Efficient Network Coded Data Transmissions in Disruption Tolerant Networks

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***Abstract*—Most routing protocols in disruption tolerant networks (DTN) use redundant transmissions to explore the diversities in routing paths in order to reduce data transmission delay. However, mobile nodes in DTN usually have limited energy and may prefer fewer transmissions for longer lifetime. Hence, it is vital to carefully balance the tradeoff between data transmission delay and the amount of transmissions among mobile nodes.**

In this paper, we consider the problem to route a batch of data packets in DTN. By making an analogy between the routing protocol and low-density erasure codes, we investigate the information-theoretical optimal number of data transmissions in delivering data. With such insights, we propose E-NCP, an efficient protocol in DTNs based on network coding, that reduces data transmissions significantly, while increasing data transmission delay only slightly as compared to the protocol with the best performance. With extensive theoretical analysis and simulations, we show that network coding facilitates a better tradeoff between resource usage and protocol performance, and that our protocol offers unique advantages over replication-based protocols.

I. Introduction

Disruption tolerant networks (DTN), or *opportunistic networks*, represent a class of networks where connections among wireless nodes are not contemporaneous, but intermittent over time. Such networks usually have sparse node densities, with short communication ranges on each node. Connections among nodes may be disrupted due to node mobility, energy-conserving sleep schedules, or environmental interference.

In such networks, an opportunistic link may be temporarily established when a pair of nodes "meet" — when they move into the communication ranges of each other. A possible data propagation path from the source to the destination, referred to as an *opportunistic path*, is composed of multiple opportunistic links, possibly established over different time instances. Clearly, more than one such opportunistic paths may exist.

In DTNs, a source may transmit data directly to its destination when they are connected by an opportunistic link. Although such a direct-transmission protocol consumes the minimum amount of network resources, it may incur an exceedingly long transmission delay. On the other extreme, epidemic routing has been proposed [1] to flood data packets to all nodes in the network, essentially exploring all opportunistic paths from the source to the destination, and attaining the shortest data transmission delay. However, most mobile nodes in DTNs have limited energy and may prefer fewer transmissions than flooding to conserve energy, and to prolong network lifetime. For these reasons, probabilistic routing [2], [3] and spray-and-wait [4], [5] are proposed to achieve tradeoffs between network resource consumption and protocol performance by focusing on routing a single packet in a network with unlimited bandwidth and node buffer capacity.

Motivated by the need to transmit a large amount of data such as a file in DTN, we consider the DTN routing problem under more realistic network settings, where limited transmission opportunities and relay buffers are insufficient to accommodate all data to be transmitted. We observe that there exist an analogy between DTN routing and erasure codes, as the amount of transmissions in DTNs is similar to the density of erasure codes. The existence and optimality of *low-density* erasure codes indicates the existence of an information-theoretical optimal number of data transmissions in DTNs.

Randomized network coding [6], [7] allows intermediate nodes to perform coding operations besides simple replication and forwarding. Using the paradigm of network coding in DTN routing, a node may transmit a coded packet — as a random linear combination of existing data packets — to another node when the opportunity arises. Intuitively, when replication is used to minimize transmission delay, a node should transmit a packet with the minimum number of replicas in the network, since it is the packet with the longest expected delay. Unfortunately, one does not have precise global knowledge of which packet has the minimum number of replicas in opportunistic networks. When network coding is used, however, a node can transmit a coded packet as a combination of all packets in its buffer such that their information can be propagated simultaneously to the destination.

Based on these important insights, we propose *E-NCP*, an efficient protocol based on network coding to dramatically decrease the amount of data transmissions in DTNs, while only increasing the data transmission delay slightly as compared to epidemic routing. We show that, utilizing network coding, our protocol achieves better performance than a protocol based on replication. Based on E-NCP, we examine the influence of network parameters on protocol performance and resource consumption with extensive theoretical analysis. Our analysis provides further insights on the difference between coding-based and replication-based efficient protocols. Moreover, our analytical results can be used as a guideline to tune protocol parameters to achieve the intended tradeoff between performance and resource consumption, and to cater to diverse

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application requirements.

The remainder of the paper is organized as follows. We discuss related work in Sec. II. Sec. III introduces the network model. In Sec. IV, we describe E-NCP, our new protocol based on network coding. Sec. V shows the analogy between DTN routing and erasure codes, and demonstrates the asymptotic benefits of E-NCP. Sec. VI uses detailed mathematical analysis to validate the benefits of E-NCP and shows the advantage of network coding over replication. In Sec. VII, we use experiments to show the effectiveness of E-NCP and validate our analysis. We conclude the paper in Sec. VIII.

II. Related Work

A variety of routing protocols have been designed for disruption tolerant networks, based on different sets of assumptions. Some (*e.g.*, [8]) assume *a priori* knowledge on connectivity patterns, or that historical mobility patterns can be used to predict future message delivery probabilities [9]. Others assume control over node mobility [10]. In this paper, we propose a network coding based efficient routing protocol with neither *a priori* knowledge of network connectivity, nor control over node mobility.

Previous studies have proposed to use erasure coding to address network disruptions in DTNs, with no information of node mobility patterns [11], or with prior knowledge of network topologies [12]. Unlike network coding, in such source-based erasure coding approaches, different upstream nodes may transmit duplicates of coded data to the same node, and may unnecessarily consume additional bandwidth.

It has been shown that network coding can improve the throughput in wireless communication [13], by exploring the broadcast nature of the wireless medium. However, in disruption tolerant networks considered in this paper, a node seldom has more than one neighbors, and such wireless coding opportunities rarely occur.

Deb *et al.* [14] showed that a gossip protocol based on network coding can broadcast multiple messages among nodes with a shorter period of time, as compared to that without network coding. With the same spirit, the benefit of network coding in wireless broadcast communication has been investigated in [15], [16]. In contrast to their work, we show that network coding can efficiently utilize multiple opportunistic paths in the case of unicast communication in DTNs.

Different routing protocols such as probabilistic routing [2], [3] and spray-and-wait [4], [5] have been proposed to attain different tradeoffs between data transmission delay and resource consumption. All these proposals focus on the spreading of one single packet in a network with abundant bandwidth and node buffers. In contrary, our work investigates the more realistic scenario where *network resources* are limited, as opposed to the amount of data to be transferred.

Zhang *et al.* [17] extended spray-and-wait [4], [5] from a single packet to a batch of data packets in the network by considering all data as a "super packet" and limiting the redundancy of such a "super packet." Our work differs from theirs in that we reduce the amount of transmissions within a batch of data, in the same spirit as decreasing the density of erasure codes. More importantly, they have conducted solely simulation-based evaluation without an optimality study. In contrast, we investigate the information-theoretical optimal number of transmissions, and based on such an insight, show that our protocol has almost the same data transmission delay as epidemic routing, while dramatically decreasing the amount of transmissions in the network.

III. Network Model

In this paper, we consider unicast communication from a source to a destination in a disruption tolerant network with N wireless nodes, moving within a constrained area. The source has K packets to be transmitted to the destination. A transmission opportunity arises when a pair of nodes "meet," *i.e.*, they are within the communication range of each other. To facilitate the analysis without loss of generality, we assume that when nodes a and b meet, the transmission opportunity is only sufficient to completely transmit one data packet[1]. With respect to the buffering capacities, while the source and the destination are able to accommodate all K packets, we assume that the buffer on each of the intermediate relay nodes is only able to hold B packets, where $1 \leq B \leq K$. Of course, any of the packets in the buffer can be purged at any time, upon the receipt of ACKs or the expiration of Time-to-Live (TTL) in packets.

We assume that the time between two consecutive transmission opportunities (when nodes meet) is exponentially distributed with a rate of λ. In the literature, the majority of previous work makes such an assumption, either explicitly [5], [17], [19] or implicitly [3], [4]. Although measurement-based studies (*e.g.*, [20]) have shown that such inter-meeting time may follow heavy-tail distributions in some applications, more recent studies have shown that the exponential distribution is in fact more prevalent both in theory and in many practical systems [21]. Therefore, we opt for more mathematically tractable models in our analysis, and believe that insights obtained from our analysis are also useful under other realistic mobility models. With a similar preference for mathematical tractability, we assume that there does not exist background traffic beyond the unicast communication under consideration, and leave the more general case with background traffic to our future work.

IV. E-NCP: An Efficient Protocol based on Network Coding

In this section, we introduce a new protocol based on network coding in disruption tolerant networks, hereafter referred to as *E-NCP* for brevity, to transmit a batch of data packets from a source to a destination in disruption tolerant networks, with limited relay buffers and transmission opportunities. The upshot of E-NCP is that it is able to achieve similar data

[1]It is straightforward to extend this to the general case where an arbitrary number of data packets can be delivered when the opportunity arises, as we illustrate in [18].

transmission delay as epidemic routing, but with much fewer transmissions.

A. Epidemic Routing Based Protocols

For the purpose of comparison, we review epidemic routing based protocols that we have investigated in previous work to transmit a batch of data packets with network coding [22], referred to as *Network Coding based Epidemic Routing* (NCER) hereafter. We omit the description of naive epidemic routing based on replication, since we have shown that it has inferior performance to NCER [22].

We first describe the details of NCER. When two nodes a and b meet, they transmit coded packets to each other. A coded packet x is a linear combination of K source packets $E_1, \ldots, E_K$ in the form $x = \sum_{i=1}^{K} \alpha_i E_i$, where α_i are coding coefficients. Suppose that node a holds m coded packets in its buffer, node a encodes all coded packets in its buffer, namely $x_1, \ldots, x_m$, to generate a coded packet x_a:

$$x_a = \sum_{i=1}^{m} \beta_i x_i, \tag{1}$$

where all multiplication and addition operations are defined on a Galois Field (such as GF(2^8) when the operations are performed on each byte), and β_i is randomly chosen from the field. It is easy to see that x_a is also a linear combination of the K original packets, and the coefficients can be derived. Node a then transmits x_a along with its coding coefficients over the original packets to node b. When node b receives x_a, it stores x_a in its buffer if space is available. Otherwise, node b encodes x_a with each packet in its buffer as follows:

$$x'_i = x'_i + \gamma_i x_a, \tag{2}$$

where x'_i represents the ith coded packet in the buffer of node b, and γ_i is randomly chosen from the Galois Field.

The destination obtains a coded packet when it meets another node, and attempts the decoding process to retrieve K source packets as long as K coded packets have been collected. Because the coding coefficients and the coded packet are known, each coded packet represents a linear equation with the K source packets as unknown variables. Decoding the K source packets is equivalent to solving the linear system composed of K coded packets. The *decoding matrix* represents the coefficient matrix of such a linear system. When the rank of the decoding matrix is K, the linear system can be solved and the K source packets are decoded. Otherwise, there exists linear dependence among coded packets, and the node will continue to obtain more coded packets until decoding is successful.

B. E-NCP: An Efficient Network Coding Based Protocol

In NCER, two nodes exchange (coded) packets whenever they meet until an ACK from the destination indicating all K packets have been received or the TTLs in packet headers expire. We propose *E-NCP*, a new protocol based on network coding that we have designed to optimize efficiency in the amount of packet transmissions.

Our design is motivated by the following fundamental question: what is the minimal number of transmissions that should be made by the source and the relays to achieve the minimal transmission delay? To deliver K data packets from the source to the destination, it is easy to see from the information-theoretical perspective that the source needs to transmit at least K coded packets to either relay nodes or directly to the destination. Furthermore, to achieve the minimal transmission delay, the destination should decode all K source packets after obtaining K coded packets. Hence, the relay nodes should disseminate and mix the K coded packets from the source such that the destination can decode all packets by obtaining K coded packets from *any* K relay nodes with high probability.

We propose the following efficient protocol, the motivation of which will be clear later in Sec. V. The source transmits slightly more than K *coded* packets into the network such that these coded packets are sufficient to decode the original packets with high probability. All these coded packets are referred to as *pseudo source packets*. Each pseudo source packet is then disseminated to L random nodes in the network in the same spirit as "Binary Spraying" in [5]. Note that we also encode them together during the dissemination whenever possible. Spyropoulos *et al.* [5] have shown that "Binary Spraying" is the optimal spraying method with the minimal packet transmission delay under a homogeneous random mobility model such as ours. By adjusting L, referred to as the *maximal spray counter* hereafter, we can tune the tradeoff between the number of relay transmissions and the packet transmission delay. An important question is whether there is a critical threshold such that the protocol performance will degrade dramatically if L is smaller than the threshold. We postpone our analysis in response to this question to Sec. V.

The protocol proceeds as follows. The source maintains a counter S with an initial value K' slightly larger than K. When the source meets a relay node, if $S > 0$, it generates a coded packet (a pseudo source packet), a random linear combination of all packets, using the algorithm presented in Sec. IV-A, and transmits the packet to the relay node. We order the pseudo source packets from the source with indices $1, \ldots, K'$. Each packet from the source carries its index i and spray counter l, which is initialized to the maximal spray counter L. The source decreases S by one after each transmission to a relay node and stops transmitting if $S = 0$.

The relay nodes implement the "Binary Spraying" protocol for each pseudo source packet, while encoding them together whenever possible. Every relay node, *e.g.*, node a, keeps a list of tuples: $\langle i, l \rangle$, where i and l denote the index of a pseudo source packet in the node's buffer and the value of the packet's spray counter. Such lists are referred to as *spray lists* and are empty initially. When node a meets b, it checks the spray lists in both nodes. If node a finds in its own list a tuple $\langle i, l \rangle$ with $l \geq 2$ and there is no tuple with i as the first element in node b, node a transmits a coded packet to node b; otherwise, node a skips the transmission opportunity.

If node a decides to transmit, it generates a coded packet

as a random linear combination of all coded packets in its buffer, using the algorithm in Sec. IV-A, and sets the packet index i and the new spray counter $\lfloor l/2 \rfloor$ to be carried in the coded packet. Node a then updates its tuple with $\langle i, \lceil l/2 \rceil \rangle$. Upon receiving a coded packet, node b stores or encodes the coded packet with the algorithm in Sec. IV-A. Furthermore, node b inserts a new tuple into its list: $\langle i, l \rangle$, where i and l are the packet index and the spray counter carried in the incoming coded packet, respectively. We note that, the source and relay nodes always generate a coded packet to be directly transmitted to the destination when the opportunity arises, regardless of the counter S or the spray lists.

In a similar fashion, we can design an efficient variant of the protocol based on replication, referred to as E-RP, for later comparison with E-NCP. E-RP works similarly as E-NCP, except that different source packets are replicated separately among relay nodes rather than mixed together as in network coding. The source transmits the K packets into the network. Each packet carries a spray counter and implements the "Binary Spraying" protocol. When two nodes meet, they choose the packet with the largest spray counter in the buffer to exchange. If the node buffer is full, it chooses the packet with the smallest spray counter, compares its spray counter with that carried in the incoming packet, and drops the one with a smaller spray counter. The goal of this protocol is to assign higher priorities to packets with larger spray counters, since it is easy to see that such packets have smaller numbers of replicas in the system, and need to be replicated to reduce the overall transmission delay. We remark, from another perspective, E-RP is essentially a variant of the "Binary Spraying" protocol proposed in [5] on a batch of packets with the following slight modification. We replicate the packet with the minimal number of replica based on local information, when more than one packets in the batch are competing for a transmission opportunity.

V. E-NCP Asymptotic Efficiency

In this section, we analyze the asymptotic efficiency of E-NCP in terms of its scaling behavior in the number of transmissions and its requirement on the size of relay buffers.

A. Amount of Packet Transmissions

To analyze the amount of transmissions generated by E-NCP, we first state the following obvious fact: under the homogeneous random mobility model, the L nodes selected by the "Binary Spraying" protocol are uniformly distributed among the N relay nodes. This is easy to see since each node has the same probability to meet another node. We further assume $K = \Theta(N)$ throughout this section since the source transmits a large amount of data to its destination. We are now ready to characterize the asymptotic optimal value of L.

Theorem 1: If each node has buffer size K, the maximal spray counter L should be $\Theta(\log K)$ in order for the destination to decode all K source packets with any K coded packets with high probability.

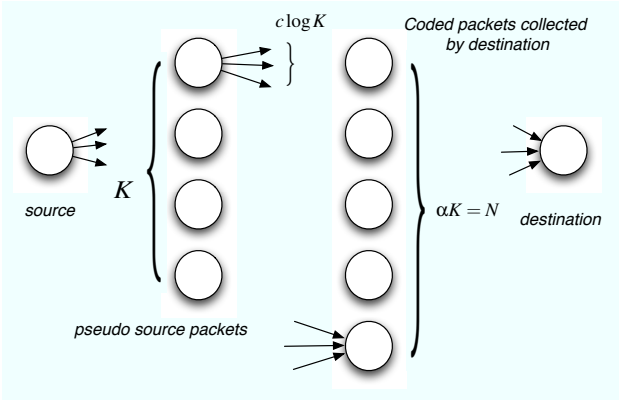


Fig. 1. The network-flow formulation in Theorem 1, where c and α are constants.

Proof: We reduce our problem to the problem studied in [23] by a network-flow formulation as shown in Fig. 1. The slightly more than K coded packets from the source can be equivalently considered as K linearly independent pseudo source packets. With E-NCP, the coded packets in relay nodes are the random linear combination of the K pseudo source packets. Moreover, as we shown previously, the information of a pseudo source packet is disseminated to L uniformly random relay nodes by the transmissions corresponding to the spray counter indexed by this pseudo source packet. Furthermore, because each relay node has buffer size K, it reserves the information of all received packets. Therefore, it is not hard to see that the transmissions of different pseudo source packets to their L relay nodes are independent. Hence, Theorem 1 and 2 in [23] apply here. They show that a source packet needs to be disseminated to only $\Theta(\log K)$ random nodes in the network in order for the destination to decode all source packets with *any* K coded packets from any K nodes with high probability. Hence, L should be $\Theta(\log K)$. ■

Consequently, we have the following corollary on the amount of transmissions made by the relay nodes in E-NCP. Combining it with Lemma 1 below, we conclude that, asymptotically, E-NCP is significantly more efficient than NCER in transmissions.

Corollary 1: In E-NCP, the relay node transmits $\Theta(\log K)$ data packets.

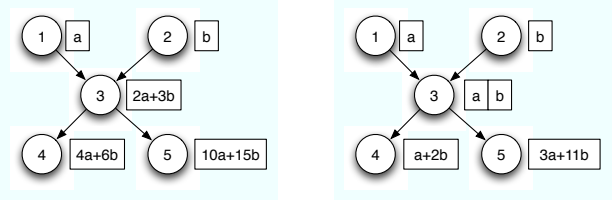
Proof: There are K pseudo source blocks, each consuming L transmissions. Hence, the total relay transmissions in the network is KL. Therefore, the average transmissions for each relay node is

$$T_{trans} = KL/N \tag{3}$$

Furthermore, with the result of Theorem 1, $L = \Theta(\log K)$, and the assumption $N = \Theta(K)$, we concludes that each relay nodes needs to transmit $\Theta(\log K)$ times. ■

Lemma 1: In NCER, each relay node transmits at least $\Theta(K)$ data packets.

Proof: The destination needs to obtain at least K coded packets from K meetings with other nodes to decode all data.



(a) (b)

Fig. 2. (a) Node 3 has buffer size 1 and transmits 2 linearly dependent coded packets. (b) Node 3 has buffer size 2 and transmits 2 linearly independent coded packets.

During such time period, each relay node behaves identically to the destination and meets at least K nodes on average. Furthermore, in NCER, each relay node transmits a coded packet whenever it meets another node. Hence, a relay node transmits at least K coded packets on average. ■

Moreover, in the analysis of Theorem 1, we show the connection between E-NCP and *low-density* distributed erasure codes [23]. Hence, the optimality results of [23] indicate that E-NCP is asymptotically optimal. On the other hand, NCER, in the same spirit of epidemic routing, floods the information of each pseudo source packet to $\Theta(K)$ relay nodes, and is comparable with *dense* erasure codes. This analogy confirms that E-NCP is more efficient than NCEP.

B. Buffer Requirement

Next, we discuss the asymptotic buffer requirement of E-NCP. As an example to illustrate the impact of relay buffers, shown in Fig. 2(a), we first consider the case when node 3 has buffer size 1. In this case, node 3 can only transmit linearly dependent coded packets to node 4 and 5 even when it has received 2 independent packets a and b. Hence, the independence between packet a and b are lost in node 4 and node 5. In contrast, if node 3 has buffer size 2, as shown in Fig. 2(b), node 4 and 5 can obtain linearly independent coded packets, and the information of packet a and b on node 4 and 5 are independent. In general, it is evident that if a relay node obtains the information of m independent packets, a buffer size of m is required to disseminate them to other relay nodes independently.

Therefore, in general, the buffer requirement of E-NCP can be much reduced from the what is stated in Theorem 1, where we have assumed each relay node has buffer size K to ensure that the spreading processes of different pseudo source packets are independent for ease of illustration. In fact, since each of the K pseudo source packet is sprayed to $\Theta(\log K)$ nodes, $\Theta(K \log K)$ packets are sprayed into the network. Furthermore, we have $N = \Theta(K)$ nodes in the network. Hence, each relay node receives $\Theta(\log K)$ pseudo source packets and its buffer size should be $\Theta(\log K)$ to ensure all coded packets transmitted from it are linearly independent. We moreover note that, in practice, the buffer requirement for E-NCP can be significantly reduced further and is only slightly larger than 1. We postpone such detailed analysis to Sec. VI-B.

Furthermore, we remark on the difference of the buffer requirements between NCER and E-NCP. NCER requires buffers with size 1 [22], whereas E-NCP needs buffers with size slightly larger than 1 as we will show in Sec. VI-B and Sec. VII-A. The reason is that the optimal low-density erasure codes [23] govern the information-theoretical optimal bound for efficient decoding: only $\Theta(K \log K)$ random opportunistic coded data transmissions are necessary. In NCER, even when the buffer size is 1, the relay nodes keep transmitting and pushing the information towards the destination. However, in E-NCP, the relay nodes transmit much fewer data packets. Hence, the transmissions should be sufficiently linearly independent and the relay buffer sizes should be slightly larger.

Finally, if relay nodes have abundant buffers, we have similar asymptotic analysis for E-RP, since the major difference between E-RP and E-NCP is that E-RP store all received packets on every node, whereas E-NCP encodes them together. We omit such analysis due to space constraint.

VI. Detailed Mathematical Analysis of E-NCP

Although the above asymptotic analysis demonstrates the clear benefit of E-NCP over NCER, it cannot answer a few important questions. For instance, what is the performance gap between E-NCP and NCER? What is the advantage of E-NCP over E-RP? In the following, we introduce detailed analytical models for E-NCP and E-RP to evaluate their performance.

A. Delivery Delay vs. Maximal Spray Counter

To facilitate our analysis, we refer to the random time duration between two consecutive meetings between a node and two other nodes as an inter-meeting time slot, or simply *time slot*, and denote it by T_{slot}. The expected length of a time slot is given by

$$E[T_{slot}] = 1/(\lambda N), \tag{4}$$

since a node meets another relay node with expected time $1/\lambda$ and there are N relay nodes in the network.

Let X be the random number of time slots until a message is delivered from the source to the destination. Let T be the total transmission delay associated with X. Then T is a random sum of X iid random variables, and its expected value has the product form as follows:

$$E[T] = E[X] \cdot E[T_{slot}] \ . \tag{5}$$

We will derive $E[X]$ for E-NCP and E-RP later in Sec. VI-A1 and VI-A2, respectively.

Before delving into the details of analysis, we review the details of spraying a packet into the network. As shown in [5], the source sprays L copies of a packet in approximately

$$E[N_L] = \lceil \log L \rceil \tag{6}$$

time slots in the "Binary Spraying" protocol. To simply the analysis, we assume that the L copies of a packet are all sprayed instantly in $E[N_L]$ time slots after the time slot that the source transmits this packet into the network.

For brevity, we refer to the jth (pseudo) source packet as the jth packet. In general, spraying packets with different maximal spray counters may lead to shorter transmission delays. We use L_j to denote the maximal spray counter for the jth packet.

1) E-NCP: We assume that a node receives information of the jth packet if it receives a coded packet as a combination of the jth packet and other packets. Since the destination can obtain a coded packet, a combination of all packets, from a relay node, we assume that the destination collects the information of different packets independently. Such an assumption may not be completely accurate since the destination can increase the rank of its decoding matrix by at most 1 when it meets a relay node. We will show that the analytical results obtained under such an assumption and other assumptions are still close to the simulation results in Sec. VII without the assumptions.

Although the maximal spray counter of the jth packet is L_j, network coding has the *side effect* to disseminate the information of the jth packet to more places when spraying the information of other packets, since a node transmits a coded packet as a combination of all packets in its buffer. However, we assume that there are only L_j "useful" copies of the jth packet because other copies made by network coding as a side effect may be dependent and useless for decoding.

Let $Y_{i,j}$ be a random variable that assumes the value 1 if the information of the jth packet is collected when the destination visits the ith random node, and the value 0 otherwise. To derive $E[X]$, we first compute the probability $\Pr(Y_{i,j} = 1)$. The source transmits the jth packet only after the jth meeting with a relay node. Furthermore, as stated previously, we assume the L_j copies of each packet are sprayed after a time lag of $E[N_{L_j}]$ time slots. Therefore, with the definition of

$$L'_j = j + E[N_{L_j}], \tag{7}$$

where $E[N_{L_j}]$ is given in (6), we have the following. If $i \leq L'_j$, the destination has probability 0 to obtain the jth packet when meeting the ith relay node, because the jth packet has not entered the network. If $i > L'_j$, the destination has probability L_j/N to obtain the jth packet when meeting a relay node, since the L_j copies of the jth packet are uniformly distributed among N relay nodes. Hence, the destination does not obtain the jth packet if it fails to obtain the packet in the $i - L'_j$ visits to relay nodes after the L'_jth visit, *i.e.*, $\Pr(Y_{i,j} = 0) = (1 - L_j/N)^{i-L'_j}$. Therefore, we have $\Pr(Y_{i,j} = 1) = 1 - (1 - L_j/N)^{i-L'_j}$. In summary, we have

$$\Pr(Y_{i,j} = 1) = \begin{cases} 0 & \text{if } i \leq L'_j, \\ 1 - (1 - L_j/N)^{i-L'_j} & \text{if } i > L'_j. \end{cases} \tag{8}$$

where L'_j is given in (7).

Next, we derive the expected time slots required to recover all data, assuming the source transmits K' pseudo source packets into the network, where $K' \geq K$. It is easy to see that the destination cannot recover all original K source packets, if it visits less than K nodes because it can increase the rank of its decoding matrix at most 1 during each visit, *i.e.*, $\Pr(X \geq i) = 1$, if $i < K$. If more than K relay nodes are visited, the destination recovers all data as long as it obtains the information of at least *any* K out of K' pseudo source packets, because these pseudo source packets are "equivalent": *any* K out of K' pseudo source packets are sufficient to decode all original K source packets with high probability. We define *index sets* with elements representing the set of Z pseudo source packets the destination obtains when it decodes all data, where $K \leq Z \leq K'$. We denote the m-th index set of size Z as $J_{Z,m}$, where the Z elements of $J_{Z,m}$ are chosen, uniformly at random, from $\{1, 2, \ldots, K'\}$:

$$J_{Z,m} = \{1 \leq j_1^m < j_2^m <, \ldots, < j_Z^m \leq K'\}. \tag{9}$$

Therefore, we have the probability that the destination obtains the information of the Z packets defined in $J_{Z,m}$ is $\Pi_{j \in J_{Z,m}} \Pr(Y_{i,j} = 1) \Pi_{j' \notin J_{Z,m}} (1 - \Pr(Y_{i,j'} = 1))$ if the destination visits i relay nodes. Henceforth, we have $X \geq i$ if the destination does not obtain *any* set of Z packets indexed by $J_{Z,m}$ after it visits $i - 1$ relay nodes, where $Z \geq K$. That is

$$\Pr(X \geq i) = \begin{cases} 1 & \text{if } i < K, \\ 1 - [\sum_{Z=K}^{K'} \sum_{m=1}^{\binom{K'}{Z}} \\ \quad \Pi_{j \in J_{Z,m}} \Pr(Y_{i-1,j} = 1) \cdot \\ \quad \Pi_{j' \notin J_{Z,m}} (1 - \Pr(Y_{i-1,j'} = 1))] & \text{if } i \geq K, \end{cases} \tag{10}$$

where $\Pr(Y_{i-1,j} = 1)$ and $\Pr(Y_{i-1,j'} = 1)$ are given in (8), and $J_{Z,m}$ is defined in (9).

It is clear that there are $\sum_{Z=K}^{K'} \binom{K'}{Z}$ such index sets, and it is computationally prohibited to enumerate all these index sets to compute the sum in (10). Let p_j denote $\Pr(Y_{i-1,j} = 1)$. The sum in (10), denoted by S_p, is equivalent to

$$S_p = \sum_{K \leq a_1 + \ldots + a_{K'} \leq K'} \Pi_{j=1}^{K'} p_j^{a_j} (1 - p_j)^{1 - a_j} \tag{11}$$

where a_j is either 0 or 1. S_p is similar to a sum of multinomial items without the multinomial coefficients and with the additional constraints on a_j. Hence, we can use a similar efficient algorithm in [24] to compute (11) with a complexity of $O((K' - K) K'^2 (\log K')^2)$ by dynamic programming and FFT. We present the details in [18] due to space constraint.

We then have $E[X] = \sum_{i=1}^{\infty} \Pr(X \geq i)$ [25], where $\Pr(X \geq i)$ is given in (10). Therefore, we can obtain the expected transmission delay $E[T]$ by (5).

With the analytical relation from the maximal spray counters L_j to the expected transmission delay $E[T]$, we can formulate an optimization problem to find the optimal L_j to minimize $E[T]$, if L_{total} transmissions are permitted in the network.

$$\begin{aligned} \text{minimize} \quad & E[T] \\ \text{subject to} \quad & \sum_{j=1}^{K} L_j = L_{total} \\ & L_j \geq 0 \quad \text{for } j = 1, \ldots, K. \end{aligned} \tag{12}$$

2) E-RP: For E-RP, the destination can choose to obtain only one packet when meeting a relay node with multiple packets. Therefore, the assumption that the destination can collect different packets independently is much less accurate than that in E-NCP. Hence, we use a different modeling idea. We model the network with state $\{R_i, M_i\}$, where R_i and M_i

are the expected numbers of packets on the destination and a relay node, respectively, at time slot i. Hence, the transmission time can be approximated by the duration from the beginning to the time that R_i reaches K. To simplify the analysis, we assume that the source replicates the same number of copies L for all packets.

First, we compute the expected number of packets M_i on a relay node at time slot i. To do so, we first derive the total number of different packets D_i in the network. As described in Sec. VI-A1, the jth packet enters the network after time slot $L'_j = j + E[N_L]$. Therefore, there is no packets in the network in the first $E[N_L]$ time slots. *i.e.*, $D_i = 0$, for $i = 1, \ldots, E[N_L]$. Afterwards, at each time slot, L copies of a new packet are injected into the network. Hence, at time slot i, there are $i - E[N_L]$ different packets in the network. Because there are K source packets, after time slot K, the source no longer sprays new packets. Therefore, after time slot $K + E[N_L]$, the total number of different packets in the network is K. Therefore, in summary, we have

$$D_i = \begin{cases} 0 & \text{if } i \leq E[N_L], \\ i - E[N_L] & \text{if } E[N_L] < i \leq K + E[N_L], \\ K & \text{if } i > K + E[N_L], \end{cases} \quad (13)$$

where $E[N_L]$ is given in (6).

In E-RP, each of the D_i packets has L copies. Furthermore, under our homogeneous mobility model, all these packets are uniformly distributed among N relay nodes. Therefore, we have the expected number of packets M_i on a relay node at time slot i as follows:

$$M_i = D_i L / N. \quad (14)$$

We then compute the probability $\Pr(R_i, M_i, D_i)$ that the destination obtains a new packet from a relay node at time slot i. In our protocol, the M_i expected number of packets are uniformly distributed among the D_i packets at time slot i. We further assume that the R_i packets on the destination are uniformly distributed among the D_i packets as well. Hence, we derive the probability $\Pr(R_i, M_i, D_i)$ as follows. First, if $R_i < M_i$, the destination can always obtain a new packet from the relay node. Second, if $R_i \geq M_i$, the destination cannot obtain a new packet from a relay node only if the destination contains all packets on the relay node, which has the probability $\binom{R_i}{M_i}/\binom{D_i}{M_i}$ under the assumption of uniform packet distribution. Hence, we have

$$\Pr(R_i, M_i, D_i) = \begin{cases} 1 & \text{if } R_i < M_i, \\ 1 - \binom{R_i}{M_i}/\binom{D_i}{M_i} & \text{if } R_i \geq M_i, \end{cases} \quad (15)$$

where D_i and M_i are derived in (13) and (14), respectively. We compute R_i later in this section.

Note, when the expected number of packets M_i at a relay node is smaller than 1, the destination can obtain M_i fraction of a packet at most. Therefore, we have the expected number of packets S_i that the destination can obtain from a relay node:

$$S_i = \min(M_i, 1) \cdot \Pr(R_i, M_i, D_i). \quad (16)$$

Therefore, with the expected number of packets S_i from a relay node at time slot i, the expected number of packets on the destination at time slot $i + 1$ is

$$R_{i+1} = R_i + S_i. \quad (17)$$

Clearly, at time slot 1, we have $R_1 = 0$. Hence, we can compute R_i for any time slot i recursively.

The destination obtains all K packets when R_i reaches K. Therefore, the expected number of time slots $E[X]$ that the destination spends to collect all packets is

$$E[X] = \arg\min_i \{K - R_i < \epsilon\}, \quad (18)$$

where ϵ is a positive number close to 0. With $E[X]$, we obtain the expected transmission delay $E[T]$ of all K packets by (5).

B. Delivery Delay vs. Relay Buffer Size

In Sec. V-A, we show that the relay buffer should be $\Theta(\log K)$ for E-NCP. In this section, we show E-NCP runs efficiently when relay buffer sizes are close to 1. To simplify the analysis, we assume the maximal spray counters of all pseudo source packets are all L. Similarly as in Sec. VI-A1, we ignore the side effect of network coding in spraying packet information and consider only the original L copies of a pseudo source packet.

In the "Binary Spraying" protocol, the L copies of a packet is spread in $E[N_L] = \lceil \log(L) \rceil$ time slots on average as discussed previously in this section. Hence, there are $\lceil \log(L) \rceil$ packets in transmitting on average at any instance of time. Furthermore, because each packet has at most L copies, and there are N nodes in the network, the probability that a node has one packet in its buffer is at most $\lceil \log(L) \rceil \cdot L/N$. Furthermore, all pseudo source packets are assumed to be spread independently. Therefore, the probability that there are more than M packets on a node is $(\lceil \log(L) \rceil \cdot L/N)^M$. By the union bound [25], the probability of the event E that there are M packets on one of N nodes is

$$\begin{aligned} \Pr(E) &\leq N \cdot (\lceil \log(L) \rceil \cdot L/N)^M \\ &= (\lceil \log(L) \rceil \cdot L)^M / N^{M-1} \\ &= O(1/N^{M-1}) \end{aligned} \quad (19)$$

The second equality in (19) holds because $\lceil \log(L) \rceil \cdot L$ is insignificant as compared with N, since we have show that L needs to be $\log(K)$ to guarantee the protocol performance. Eq. (19) implies that there are unlikely more than 2 packets arriving on any node at any instance of time. We conclude that as long as the buffer size is equal to or larger than 2, the probability that a relay node transmits linearly dependent coded packets is low. Therefore, E-NCP requires relay buffer size slightly larger than 1.

We note that the above analysis applied to E-RP as well: the probability that there are more than 2 packets arriving at a relay node is very low for E-RP. However, there is one fundamental difference between them. As discussed in Sec. V-A, each node receives $\Theta(\log K)$ (pseudo) source packets. To hold all of them in E-RP, the relay buffer size needs to be at

least $\Theta(\log K)$. On the other hand, in E-NCP, these $\Theta(\log K)$ pseudo source packets can be encoded into one coded packet such that E-NCP requires buffer size close to 1.

Finally, we give a simple analytical lower bound of the transmission delay for E-RP. It is easy to see that the expected number of packets on a node is upper-bounded by buffer sizes. Hence, we replace M_i in (14) by

$$M_i = \min\{D_i L/N, B\}. \tag{20}$$

All other computations are identical with the analysis of E-RP under abundant buffers as described in Sec. VI-A2.

VII. Performance Evaluation

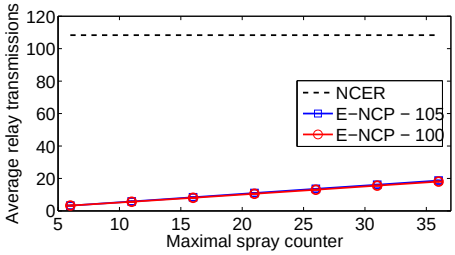
In this section, we demonstrate the advantage of E-NCP and validate our theoretical analysis by experiments. We have developed a discrete-event simulator with the implementation of network coding, the original epidemic routing based protocols, and our efficient protocols. To mitigate randomness in simulations, we show, for each data point in all figures, the average and the 95% confidence intervals from 100 independent experiments. We set the node inter-meeting rate λ to 0.005 and the number of packets K to 100 in most experiments unless explicitly pointed out. We use GF(2^8) as the Galois fields where network coding is operated in all simulations.

A. Advantages of E-NCP

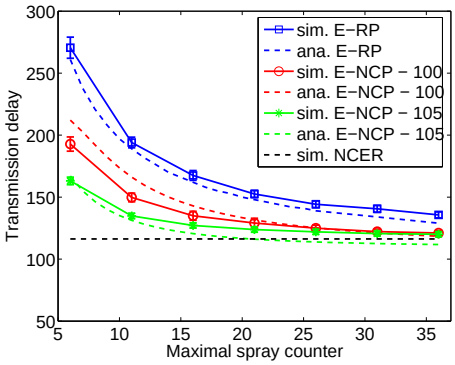
Fig. 3 shows the average number of relay transmissions and the transmission delay as functions of the maximal spray counter. We set the maximal spray counters for all source packets to be identical and vary the value from 1 to 36. Furthermore, we set the number of source packets to 100, the number of relay nodes to 200, the number of pseudo source packets to 100 or 105, and the maximal relay buffer size to 100. To serve as comparison with E-NCP, we also show the simulation result of NCER. The analytical result of NCER [22] is omitted since it is not the focus of this paper.

As expected, Fig. 3(a) shows that the amount of relay transmissions increases linearly as the maximal spray counter increases, matching perfectly with the analytical result of Eq. (3), which is omitted in the figure for clarity. More importantly, for the range of spray counters under consideration, E-NCP significantly reduces the amount of transmissions and achieves near optimal performance, as compared with NCER. From Fig. 3(b), we observe that the data transmission delay decreases significantly when the maximal spray counter increases. Furthermore, E-NCP approaches the performance of NCER, when the maximal spray counter is close to the logarithm of the total number of data packets. This observation agrees with Theorem 1.

We further observe that our analysis is close to the simulations in Fig. 3(b). Moreover, both simulations and analysis show the significant advantage of E-NCP over E-RP. This is because the probability that the destination increases the rank of its decoding matrix in E-NCP is much higher than the probability that the destination obtains a new packet in E-RP.



(a)



(b)

Fig. 3. (a) Average number of transmissions by a relay node vs. maximal spray counter. (b) Data transmission delay vs. maximal spray counter.

Finally, we remark on the difference between E-NCP with 105 and 100 pseudo source packets. We observe that although the amount of transmissions in the former case is only 5% larger than the latter case, the transmission delay of the former case is much shorter, if the maximal spray counters are smaller than 10. This implies that it is more desirable to transmit slightly more pseudo source packets.

B. Impact of Relay Buffer Sizes

Next, we investigate the impact of the relay buffer size on the data transmission delay of E-NCP and E-RP. We set the maximal spray counter to 25 while varying the relay buffer size from 1 to 20. We further set the number of pseudo source packets to 105 in E-NCP. All the other settings are the same as the previous experiments. Fig. 4 shows that as long as the relay buffer size is larger than 1, the performance of E-NCP is almost the same as NCER. This confirms our analysis in Sec. VI-B that the relay buffer sizes can be very small for E-NCP. On the other hand, the transmission delay of E-RP increases dramatically when the relay buffer size is smaller than 10 as shown by both the simulation result and the analytical lower bound.

C. Optimal Spray Counters

In E-NCP, the source transmits packets at different times. Intuitively, the packets transmitted later will benefit from more replications, since the destination has less opportunity to obtain them. In this section, we quantitatively study this effect, using the optimization formulation (12). We set the number of total source packets K to 100, the number of pseudo source packets

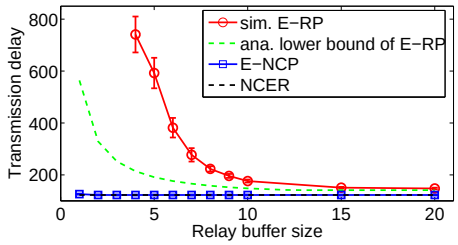
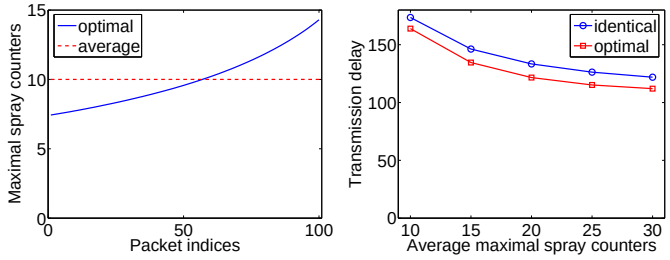


Fig. 4. Packet transmission delay under different sizes of relay buffers.



(a) (b)

Fig. 5. (a) The optimal maximal spray counters when the average maximal spray counter is 10. (b) The data transmission delay under optimal maximal spray counters vs. the data transmission delay under identical maximal spray counters.

K' to 100, and the average maximal spray counter L from 10 to 30. Then the sum of all maximal spray counters is $L_{total} = L \cdot K$. Furthermore, we set the initial search point for the optimization problem (12) to $L_j = L$, for $j = 1, \ldots, K'$.

Fig. 5(a) shows the optimal maximal spray counters solved by (12), when the average maximal spray counter L is 10. As expected, the packets transmitted later are assigned with more copies. However, Fig. 5(b) shows that the improvements of the optimal maximal spray counters over identical maximal spray counters is marginal (less than 10%). Therefore, in practice, it may be preferable to use identical maximal spray counters to simplify protocol setup. In this regard, our analysis leading to (12) provides design guidelines for the tradeoff between performance and the ease of implementation.

VIII. Conclusion

In this paper, we demonstrate the analogy between DTN routing and erasure codes. Based on this insight, we explore the information-theoretical optimal scaling of data transmissions, and propose an efficient network coding based protocol that significantly decreases the amount of resource used in transmitting a batch of data packets, while only increasing the data transmission delay slightly. We evaluate the proposed E-NCP protocol with extensive analysis and simulation. Our theoretical analysis results yield further insights into the difference between coding based and replication based protocols, and provide guidelines in tuning protocol parameters to attain the best tradeoff to accommodate a diverse set of application requirements.

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