$	
eig			$O(M^3)$

TABLE I

APPROXIMATE COMPLEXITY OF MAP, MMSE [2], [3], AND RDMAP DETECTORS FOR EACH CODED BIT DECISION

We consider two simulated examples. The first example is a system with $N = 6$ transmitting and $M = 3$ receiving antennas. The BER curves for the RDMAP, MMSE, and MAP detectors are shown in Figure 3. For the real signal vector of dimension $N' = 12$, the group size for RDMAP detector was set to $N_G = 4$. The RDMAP detector had a performance improvement over the MMSE detector of slightly more than $1dB$ at nominal BER of 10^{-3}. In the second example, we consider a system with $N = 10$ transmitting antennas and $M = 4$ receiving antennas. The BER curves for the RDMAP detector are shown in Figure 4 for different groups sizes $N_G = 1, 2, 4, 5$, where the $N_G = 1$ case corresponds to the MMSE detector. The MAP detector performance has been omitted as each decision requires on order of 2^{20} operations. The RDMAP detector performance improves with increasing N_G, with a performance improvement of approximately $2dB$ over the MMSE detector for $N_G = 5$ case.

V. SUMMARY AND CONCLUSIONS

In this paper, we proposed a novel RDMAP group detector that operates within a turbo processing BLAST receiver. This detector can be applied to systems having fewer receiver antennas than transmitter antenna. The RDMAP detector allows a tradeoff between complexity and performance through the MAP group size and includes as special cases, both the MAP detector and MMSE detector in [2], [3]. A novel grouping algorithm is developed for the RDMAP detector. For systems with an excess number transmitter antennas, the proposed detector has a significant performance improvement over the MMSE detector in [2], [3] with a relatively small MAP group.

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