Optimal Multi-antenna Relay Beamforming with Per-Antenna Power Control

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***Abstract*—We consider amplify-and-forward multi-antenna relaying between a single pair of source and destination under per-antenna power constraints. Our objective is to obtain the optimal relay processing matrix to minimize the maximum individual antenna power for a given received SNR target. The problem is not convex, but it can be shown to satisfy strong Lagrange duality. We reveal a prominent structure of this problem, by establishing its duality with direct point-to-point SIMO beamforming with an uncertain noise. This enables us to derive a semi-closed form expression for the optimal relay processing matrix that depends on a set of dual variables, thus converting the original optimization of a $N \times N$ matrix with $(N+1)$ constraints, to a dual problem with $(N+1)$ variables and three constraints. We further show that the dual problem has a semi-definite programming form, so that the proposed solution has polynomial worst-case complexity.**

I. Introduction

We study the optimal design of amplify-and-forward (AF) multi-antenna relaying to assist dual-hop data transmission. A processing matrix is used at the relay to linearly combine received signals to forward to the destination. The central question for the design is how to optimize this relay processing matrix for a given performance metric. This involves finding both the structure of the optimal processing matrix and the jointly optimal power allocation.

For transmission between a single pair of source and destination, optimal relay design has been studied under different performance criteria, such as capacity, diversity gain, and relay power minimization under quality-of-service constraints [1]-[4]. For many cases studied, the processing matrix inherits a beamforming structure characterized by the channels at the first and second hops. The relaying design for multiple sources and/or destinations are also studied in [5]-[7]. Either numerical methods are proposed to obtain approximate solution for the optimal processing matrix, or suboptimal structure is imposed, as the explicit solution for the optimal processing matrix cannot be obtained. All these existing results are obtained based on a sum-power constraint at the relay.

In a practical system, each antenna is limited by its own front-end power amplifier, so that a realistic multi-antenna relay processing design is constrained by a per-antenna power budget. With this constraint, the relay design becomes more challenging. Even for the scenario of a single pair of source and destination, none of the techniques developed in [1]-[4] is applicable. Furthermore, the structure of the processing matrix under the per-antenna power constraint becomes unclear. For SNR maximization, for example, the processing matrix may no longer possess the rank-one beamforming matrix structure as found in [3].

In this work, we consider the more practical per-antenna power constraint. For dual-hop AF multi-antenna relaying between a single pair of source and destination, given a received SNR target, we design the optimal relay processing matrix to minimize the maximum power consumption among the relay transmit antennas. The solution can be applied to solve an alternate design objective, to maximize the received SNR with a uniform per-antenna power constraint. Our approach is inspired by the framework in [8] for direct downlink communication, where the optimal transmit beamforming design is obtained under per-antenna power constraints. However, different from downlink beamforming, multi-antenna relaying leads to a unique structure for the received SNR, which depends on the channels over two hops and the additional noise amplification, in addition to the per-antenna power control at the relay. This complicates the optimization problem with new challenges.

Nonetheless, we show that the originally non-convex problem can still be transformed into an equivalent problem with zero duality gap. Interestingly, through the Lagrange dual method, we establish a duality between multi-antenna relay beamforming and direct point-to-point SIMO beamforming with uncertain noise and a channel vector formed by concatenating the two-hop relay channel vectors. This enables us to derive a semi-closed form expression for the optimal relay processing matrix. With N relay antennas, this solution not only reveals the structure of the optimal processing matrix, but also allows us to convert the original optimization problem with $N \times N$ variables in the processing matrix and $(N+1)$ constraints, to one with $(N+1)$ variables and three constraints. We further show that the dual problem has a semi-definite programming (SDP) form, which has polynomial worst-case complexity [9]. This greatly reduces the computation complexity in determining the final solution.

Notations: $\|\cdot\|$ denotes the Euclidean norm of a vector. $E\{\cdot\}$ denotes statistical expectation. $\otimes$ stands for the Kronecker product. Hermitian and transpose are denoted as $(\cdot)^H$ and $(\cdot)^T$, respectively. Conjugate is denoted as $(\cdot)^*$.

II. Problem Formulation

A. System Model

We consider a dual-hop AF relaying system where a source and a destination each equipped with a single antenna

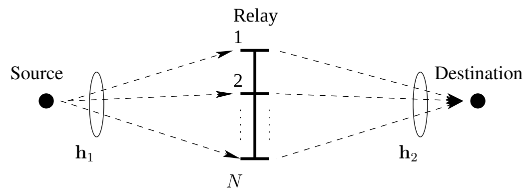

Fig. 1: AF multi-antenna relaying.

communicate through a relay equipped with N antennas, as illustrated in Fig. 1. Half-duplex transmission is assumed, and the relaying takes place in two phases. In the first phase, the source sends the signal to the relay. The received signal vector at the relay is given by $\mathbf{y}_r = \mathbf{h}_1\sqrt{P_o}s + \mathbf{n}_r$, where s is the transmit signal from the source with unit power, P_o is the given transmit power at the source, $\mathbf{h}_1 = [h_{1,1}, \cdots, h_{1,N}]^T$ is the $N \times 1$ complex channel vector between the source and the relay, and $\mathbf{n}_r$ is the $N \times 1$ complex additive white Gaussian noise (AWGN) vector with covariance $\sigma_r^2\mathbf{I}$. In the second phase, the relay forwards a processed version of signals to the destination. The received signal at the destination is given by

$$y_d = \mathbf{h}_2^T\mathbf{W}\mathbf{h}_1\sqrt{P_o}s + \mathbf{h}_2^T\mathbf{W}\mathbf{n}_r + n_d \tag{1}$$

where $\mathbf{h}_2 = [h_{2,1}, \cdots, h_{2,N}]^T$ is the $N \times 1$ complex channel vector between the relay and the destination, $\mathbf{W}$ is the $N \times N$ complex relay processing matrix, and n_d is the AWGN at the destination receiver with variance σ_d^2. The received signal-to-noise ratio (SNR) at the destination is given by

$$\mathrm{SNR} = \frac{P_o|\mathbf{h}_2^T\mathbf{W}\mathbf{h}_1|^2}{\sigma_r^2\|\mathbf{h}_2^T\mathbf{W}\|^2 + \sigma_d^2}. \tag{2}$$

The end-to-end performance such as data rate or BER is a function of the received SNR given above. The optimization of the relay processing matrix $\mathbf{W}$ for a given performance metric can then be converted directly to that for the SNR metric.

We point out that, although we limit to single antenna setting at the source and destination, the results developed in this work is directly applicable to the case of MIMO relay beamforming where source and/or destination are equipped with multiple antennas, provided that the source beamforming vector and the destination combining vector are fixed.

B. Relay Per-Antenna Power Limitation

Our goal is to design an optimal $\mathbf{W}$ at the relay to maximize the received SNR at the destination, subject to a given relay power constraint. For a sum-power constraint over N antennas at the relay, this problem has been investigated in the past [3], [4]. In this paper, we focus on per-antenna power constraints that are more realistic in a practical system. The per-antenna power constraint at the output of the relay is given by

$$\mathrm{E}\{|[\mathbf{W}\mathbf{y}_r]_i|^2\} = \left[P_0\mathbf{W}\mathbf{h}_1\mathbf{h}_1^H\mathbf{W}^H + \sigma_r^2\mathbf{W}\mathbf{W}^H\right]_{i,i} \le P_i \tag{3}$$

for $i = 1, \cdots, N$, where P_i is the power budget at antenna i.

Alternatively, we consider the min-max problem of the relay per-antenna power for a given SNR target γ_o at the destination. This optimization problem can be formulated as

$$\min_{\mathbf{W}} \max_i \; P_i \tag{4}$$

$$\text{subject to} \quad \frac{P_o|\mathbf{h}_2^T\mathbf{W}\mathbf{h}_1|^2}{\sigma_r^2\|\mathbf{h}_2^T\mathbf{W}\|^2 + \sigma_d^2} \ge \gamma_0, \tag{5}$$

$$\left[P_0\mathbf{W}\mathbf{h}_1\mathbf{h}_1^H\mathbf{W}^H + \sigma_r^2\mathbf{W}\mathbf{W}^H\right]_{i,i} \le P_i \tag{6}$$

$$\text{for } i = 1, \cdots, N$$

where $[\cdot]_{i,i}$ denotes the ith diagonal entry of a matrix. It can be shown that SNR maximization under per-antenna power constraints with the same power budget can always be solved through the above optimization problem. The details are omitted due to page limitation.

III. Optimal Relay Beamforming Design

In this section, we provide the solution to the optimization problem in (4). We first show that the problem can be transformed into a formulation, for which the Lagrange dual method [10] can be applied to obtain the solution. The dual method leads to the establishment of the duality of multi-antenna relay beamforming to SIMO beamforming in direct point-to-point communication. Finally, we provide an SDP formulation as the numerical method to determine $\mathbf{W}$.

A. Feasibility Condition

The feasibility of the optimization in (4) depends on the existence of $\mathbf{W}$ to satisfy the SNR constraint (5). It is determined by the values of the given transmit power P_o, the SNR target γ_o, and the channel condition $\mathbf{h}_1, \mathbf{h}_2$. We now state the feasibility condition for (4).

Proposition 1: The multi-antenna relay beamforming problem (4) is feasible only if the source transmit power P_0 and destination target SNR γ_o satisfy

$$\frac{\gamma_o}{P_0}\mathbf{h}^H\mathbf{R}_g^{-1}\mathbf{h} < 1 \tag{7}$$

where $\mathbf{h} \triangleq \mathrm{vec}(\mathbf{h}_1\mathbf{h}_2^T) = \mathbf{h}_2 \otimes \mathbf{h}_1$, and $\mathbf{R}_g \triangleq (\mathbf{h}_2\mathbf{h}_2^H) \otimes \mathbf{I}\sigma_r^2$.

Proof: To guarantee (5) has a solution, we first need to find an upper bound, $\mathrm{SNR}_{\mathrm{up}}$, of the SNR in (2). Then, we can show that (5) is feasible only if there exists $\mathbf{W}$, such that $\mathrm{SNR}_{\mathrm{up}} > \gamma_o$. This condition eventually leads to (7). Details are omitted due to page limitation. ∎

B. Optimization via Lagrange Dual Method

It is easy to see that the optimization in (4) is equivalent to

$$\min_{\mathbf{W}} \; P_r \tag{8}$$

$$\text{subject to} \quad (5) \quad \text{and}$$

$$\left[P_0\mathbf{W}\mathbf{h}_1\mathbf{h}_1^H\mathbf{W}^H + \sigma_r^2\mathbf{W}\mathbf{W}^H\right]_{i,i} \le P_r \tag{9}$$

$$\text{for } i = 1, \cdots, N.$$

In other words, it is equivalent to the problem of uniformly minimizing per-antenna power*. In the following, we carry

*It can be shown that the same framework also applies to the power minimization problem where each antenna has a power limit P_i, and the objective is to minimize the fraction η used on each antenna, *i.e.,* ηP_i.

the discussion assuming the problem has a feasible solution. Since (5) is not a convex constraint, the optimization problem (8) is not convex. Nonetheless, we show in the following that the optimization can be solved in the Lagrange dual domain.

Proposition 2: The optimization problem in (4) has zero duality gap.

Proof: We here provide an outline of the proof. We first show that the constraint function in (9) is convex. To see this, let $\mathbf{W}^H = [\mathbf{w}_1, \cdots \mathbf{w}_N]$. Then, (9) can be rewritten as

$$\left[P_0\mathbf{W}\mathbf{h}_1\mathbf{h}_1^H\mathbf{W}^H + \sigma_r^2\mathbf{W}\mathbf{W}^H\right]_{i,i} = \mathbf{w}_i^H(P_0\mathbf{h}_1\mathbf{h}_1^H + \sigma_r^2\mathbf{I})\mathbf{w}_i$$

which shows that the constraint function is convex w.r.t. $\mathbf{w}_i$.

Only (5) is a non-convex constraint function. However, we show that (5) can be converted into a second-order cone programming (SOCP) constraint [10]. Denote $\mathbf{w} \triangleq \mathrm{vec}(\mathbf{W}^H)$. Omitting details, we can show that the constraint in (5) can be rewritten as

$$\frac{P_o|\mathbf{h}_2^T\mathbf{W}\mathbf{h}_1|^2}{\sigma_r^2\|\mathbf{h}_2^T\mathbf{W}\|^2 + \sigma_d^2} = \frac{P_o|\mathbf{w}^H\mathbf{h}|^2}{\|\mathbf{R}_g^{1/2}\mathbf{w}\|^2 + \sigma_d^2} \tag{10}$$

where $\mathbf{h} = \mathbf{h}_2 \otimes \mathbf{h}_1$, and $\mathbf{R}_g = (\mathbf{h}_2\mathbf{h}_2^H) \otimes \mathbf{I}\sigma_r^2$. Then, the inequality in (5) can be rewritten as

$$\sqrt{P_o}|\mathbf{w}^H\mathbf{h}| \geq \sqrt{\gamma_o}\left\| \begin{matrix} \mathbf{R}_g^{1/2}\mathbf{w} \\ \sigma_d \end{matrix} \right\| \tag{11}$$

which is a SOCP constraint, implying strong duality when the dual problem is formulated in the conic form [10]. Furthermore, in [11], it is shown that this dual problem is equivalent to the dual formulation of the original problem. Therefore, the optimization problem in (8) has zero duality gap to its dual problem. ■

Following Proposition 2, we solve the optimization problem (8) through the Lagrange dual. Let the Lagrangian for (8) be

$$L(P_r, \mathbf{W}, \mathbf{\Lambda}, \nu) = P_r - \nu\left\{\frac{P_0}{\gamma_0}\left|\mathbf{w}^H\mathbf{h}\right|^2 - \|\mathbf{R}_g^{\frac{1}{2}}\mathbf{w}\|^2 - \sigma_d^2\right\}$$
$$+ \sum_{i=1}^{N}\lambda_i\left\{\left[P_o\mathbf{W}\mathbf{h}_1\mathbf{h}_1^H\mathbf{W}^H\right]_{i,i} + \left[\sigma_r^2\mathbf{W}\mathbf{W}^H\right]_{i,i} - P_r\right\} \tag{12}$$

where $\mathbf{\Lambda} \triangleq \mathrm{diag}(\lambda_1, \cdots, \lambda_N)$ is the diagonal matrix of Lagrange multipliers corresponding to the per-antenna power constraints, and ν is the Lagrange multiplier corresponding to the received SNR target. The dual problem is given by

$$\max_{\mathbf{\Lambda},\nu}\min_{P_r,\mathbf{W}} L(P_r, \mathbf{W}, \mathbf{\Lambda}, \nu) \tag{13}$$
$$\text{subject to } \mathbf{\Lambda} \succeq 0, \quad \nu \geq 0.$$

C. Duality with Direct Point-to-Point SIMO Beamforming

In this section, we show that the optimization in (13) can be transformed into the dual power minimization problem of SIMO beamforming in direct point-to-point communication

For the SIMO beamforming problem under consideration, the transmitter has a single antenna with transmit power $\tilde{P}$. The receiver has N^2 antennas with a receiver noise covariance matrix $\tilde{\mathbf{\Sigma}}$. Assume that the channel is given by $\mathbf{h}$, and $\tilde{\mathbf{w}}$ is the receiver beamforming vector. The objective is to jointly optimize $\tilde{\mathbf{w}}$ and $\tilde{P}$ to satisfy a given received SNR target γ_o:

$$\min_{\tilde{\mathbf{w}}} \quad \tilde{P} \tag{14}$$
$$\text{subject to} \quad \frac{\tilde{P}\left|\tilde{\mathbf{w}}^H\mathbf{h}\right|^2}{\tilde{\mathbf{w}}^H\tilde{\mathbf{\Sigma}}\tilde{\mathbf{w}}} \geq \gamma_o.$$

We establish the duality between the multi-antenna relay beamforming and the direct point-to-point SIMO beamforming with uncertain noise and the same SNR requirement in the following result.

Theorem 1: The Lagrange dual problem associated with (4) is equivalent to the following problem:

$$\max_{\mathbf{\Lambda}}\min_{\nu,\tilde{\mathbf{w}}} \quad \nu\sigma_d^2 \tag{15}$$
$$\text{subject to} \quad \frac{\nu P_0\left|\tilde{\mathbf{w}}^H\mathbf{h}\right|^2}{\tilde{\mathbf{w}}^H\mathbf{\Sigma}\tilde{\mathbf{w}}} \geq \gamma_o \tag{16}$$
$$\mathrm{tr}\left(\mathbf{\Lambda}\right) \leq 1, \quad \mathbf{\Lambda} \text{ is diagonal} \tag{17}$$
$$\mathbf{\Lambda} \succeq 0, \nu \geq 0 \tag{18}$$

where $\mathbf{\Sigma} \triangleq \mathbf{\Lambda} \otimes \left(P_o\mathbf{h}_1\mathbf{h}_1^H + \sigma_r^2\mathbf{I}\right) + \nu\left(\mathbf{h}_2\mathbf{h}_2^H \otimes \sigma_r^2\mathbf{I}\right)$ is the receiver noise covariance matrix, and $\mathbf{\Lambda} \triangleq \mathrm{diag}(\lambda_1, \cdots, \lambda_N)$. Furthermore, the problem (15) can be interpreted as a point-to-point SIMO beamforming problem (14) with a dual transmit power $\tilde{P} = \nu\sigma_d^2$, the dual channel $\mathbf{h} = \mathrm{vec}(\mathbf{h}_1\mathbf{h}_2^T)$, and the noise covariance matrix $\tilde{\mathbf{\Sigma}} = \frac{\sigma_d^2}{P_0}\mathbf{\Sigma}$, for all diagonal $\mathbf{\Lambda} \succeq 0$, such that the SNR constraint (16) is satisfied.

Proof: The Lagrangian for (8) is given in (12). Omitting details, we can show that

$$\sum_{i=1}^{N}\lambda_i\left[\mathbf{W}(P_o\mathbf{h}_1\mathbf{h}_1^H + \sigma_r^2\mathbf{I})\mathbf{W}^H\right]_{i,i}$$
$$= \mathbf{w}^H\left[\mathbf{\Lambda} \otimes (P_o\mathbf{h}_1\mathbf{h}_1^H + \sigma_r^2\mathbf{I})\right]\mathbf{w}. \tag{19}$$

Thus, the Lagrangian in (12) can be rewritten as

$$L(P_r, \mathbf{W}, \mathbf{\Lambda}, \nu) = \nu\sigma_d^2 + P_r[1 - \mathrm{tr}(\mathbf{\Lambda})]$$
$$+ \mathbf{w}^H\left[\mathbf{\Lambda} \otimes (P_o\mathbf{h}_1\mathbf{h}_1^H + \sigma_r^2\mathbf{I}) - \nu\frac{P_0}{\gamma_0}\mathbf{h}\mathbf{h}^H + \nu\mathbf{R}_g\right]\mathbf{w} \tag{20}$$

Solving the inner minimization of the dual problem (13), the dual problem can now be expressed as

$$\max_{\mathbf{\Lambda}}\max_{\nu} \quad \nu\sigma_d^2, \tag{21}$$
$$\text{subject to} \quad \mathbf{\Sigma} \succeq \frac{\nu P_0}{\gamma_0}\mathbf{h}\mathbf{h}^H, \tag{22}$$
$$\text{(17) and (18)}$$

where $\mathbf{\Sigma} = \mathbf{\Lambda} \otimes \left(P_o\mathbf{h}_1\mathbf{h}_1^H + \sigma_r^2\mathbf{I}\right) + \nu\left(\mathbf{h}_2\mathbf{h}_2^H \otimes \sigma_r^2\mathbf{I}\right)$. From Proposition 2, the optimal solution of the dual problem (21) is the same as that of the original relay beamforming problem (4). To show (15) and (21) are equivalent, we adopt the technique in [8]. By [8, Lemma 1], the SNR constraint (22) is equivalent to $\frac{\nu P_0}{\gamma_0}\mathbf{h}^H\mathbf{\Sigma}^{\dagger}\mathbf{h} \leq 1$, where $(\cdot)^{\dagger}$ denotes the matrix pseudo-inverse, since the matrix $\mathbf{\Sigma}$ may not be strictly positive definite. Thus, the dual problem (21) is equivalent to

$$\begin{aligned} \max_{\mathbf{\Lambda}} \max_{\nu} \quad & \nu \sigma_d^2 \\ \text{subject to} \quad & \frac{\nu P_0}{\gamma_0} \mathbf{h}^H \mathbf{\Sigma}^\dagger \mathbf{h} \leq 1 \\ & \text{(17) and (18).} \end{aligned} \tag{23}$$

To show (15) and (23) are equivalent, we note that the inner minimization part in (15) can be interpreted as a direct point-to-point SIMO beamforming problem given in (14), for which the solution is known. The optimal receiver beamforming vector $\tilde{\mathbf{w}}^o$ is thus given by

$$\tilde{\mathbf{w}}^o = \tilde{\mathbf{\Sigma}}^\dagger \mathbf{h} = \frac{P_o}{\sigma_d^2} \mathbf{\Sigma}^\dagger \mathbf{h}. \tag{24}$$

Substituting (24) into the SNR constraint (16), we obtain

$$\frac{\nu P_0}{\gamma_0} \mathbf{h}^H \mathbf{\Sigma}^\dagger \mathbf{h} \geq 1. \tag{25}$$

Compared to (23), the SNR constraint is reversed and the minimization over ν is also reversed as maximization. However, with a fixed noise covariance matrix $\mathbf{\Sigma}$, at optimality, it can be shown that the received SNR constraint in both problems are reached with equality, and the two problems lead to the same optimal beamforming vector solution [8], [11]. Thus, the optimal ν^o in both cases is the solution of $\frac{\nu P_0}{\gamma_0} \mathbf{h}^H \mathbf{\Sigma}^\dagger \mathbf{h} = 1$. Therefore, the Lagrange dual of the relay beamforming problem (4) is equivalent to the problem (15). ■

Corollary 1: The min-max per-antenna power P_r^o in the relay beamforming problem (4) is obtained through the dual point-to-point SIMO beamforming problem (15) as

$$P_r^o = \frac{\sigma_d^2 \gamma_o}{P_o \mathbf{h}^H \mathbf{\Sigma}^{o\dagger} \mathbf{h}} \tag{26}$$

where $\mathbf{\Sigma}^o$ is the value of $\mathbf{\Sigma}$ under the opitmal $(\mathbf{\Lambda}^o, \nu^o)$.

Proof: At optimality, the minimum per-antenna power P_r^o in (4) is the same as the value of the objective function in (15). From (24), P_r^o in (26) follows. Details are omitted due to page limitation. ■

D. The Semi-Closed Form Solution for the Optimal $\mathbf{W}^o$

Since (4) and (15) lead to the same beamforming vector solution, up to a scaling factor, the optimal $\mathbf{w}^o$ at the relay can be determined through the SIMO beamforming problem (15). Let

$$\mathbf{w}^o = \beta \tilde{\mathbf{w}}^o \tag{27}$$

where, without loss of generality, β is a real scaling factor. To determine β, note that the SNR constraint (5) is met with equality at the optimality. From (10), we have

$$\frac{P_0 \mathbf{w}^{oH} \mathbf{h}\mathbf{h}^H \mathbf{w}^o}{\mathbf{w}^{oH} \mathbf{R}_g \mathbf{w}^o + \sigma_d^2} = \gamma_o. \tag{28}$$

Substituting (27) into this equation, and combining with equation (24), we obtain

$$\beta = \frac{\sigma_d^2}{P_o} \sqrt{\frac{\sigma_d^2}{\frac{P_0}{\gamma_0} \left| \mathbf{h}^H \mathbf{\Sigma}^{o\dagger} \mathbf{h} \right|^2 - \left\| \mathbf{R}_g^{\frac{1}{2}} \mathbf{\Sigma}^{o\dagger} \mathbf{h} \right\|^2}} \tag{29}$$

By reversing the operation $\mathbf{w}^o = \text{vec}(\mathbf{W}^{oH})$, we now have obtained the explicit closed form solution (once $\mathbf{\Lambda}^o$ and ν^o are given) for the optimal relay processing matrix $\mathbf{W}^o$ of the relay beamforming problem (4).

E. Determining $\mathbf{\Lambda}^o$ and ν^o through SDP

To determine the optimal relay processing matrix $\mathbf{W}^o$, we need to obtain the optimal $\mathbf{\Lambda}^o$ and ν^o. This can be done by directly solving the Lagrange dual problem (21).

Proposition 3: The dual problem (21) is an SDP problem.

Proof: Define $\mathbf{s} \triangleq [0, \cdots, 0, -\sigma_d^2]^T$, $\mathbf{a} \triangleq [1, \cdots, 1, 0]^T$, and $\mathbf{x} \triangleq [\lambda_1, \cdots, \lambda_N, \nu]^T = [x_1, \cdots, x_N, x_{N+1}]^T$, where $\mathbf{s}, \mathbf{a}, \mathbf{x} \in \mathbf{R}^{(N+1)\times 1}$. The constraint (22) can be expressed as

$$-\mathbf{\Lambda} \otimes \mathbf{R}_r - \nu \left(\mathbf{R}_g - \frac{P_0}{\gamma_0} \mathbf{h}\mathbf{h}^H \right) \preceq 0 \tag{30}$$

where $\mathbf{R}_r \triangleq P_o \mathbf{h}_1 \mathbf{h}_1^H + \sigma_r^2 \mathbf{I}$. Observing that $\mathbf{\Lambda}$ is a diagonal matrix, we obtain

$$-\mathbf{\Lambda} \otimes \mathbf{R}_r = \sum_{i=1}^N \lambda_i \mathbf{F}_i, \tag{31}$$

where $\mathbf{F}_i$ is a block diagonal matrix, whose ith diagonal block is $-\mathbf{R}_r$ and all other $(N-1)$ diagonal blocks are $\mathbf{0}_{N\times N}$. Thus, the constraint (30) can be further expressed as $\sum_{i=1}^{N+1} x_i \mathbf{F}_i \preceq 0$, where $\mathbf{F}_{N+1} \triangleq \frac{P_0}{\gamma_0} \mathbf{h}\mathbf{h}^H - \mathbf{R}_g$. Therefore, the dual problem (21) can be transformed into the following SDP

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{s}^T \mathbf{x} \\ \text{subject to} \quad & \mathbf{x} \succeq 0, \quad \mathbf{a}^T \mathbf{x} - 1 \leq 0, \quad \sum_{i=1}^{N+1} x_i \mathbf{F}_i \preceq 0 \end{aligned} \tag{32}$$

where $\mathbf{F}_1, \cdots, \mathbf{F}_{N+1}$ are all Hermitian matrices. ■

The significance of the SDP formulation is two-fold: first, we now convert the optimization problem (4) with N^2 variables and $(N+1)$ constraints to one with $(N+1)$ variables and three constraints; second, the SDP algorithm has a polynomial worst-case complexity, and performs very well in practice [9]. Thus, we greatly reduce the computation complexity in finding the solution of the original optimization problem. The SDP problem above can be solved using standard SDP software such as SeDuMi or CVX.

IV. Numerical Comparisons

Using the proposed optimization solution, we compare the performance under per-antenna power minimization with that under sum-power P_{sum} minimization at the MIMO relay. For relay sum-power minimization with a given destination SNR target, we can use the result in [3] to derive the minimum sum-power as

$$P_{\text{sum}} = \frac{\left(\sigma_r^2 + P_o\|\mathbf{h}_1\|^2\right)\sigma_d^2\gamma_o}{(P_o\|\mathbf{h}_1\|^2 - \sigma_r^2\gamma_o)\|\mathbf{h}_2\|^2}. \tag{33}$$

It is easy to see that the feasibility condition for the equation above is $P_o > \frac{\sigma_r^2\gamma_o}{\|\mathbf{h}_1\|^2}$.

In the following numerical results, the noise powers at the relay and at the destination are set as $\sigma_r^2 = \sigma_d^2 = 0.1W$. The source transmitted power P_0 is set to $10dB$ above the noise power. The relay has $N = 4$ antennas. The entries of $\mathbf{h}_1$ and $\mathbf{h}_2$ are assumed i.i.d. zero-mean Gaussian with variance 1. First, we explore the average per-antenna power usage for different required SNR γ_0 under both power objectives. As demonstrated in Fig. 2, the higher required SNR, the higher average per-antenna power usage in both cases. As expected, the sum-power objective uses less power on average than per-antenna power objective due to the flexibility of power distribution among antennas.

Next, we study the statistical behavior of per-antenna power usage under both types of power minimization. Fig. 3 demonstrates the PDF of the 1st antenna's power usage. As we see the variance of per-antenna power usage under the per-antenna case is much smaller than that under the sum-power case; wider tails at both ends of the PDF curves for the sum-power case can be seen.

Finally, we compare the PDF of the maximum power usage among all antennas under both types of power minimization objectives at the MIMO relay in Fig. 4. The shift of power profile of the maximum power consumption among antennas under the two cases is evident, where lower maximum power usage for the per-antenna objective can be clearly seen.

V. Conclusion

In this work, we have considered the jointly optimal design of multi-antenna relay processing matrix and per-antenna power control. We have established the duality of MIMO relay beamforming and direct point-to-point SIMO beamforming. This enables us to obtain a semi-closed form solution of the optimal relay processing (beamforming) matrix. The solution not only reveals the structure of the optimal processing matrix, but also allows us to significantly lower the computation complexity of the optimal design, through drastic reduction in the number of optimization variables and constraints, as well as an efficient SDP formulation for the dual problem.

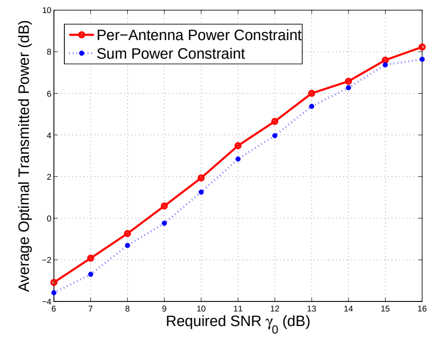

Fig. 2: Average per-antenna power usage on Antenna 1 vs. required SNR γ_0.

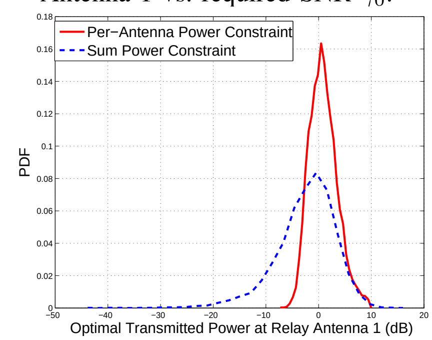

Fig. 3: PDF of power usage on Antenna 1 ($\gamma_o = 10$dB).

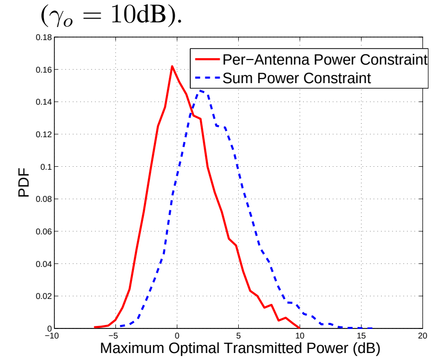

Fig. 4: PDF of maximum power usage among antennas ($\gamma_o = 10$dB).

References

[1] O. Munoz-Medina, J. Vidal, and A. Agustin, "Linear transceiver design in nonregenerative relays with channel state information," *IEEE Trans. Signal Processing*, vol. 55, pp. 2593 –2604, Jun. 2007.

[2] X. Tang and Y. Hua, "Optimal design of non-regenerative MIMO wireless relays," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 1398 –1407, Apr. 2007.

[3] B. Khoshnevis, W. Yu, and R. Adve, "Grassmannian beamforming for MIMO amplify-and-forward relaying," *IEEE J. Select. Areas Commun.*, vol. 26, no. 8, pp. 1397 –1407, Oct. 2008.

[4] V. Havary-Nassab, S. Shahbazpanahi, and A. Grami, "Joint receive-transmit beamforming for multi-antenna relaying schemes," *IEEE J. Select. Areas Commun.*, vol. 58, pp. 4966 –4972, Sep. 2010.

[5] C.-B. Chae, T. Tang, R. Heath, and S. Cho, "MIMO relaying with linear processing for multiuser transmission in fixed relay networks," *IEEE Trans. Signal Processing*, vol. 56, pp. 727 –738, Feb. 2008.

[6] B. Chalise and L. Vandendorpe, "Mimo relay design for multipoint-to-multipoint communications with imperfect channel state information," *IEEE Trans. Signal Processing*, vol. 57, pp. 2785 –2796, Jul. 2009.

[7] R. Zhang, C. C. Chai, and Y.-C. Liang, "Joint beamforming and power control for multiantenna relay broadcast channel with QoS constraints," *IEEE Trans. Signal Processing*, vol. 57, pp. 726 –737, Feb. 2009.

[8] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," *IEEE Trans. Signal Processing*, vol. 55, pp. 2646 –2660, Jun. 2007.

[9] L. Vandenberghe and S. Boyd, "Semidefinite programming," *SIAM review*, pp. 49–95, 1996.

[10] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, March 2004.

[11] A. Wiesel, Y. Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed MIMO receivers," *IEEE Trans. Signal Processing*, vol. 54, pp. 161 – 176, Jan. 2006.