Homework #4

1. Papoulis & Pillai 9-44, 11-11

2. (*From ALG 6-93*) Let $Y(t) = X(t) + W(t)$, where $X(t)$ and $W(t)$ are orthogonal random processes, and $W(t)$ is a WSS white Gaussian noise process. Let $\phi_n(t)$ be the eigenfunctions corresponding to $K_X(t_1, t_2)$. Show that $\phi_n(t)$ are also the eigenfunctions for $K_Y(t_1, t_2)$. What is the relation between the eigenvalues of $K_X(t_1, t_2)$ and $K_Y(t_1, t_2)$?

3. In a digital communication system, every n-bit codeword is encoded into a message represented by a discrete random variable M uniformly distributed between 0 and $2^n - 1$. The continuous-time communication channel is modeled as signal plus an independent additive noise, i.e., $Y(t) = S(t) + N(t)$. Suppose the noise $N(t)$ is Gaussian with known K-L expansion $\{\lambda_k, \phi_k(t)\}$ for $0 < t < T$. Then, one way to transmit the digital message is to send signals of the form $S(t) = M\phi_{k_0}(t)$, for some choice of k_0. (This gives a data rate of n/T bits per second.) The receiver processes the received signal $Y(t)$ to determine the value of M. Suppose that the receiver bases its decision exclusively on

$$Y_k = \int_0^T Y(t)\phi_k{}^*(t)dt \ ,$$

   a. Show that the receiver can safely ignore all Y_k's except Y_{k_0}.

   b. How should we choose the best k_0?

4. Papoulis & Pillai 7-18, 7-19

5. (*From ALG 4-105*) Let $Y = X + N$ where X and N are independent zero-mean Gaussian random variables with different variances.

   a. Plot the correlation coefficient between the "observed signal" Y and the "desired signal" X as a function of the signal-to-noise ratio σ_X/σ_N.

   b. Find the minimum mean square error estimator for X in terms of Y. Find the mean square error for the estimator.

6. (*From ALG 4-107*) Let X_1, X_2, X_3, $\ldots$ be the samples of a speech waveform which is represented by a process with zero mean, variance σ^2, and autocovariance $\rho_{|j-k|}\sigma^2$. Suppose we want to interpolate for the value of a sample in terms of the previous and the next sample; that is, we wish to find the best linear estimate for X_2 in terms of X_1 and X_3.

   a. Find the coefficients of the best linear estimator (interpolator).

   b. Find the mean square error of the best linear interpolator.

   c. Suppose that the samples are jointly Gaussian. Find the pdf of the interpolation error.

7. Papoulis & Pillai 13-1, 13-2, 13-3, 13-8