Two-Dimensional Blind Deconvolution Using a Robust GCD Approach*

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Abstract

In this paper we examine the applicability of the previously proposed Greatest Common Divisor (GCD) method to blind image deconvolution. In this method, the desired image is approximated as the GCD of the two-dimensional polynomials corresponding to the z-transforms of two or more distorted and noisy versions of the same scene, assuming that the distortion filters are FIR and relatively co-prime. We justify the breakdown of two-dimensional GCD into one-dimensional Sylvester-type GCD algorithms, which lowers the computational complexity while maintaining the noise robustness. A way of determining the support size of the true image is also described. We also provide a solution to deblurring using the GCD method when only one blurred image is available. Experimental results are shown using both synthetically blurred images and real motion-blurred pictures.

1 Introduction

Blind image deconvolution is the process of identifying both the true image and the blurring function from the degraded image, using partial information about the imaging system. Although this has been shown to be possible by making use of the irreducible property of 2-D polynomial factors [1], a small amount of additive noise can lead to large deviations, and a unique solution may not be found. In existing methods dealing with such problems, reliability is often traded with high computational complexity.

A novel approach has been proposed in [2] that identifies the true image from two blurred versions of the same scene. Assuming that the distortion filters are FIR, the z-transform of the images and filters can be written as two-dimensional polynomials. The problem is then transformed into estimating the GCD of 2-D polynomials corresponding to the z-transforms of the blurred images, and a robust interpolative 2-D GCD method is introduced based on a 1-D Sylvester-type GCD algorithm [3]. In the following section we will first briefly describe the method and then introduce an extension to determine the order of the GCD polynomial when the support size of the true image is unknown. A justification is provided to the process of 1-D interpolation of the 2-D GCD using Discrete Fourier Transform points on the unit circle. Section 3 shows how to apply the GCD method on a single blurred image, with the example of a real motion-blurred picture.

*This research work was supported by the Office of Naval Research under contract N-00014-89-J-1512P-5.

2 The 2-D GCD Approach with Two Blurred Images

2.1 Problem Formulation

Let $f_1(m,n)$ and $f_2(m,n)$ represent two distorted versions of a true image $p(m,n)$ in presence of noise. Thus

$$f_1(m,n) = p(m,n) * d_1(m,n) + n_1(m,n) \quad (1)$$

$$f_2(m,n) = p(m,n) * d_2(m,n) + n_2(m,n). \quad (2)$$

In z-domain, we have

$$F_1(z_1,z_2) = P(z_1,z_2)D_1(z_1,z_2) + N_1(z_1,z_2) \quad (3)$$

$$F_2(z_1,z_2) = P(z_1,z_2)D_2(z_1,z_2) + N_2(z_1,z_2). \quad (4)$$

When the two blurring functions $D_1(z_1,z_2)$ and $D_2(z_1,z_2)$ are co-prime, and the additive noise $N_1(z_1,z_2) = 0$, $N_2(z_1,z_2) = 0$, the GCD of the two-dimensional polynomials $F_1(z_1,z_2)$ and $F_2(z_1,z_2)$ is $P(z_1,z_2)$, which represents the true image. However, when $N_1(z_1,z_2) \neq 0$ and/or $N_2(z_1,z_2) \neq 0$, $P(z_1,z_2)$ only at best approximates as the common factor that divides $F_1(z_1,z_2)$ and $F_2(z_1,z_2)$.

2.2 With Known True Image Support Size

If the true image support size, or in other words the order of the GCD polynomial, is known *a priori*, the procedure as described in [2] can be carried out to approximate the GCD of $F_1(z_1, z_2)$ and $F_2(z_1, z_2)$.

In fact, in the 1-D case a Sylvester matrix $\mathbf{S}$ is constructed from the two polynomials

$$A(z) = a_0 + a_1 z + a_2 z^2 + \ldots + a_n z^n = P(z)C(z) \quad (5)$$
$$B(z) = b_0 + b_1 z + b_2 z^2 + \ldots + b_m z^m = P(z)D(z), \quad (6)$$

which contain the GCD $P(z)$ of order r and the co-prime multipliers $C(z)$ and $D(z)$. The size of $\mathbf{S}$ is $(n + m - 2r + 2) \times (n + m - r + 1)$, and its rank is one less than its number of rows. A singular value decomposition is then operated on this matrix. The singular vector corresponding to the smallest singular value gives the multiplier polynomials, from which a least-square technique is used to calculate the "best" 1-D polynomial GCD.

To process 2-D images, a interpolative scheme is used to disassemble the 2-D problem into 1-D cases. Each of the variables in the two-variable polynomials, which correspond to the z-transforms of the two blurred images, are sampled at the Discrete Fourier Transform (DFT) points on the unit circle:

$$z_1 = e^{-j\frac{2\pi m}{M}}, \quad m = 0, 1, \ldots, M-1, \qquad (7)$$

and

$$z_2 = e^{-j\frac{2\pi n}{N}}, \quad n = 0, 1, \ldots, N-1, \qquad (8)$$

where M and N are the number of rows and columns of the blurred image, respectively. Sampling a two-variable polynomial in one variable leads to a one-variable polynomial in the other. The GCD of the resulting two 1-D polynomials are found using the Sylvester-type method. These GCD polynomials are then again sampled at the DFT points, eventually generating two matrices that are scaled versions of the 2-D DFT of the original image. We obtain an approximation of the original image by equalizing these two matrices and taking the inverse 2-D DFT.

2.3 True Image Support Size Unknown

The construction of the above $\mathbf{S}$ requires the knowledge of r. However, in practice the size of the true image is often unknown to us. In this case, we can generate the resultant matrix $\mathbf{S_0}$, of $A(z)$ and $B(z)$ given by

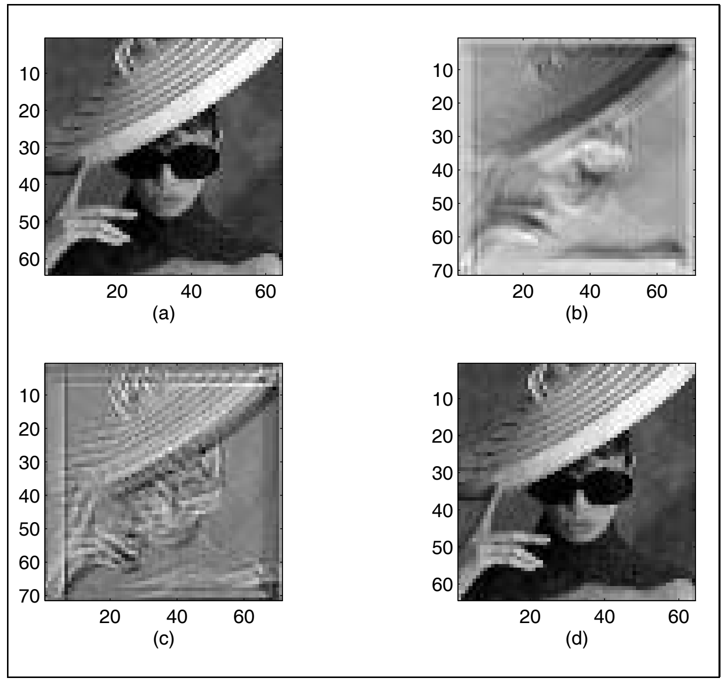

Figure 1: Blind Identification from Two Blurred Images. (a) the original image; (b) and (c) two blurred images; (d) the reconstruction from (b) and (c).

$$\begin{bmatrix}
a_n & a_{n-1} & \cdots\cdots & & a_1 & a_0 & 0 & \cdots & 0 \\
0 & a_n & a_{n-1} & \cdots\cdots & & a_1 & a_0 & \cdots & 0 \\
\vdots & & \vdots & & & & & & \vdots \\
0 & \cdots & 0 & a_n & a_{n-1} & \cdots\cdots & & a_1 & a_0 \\
0 & 0 & \cdots & 0 & b_m & b_{m-1} & \cdots & b_1 & b_0 \\
0 & \cdots & 0 & b_m & b_{m-1} & \cdots & b_1 & b_0 & 0 \\
\vdots & & & & \vdots & & \vdots & & \vdots \\
b_m & b_{m-1} & \cdots & b_1 & b_0 & 0 & 0 & \cdots & 0
\end{bmatrix}$$

of size $(n + m) \times (n + m)$.

The necessary and sufficient condition for the polynomials $A(z)$ and $B(z)$ to have a non-constant GCD is that the resultant Sylvester matrix $\mathbf{S_0}$ be singular [3]. Further more, the rank of $\mathbf{S_0}$ can be inferred by noticing the structural relationship between $\mathbf{S}$ and $\mathbf{S_0}$:

$$\mathbf{S_0} = \left[\begin{array}{cc} & r-1 \text{ rows} \\ \begin{array}{c} r-1 \\ \text{columns} \end{array} & \boxed{\quad\mathbf{S}\quad} \\ & r-1 \text{ rows} \end{array}\right] . \qquad (9)$$

It follows that $\mathbf{S_0}$ is of rank $n+m-r$. In fact, let $\mathbf{S_k}$ denote the submatrix of size $(n+m-2k)\times(n+m-k)$ obtained by striking out the first k and last k rows of $\mathbf{S_0}$, and the first k columns of $\mathbf{S_0}$, then $\mathbf{S_k}$ is of rank $n+m-2k-1$. Note that $\mathbf{S_{r-1}}$ is the same as $\mathbf{S}$. This indicates a new procedure to obtain r, and knowing r one may go back to Section 2.2 to obtain $C(z)$ and $D(z)$.

Using simulated distorted images, the 2-D GCD approach is found to be efficient and reliable when SNR $\geq$ 40dB. Figure 1 demonstrates the results of our simulation. Figures 1b and 1c are two blurred versions of a "hat" image, obtained by convolving the original 64×64 image in Figure 1a with two relatively co-prime 8×8 distortion filters and then adding uniform white noise so that SNR=40dB. The estimation using the above interpolative 2-D GCD approach is shown in Figure 1d. Run time required to reconstruct the original image in this case is approximately 1 minute on a Sun Ultra station. The percentage MSE of the reconstruction is 1.2%.

It is possible to directly extend the 1-D GCD technique described above to the 2-D case in terms of a generalized resultant matrix generated from the given 2-D polynomial coefficients. To achieve this, we only need to write the matrix corresponding to the 2-D polynomial coefficients into a 1-D vector in lexicographical order. However, this direct procedure leads to prohibitively large size matrices. For example, for images of size $N\times N$, the matrix corresponding to $\mathbf{S_0}$ will be of size $2N^2\times 2N^2$. Since computations involved in a matrix SVD decomposition are proportional to the cube of the matrix size, this direct procedure requires operations of the order of $O(N^6)$ for SVD alone. This is clearly an impossible task even for moderate value of N. By bringing down the 2-D problem to a series of 1-D polynomial GCD's, the computational complexity is reduced to $O(N^4)$ [5].

3 The 2-D GCD Approach with Only One Blurred Image

GCD by definition requires at least two blurred versions. In practice, we often have only one blurred image and want to identify the original scene. This dilemma can be solved by extending the original algorithm to first extract multiple images from the available blurred image.

When the support of the blurring function is small compared to the blurred image, the blurred image can be partitioned such that each part completely contains the blurring function. In the z-domain the blurred image is given by

$$F(z_1,z_2) = P(z_1,z_2)D(z_1,z_2), \tag{10}$$

where $P(z_1,z_2)$ represents the true image and $D(z_1,z_2)$ represents the blurring function. Partitioning the blurred image, we have

$$F(z_1,z_2) = \bigcup_{i=1}^{k} F_i(z_1,z_2) \quad , \tag{11}$$

and

$$F_i(z_1,z_2) = P_i(z_1,z_2)D(z_1,z_2)\ , \quad i=1,2,\ldots,k \quad , \tag{12}$$

where

$$P(z_1,z_2) = \bigcup_{i=1}^{k} P_i(z_1,z_2) \quad . \tag{13}$$

In this case, the blurring function $D(z_1,z_2)$ becomes the GCD among these partitioned images, and the different parts of the original scene now serve as the co-prime multiplicative factors.

A special case is linear motion blur, where the blurring function is one dimensional. In this case, each line along the motion contains the same blurring function. Figure 2a shows a blurred picture of letters taken from one frame of a video sequence of a fast moving truck. A Cannon Hi8 cam-corder was used to provide a reasonably clear image sequence. The selected frame was then digitized, and the predominant red channel was used as a gray level image. There are two blurred parts of letters on the image. The larger string is used to find the blurring function by averaging the GCD of two distant blurred lines. Then we restore the entire image area using the deduced blurring function. Figure 2b shows the result of the above algorithm. Here the large letters are clearly recognizable, and the smaller blurred string is also revealed to contain letters. Figure 2c shows the reconstructed image after median filtering and enhancement. In this figure even the smaller string of letters is recognizable as "KING OF BEERS".

4 Conclusions

The 2-D GCD blind image deconvolution method is efficient and moderately noise robust. It can be used to determine the support size of the blurring function when no such a priori information is given. The advantages of bringing down the problem to a series of 1-D polynomial GCD's and then making use of a Sylvester-type approach have been justified both theoretically and experimentally. The key to applying the

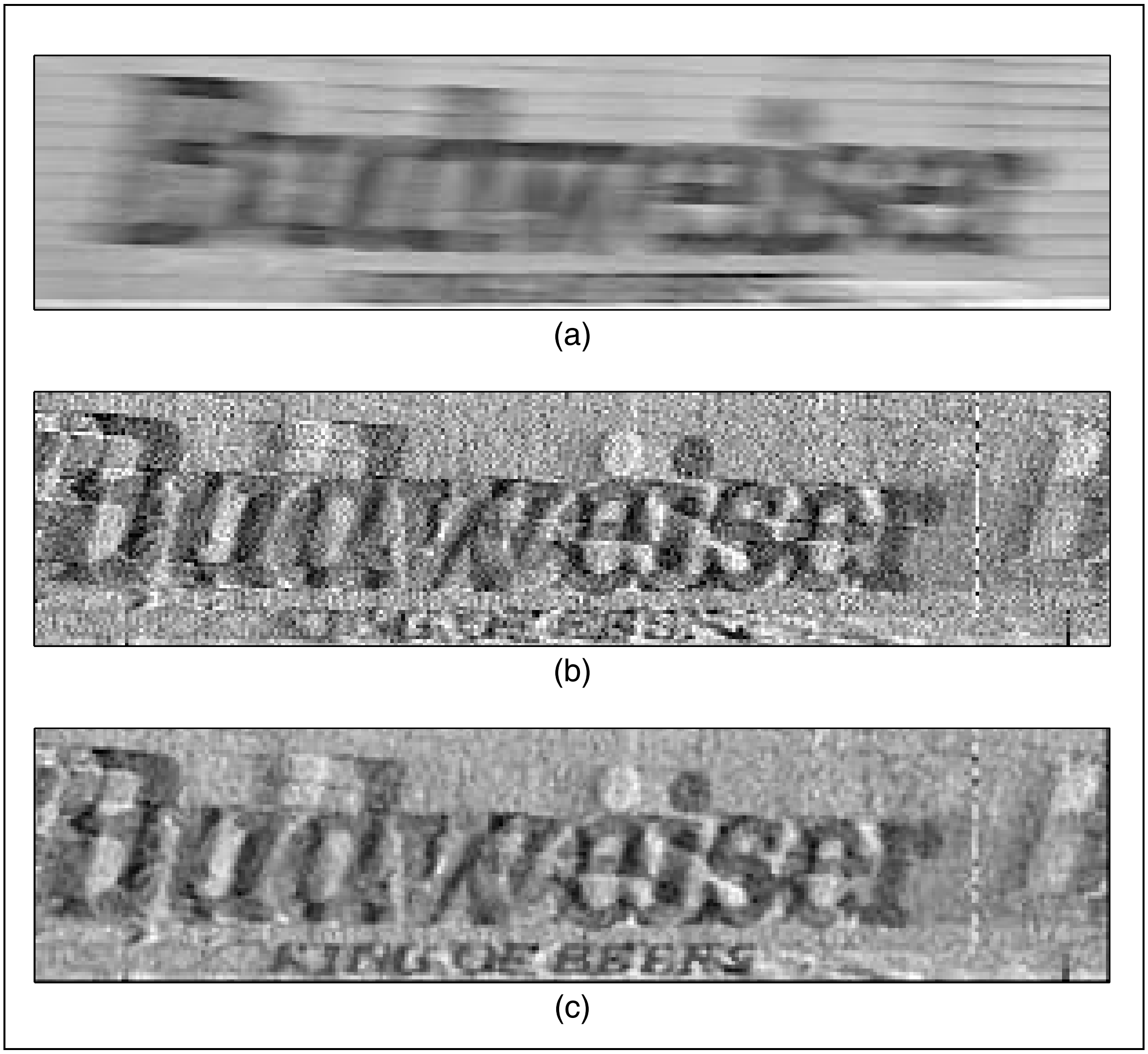

Figure 2: Deblurring of linear motion blurred image. (a) Blurred image (moving truck on the Brooklyn-Queens Expressway, April, 1997); (b) Image magnitude after GCD processing; (c) Enhanced image.

GCD method when only one blurred image is available is to extract two blurred images that contain as common part either the true image or the blurring function. Real pictures have been deblurred using such an approach, suggesting the practical value of this algorithm.

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