Decentralized Multiuser Diversity with Cooperative Relaying in Wireless Sensor Networks

Sam Vakil and Ben Liang

***Abstract*— Multiuser diversity is a phenomenon caused by channel variations among different users in a wireless network. Cooperative relaying provides another form of diversity due to the spatial separation of sensors. In this work we show how the simultaneous application of these two sources of diversity in a decentralized manner can lead to significant throughput improvement in sensor networks. To exploit this synergy we propose a family of protocols termed *Channel Aware Aloha with Cooperation* (CAAC). Different power allocation schemes for CAAC are considered, including Constant Power, Fixed Rate, and Optimal Variable Rate. In each case we derive the scaling behavior of the achievable rate. We find the optimal source and relay transmission strategies under each scheme and show that the overall system performance is significantly improved. Furthermore, we show that the Constant Power scheme is asymptotically optimal, allowing easy implementation in simple sensors.**

I. Introduction

Distributed wireless sensor networks are commonly characterized by small sensor nodes with limited energy reserve and computing power. Because of the unstable nature of wireless communication links, cross-layer scheduling techniques that account for the physical-layer characteristics, such as multiuser diversity based transmission, can significantly improve the performance of sensor networks. Furthermore, because of the limited capability of each individual sensor node, these networks can benefit from intelligent node cooperation. In this work, we consider the design of a decentralized cooperative scheme which exploits the multiuser diversity effect in a sensor network.

The concept of *multiuser diversity* is best demonstrated in the work of Knopp and Humblet [1]. In this work the authors consider the uplink of a wireless network as a multiple-access channel[1]. They prove that to maximize the sum throughput of the network, during each time slot the user with the best channel state should transmit, and other users should remain silent. Under this scheme, the diversity gain is due to the fact that increasing the number of users gives the system a higher chance for having a user near its peak channel state during each time slot. However, in their setting the authors assume that the system has *centralized* access to the uplink channel state information (CSI) from all users. This assumption of centralized access to CSI becomes harder to justify as the number of users increases, and the need for a decentralized access becomes apparent.

In sensor networks, generally it is not reasonable to assume centralized access to channel states. Telatar and Shamai [2] have been the first to address decentralized resource allocation and power control for the uplink. In [3] the authors consider a variant of the ALOHA model in which the destination can benefit from multi-packet reception, and the sensors' probability of transmission is based on a control function of the channel state. Qin and Berry in [4], [5] consider a simpler "collision model" for the reception to find an abstraction for the multiple-access system throughput performance in a fading environment. The authors show that the effect of multi-user diversity is preserved in their decentralized Channel Aware Aloha (CAA) scheme. They evaluate the throughput scaling behavior and show that this scheme is asymptotically optimal in the limit of large number of sensors.

Another type of diversity can be obtained through spatial separation of sensors. The approach of *cooperative diversity* has been introduced mainly by Laneman et al [6], [7] and Sendonaris et al [8]. In this setting each user, besides sending its own message, can detect other users' messages and relay them to the destination. This forwarding of the data can increase the achievable rate, specifically for the cases where the source-destination channel experiences deep fades, such that there is a high probability that the relay destination link can help increase the achievable rate. Hence, cooperation provides performance improvements through the use of available resources in the network, especially important when the size of devices limits the number of antennas that can be deployed in each. In sensor networks, cooperation can lead to significant increases in the network throughput, sensing coverage, and energy savings.

Much of the previous work concerning cooperative diversity has been concentrated on developing physical layer protocols to exploit spatial diversity and in-

The authors are affiliated with the Department of Electrical and Computer Engineering, University of Toronto. Email: {vakil,liang}@comm.utoronto.ca.

[1]In the multiple-access channel model, there are multiple transmitters competing for access to one channel and a single receiver.

crease point-to-point throughput. Among these protocols, *Amplify and Forward* and *Decode and Forward* [6] have been the subject of extensive research. Recently, cooperative schemes have been used to mitigate the throughput loss of random access schemes. In [9] the authors propose a cooperative scheme to mitigate the throughput loss inherent in ALOHA. In [10], we further study the problem of joint MAC-PHY design from the perspective of interference mitigation in cooperative sensor networks.

In this work, we investigate into a joint MAC-PHY layer design that unites decentralized multiuser diversity and cooperative relaying. We evaluate the benefit of such a union in the uplink of a sensor network with the assumption of Rayleigh fading and a collision model at the destination (i.e., the sink) and at each intermediate relaying sensor. We study the effect of sensor cooperation over the asymptotically optimal multiuser diversity approach proposed in [4], [5] and evaluate the increase in throughput thus obtained, through a family of protocols we term *Channel Aware Aloha with Cooperation* (CAAC). To the best of our knowledge, this is the first study on the synergy between multiuser diversity and cooperative relaying in a decentralized environment.

Our main contributions include the following. *First*, through an analytical performance evaluation framework, we derive optimal source transmission and cooperative relaying strategies with decentralized random access in CAAC. *Second*, we consider different power allocation strategies for CAAC, including Constant Power, Fixed Rate, and Optimal Variable Rate. We observe their scaling behavior and relative merits in comparison with CAA and simple cooperation without considering multiuser diversity. *Third*, we show that the throughput of CAAC with constant transmission power is asymptotically optimal and scales as $R(n\log n)$, where R is the rate function and n is the number of sensors. Hence, sensors with limited capabilities can still fully benefit from CAAC with a simple power allocation scheme.

The remainder of this paper is organized as follows. Section II presents the network model, describes the relaying protocol, and formulates the optimal design problem. In Section III, we derive the system throughput under different power settings for CAAC and evaluate its scaling behavior. The performance gains obtained by cooperation are validated in Section IV with numerical examples and simulation.

II. Model and Problem formulation

A. Network Model

We consider a cooperative multiple access wireless sensor network with n sensors indexed $\mathbb{M} = \{1, 2, \ldots, n\}$ communicating to a single sink that is reachable within one hop[2]. Sensors can cooperate in relaying another sensor's message towards the destination. The message transmission is assumed to be done in two phases called *Phase A* and *Phase B*.

In Phase A the active sensors can send their messages towards the destination and other sensors if their channel amplitude towards the destination is above a required threshold, which is to be determined. We assume that at time m sensor i has message $x_i(m)$, $m \in [oL, (o+1)L]$ in its buffer, where L represents the length of a time slot and o is the slot index. The received message by sensor j during Phase A can be represented as

$$y_j(m) = \sum_{i=1, i\neq j}^{N} \sqrt{H_{ij}(m)}x_i(m) + z_j(m), \qquad (1)$$

where $y_j(m)$ is the received message at sensor $j \in \mathbb{M} - \{i\}$, $H_{ij}(m)$ is the channel gain between the ith sensor and the potential relay j or the sink d, $z_j(m)$ is the additive white Gaussian noise at sensor j (or the sink) with power Z_j (or Z_d).

In Phase B the sensors which were not senders during the previous phase and which have been successful in the decoding of the message of the sender in Phase A are potential forwarders. These sensors may implement the *Decode and Forward* scheme [6], to forward the data to the sink. This decision is dependent on the channel amplitude of these relays towards the destination[3]. The derivation of an optimal decision threshold for the channel amplitude is part of our design goal. We represent the channel gain between the kth relay and the destination as $G_{kd}(m)$. A block-fading process has been considered for the channel gains, so for $m \in [oL, (o+1)L]$, the channel gain remains constant. We adopt the common assumption that any pair of channel gains ($H_{i_1j_1}$,$H_{i_2j_2}$) or (G_{i_1d},G_{i_2d}) are independent random variables. For the purpose of analytical simplicity we assume the channel gain random variables to have the same distribution. Throughout this work, for each sensor independent fading is assumed over different time slots.

The probability density function of a channel gain $H_{ij}(m)$, for $m \in [oL, (o+1)L]$, is represented as $f_{H_{ij}}(h)$. We consider the symmetrical channel case, and denote by $f_H(h)$ the source-destination coefficients,

[2]For clustered sensor networks, this model can be equally applied to intra-cluster communication between the sensor and the clusterhead. The application of CAAC in a multihop sensor-to-sink environment requires the additional consideration for complicated routing and interference mitigation schemes [10]; it remains an interesting open problem for future research.

[3]Channel dependent transmission of the relays has been recently studied in [11], for the single source, single relay, and single destination scenario.

$f_{H'}(h')$ the source-relay coefficients, and $f_G(g)$ the distribution of the channel gains between the relays and the sink. As defined in [5], a fading density $f_H(h)$ is well behaved if $\lim_{h\to\infty}\frac{\bar{F}_H(h)}{hf_H(h)} = 0$, where $\bar{F}_H(h) = 1 - F_H(h)$ represents the Compliment Cumulative Distribution Function. For example, the fading distributions such as Rayleigh satisfy this definition.

Each sensor has access to its own CSI (toward the sink), meaning that at the beginning of the oth slot the channel gain H_{id} is known by sensor i and the gain G_{kd} is known by the potential relay k during its relaying phase. Note that this is the same assumption as the one used in the distributed CSI analysis in [4], [5]. In practice this knowledge may be estimated by having the sink periodically broadcast a pilot signal or directly obtained via feedback from the sink. Each sensor also knows its own channel state distribution. Note that, in the decentralized environment under consideration, a sensor does *not* have access to the CSI of other sensors. Furthermore, sensor i does *not* have access to the channel state H_{ij}, since in a large network it becomes unreasonable to assume that all of the sensors have access to the state of the channel towards any other peer.

We denote by $P_{1i}(H_{id})$ the transmission power of sensor i in Phase A and $P_{2i}(G_{id})$ its transmission power in Phase B. Throughout this work we consider a *long-term average* power constraint on the sensors meaning that for each sensor

$$\mathbb{E}_{H_{id}}[P_{1i}(H_{id})] < \bar{P}_1, \quad \mathbb{E}_{G_{kd}}[P_{2k}(G_{kd})] < \bar{P}_2. \tag{2}$$

For the reception at the sink we assume a channel-aware ALOHA type model in the fading environment [5]. During each transmission block the destination can only decode the message received from one sensor successfully. Therefore, in (1) only the ith sensor can send its message. We make identical assumption as [5] for the maximum rate $R(\gamma)$ that a sensor can transmit as a function of the sensor's channel state γ, where for the ith source $\gamma_i = \frac{P(H_{id})H_{id}}{Z_d}$. We normalize Z_d to be equal to one. It is also assumed that R has zero asymptotic elasticity [5] meaning that $\limsup_{\gamma\to\infty}\frac{\gamma R'(\gamma)}{R(\gamma)} = 0$. As an example, the Shannon capacity satisfies this requirement and has been used for our analytical results.

In the two-phase model considered in this work, the transmission rate in the first phase is expressed as $R_D(\gamma_i)$, while the achieved rate via cooperation of K sensors is expressed as $R_C(\Phi)$. Since the forwarders cooperate over sending a common message, they can be considered as distributed antennas sending the same message and MIMO capacity results apply in the second phase.

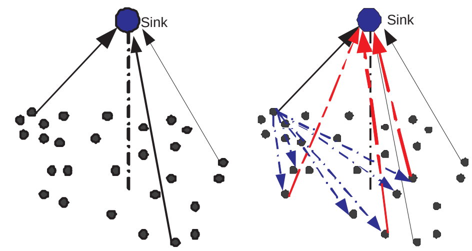


Fig. 1. Schematic of a) CAA b) CAAC.

B. CAAC Protocol Description and Optimization Problem Formulation

The CAA protocol proposed in [5] is a variation of ALOHA, in which the transmission decision is made based on the channel state as opposed to the traditional ALOHA in which sensors transmit with a predetermined probability. In their work, the authors introduce a decision threshold for the channel state of each sensor. If the sensor's channel is above the threshold it will transmit and otherwise it will remain silent. Assuming uplink down-link duality the sink can transmit periodic pilot signals and each sensor can estimate its own channel *independently* from the other sensors.

In *Channel Aware Aloha with Cooperation*, sensor i transmits when its channel state to the destination is above a threshold h_s. The arrived message at relay k is decoded with probability Pr_{dec} which is the same at different relays due to the assumption of symmetric channels. A relay k will forward its decoded message to the destination if its channel gain towards the destination is above a threshold g_{rel}. The transmission and relaying probabilities can therefore be expressed as $\text{Pr}_s = \text{Pr}(h > h_s) = \bar{F}_H(h_s)$ and $\text{Pr}_{rel} = \text{Pr}(g > g_{rel}) = \bar{F}_G(g_{rel})$, where $\bar{F}_H(a) = \int_a^\infty f_H(h)dh$ represents the compliment of the cumulative distribution function and $\bar{F}_G(\cdot)$ is similarly defined. Fig. 1 presents a schematic comparison between CAA and CAAC.

In CAAC, the average network throughput $\mu(\text{Pr}_s, \text{Pr}_{rel}, n)$ can be expressed as

$$\mathbb{E}[\mu(\text{Pr}_s, \text{Pr}_{rel}, n)] = \frac{1}{2}\big(n\text{Pr}_s(1-\text{Pr}_s)^{n-1}\times \\ \mathbb{E}_H[R_D(P(H_{id})H_{id})|H_{id} > h_s] + \mathbb{E}_\Phi[R_C(\Phi)]\big), \tag{3}$$

where Φ is the received power at the sink. The first term represents the throughput in Phase A and includes two parts, a contention probability and a conditional rate, which is representative of the direct transmission throughput. The second term in the summation represents the cooperative phase (Phase B) throughput. The expected value of this term will be evaluated conditioned on the number of forwarder relays. We quantify the number of successful relays in the decoding of the message as the relays which do not undergo collision and outage. Based on this number, the probability mass function (pmf) of forwarding relays will be shown to be binomial in section III-A. The factor $\frac{1}{2}$ takes into account the fact that the transmission has occurred over two time slots of length L as compared to a direct transmission over one slot. Then, the optimization problem can be stated as

$$(\text{Pr}_s^*, \text{Pr}_{rel}^*) = \arg \max_{\text{Pr}_s, \text{Pr}_{rel}} \mu(\text{Pr}_s, \text{Pr}_{rel}, n). \tag{4}$$

We discuss in more details the implication of (3) under different power allocation settings in the following section.

III. Opportunistic Cooperation: Scaling Behavior

We consider the following three different power allocation settings:

Constant Power (CAAC-CP): In this case the sensors transmit with a constant power over each block, and the channel state information is only used to decide when to transmit.

Fixed Rate (CAAC-FR): The sensors use channel inversion to allocate power, so that a fixed value of throughput is guaranteed.

Optimal Variable Rate (CAAC-OVR): The sensors apply optimal power allocation over the channel states. This can be considered as a classical water-filling problem [12]. Each sensor/relay only has access to its own channel state and performs water-filling independently from the other sensors.

For each power allocation setting, we present throughput analysis, parameter optimization, and the scaling behavior for large n.

A. CAAC-CP

In this case we assume that each source transmits with a constant power P_{t1} during each slot, and the relays which are successful in the decoding of the message forward the decoded version of the message with power P_{t2}. The long-term average power constraint results in

$$\int_{h_s}^{\infty} P_{t1} f_H(h) dh = P_{t1} \bar{F}_H(h_s) \leq \bar{P}_1 \Rightarrow P_{t1} \leq \frac{\bar{P}_1}{\text{Pr}_s}$$
$$\int_{h_{rel}}^{\infty} P_{t2} f_G(g) dg = P_{t2} \bar{F}_G(g_{rel}) \leq \bar{P}_2 \Rightarrow P_{t2} \leq \frac{\bar{P}_2}{\text{Pr}_{rel}}. \tag{5}$$

Since only one source can transmit successfully during each slot due to the collision assumption in this work, when this happens, the throughput R_D is only a function of the channel state for the single transmitting sensor i. Other sensors listen to sensor i's transmission and try to decode it. Note that during Phase B, other sensors will not send their own data, since this would cause another collision.

We now quantify the set of decoding relays for a source i which we denote as $\mathbb{D}(i)$. Using the same approach as [7], for each potential relay j a message from sensor i is successfully decoded, if the mutual information between i and j is above a required rate, i.e., if $\mathbb{I}(i,j) > R_{th}$, where $\mathbb{I}$ represents the mutual information and R_{th} is the required threshold for decoding. However, in our case if more than one sensor sends during Phase A a collision will happen in Phase B. Therefore, we can write

$$\begin{aligned}\text{Pr}_{\text{dec}} &= \text{Pr}(\mathbb{I}(i,j) > R_{th} | \mathbb{NC}(j)) \times \text{Pr}(\mathbb{NC}(j)) \\ &= \text{Pr}(\log(1 + \frac{\bar{P}_1}{\text{Pr}_s} H_{ij}) > R_{th}) \times n\text{Pr}_s(1 - \text{Pr}_s)^{n-1} \\ &= \text{Pr}(\frac{\bar{P}_1}{\text{Pr}_s} H_{ij} > \gamma) n\text{Pr}_s(1 - \text{Pr}_s)^{n-1},\end{aligned}$$

where the threshold $\gamma = 2^{R_{th}} - 1$ and $\mathbb{NC}(j)$ represents the event of having no collisions at j. Note that by the assumption of symmetric gains the transmission probabilities are equal for different sensors in phase A as well as phase B and the optimization is simplified.

Prior to solving the throughput optimization problem (4), we can predict the scaling behavior of the source transmission probability. As $n \to \infty$, the source transmission probability has to follow $\text{Pr}_s \to 0$, or a collision will happen and the first term in (3) tends to 0. Therefore, as $n \to \infty$, $\text{Pr}(\frac{\bar{P}_1}{\text{Pr}_s} H_{ij} > \gamma) \to 1$ and $\text{Pr}_{dec} \to n\text{Pr}_s(1 - \text{Pr}_s)^{n-1}$. This probability is maximized by replacing $\text{Pr}_s = \frac{1}{n}$, which results in $\text{Pr}_{dec} \to \frac{1}{e}$. This value for the transmission probability in Phase A maximizes the throughput of the direct transmission term as shown in [4]. Hence, it optimizes (4).

The decoding event at sensor j is a Bernoulli random variable with

$$I_{\text{dec}}^j = \begin{cases} 1, & \text{with pr.} \quad \text{Pr}_{\text{dec}} \\ 0, & \text{with pr.} \quad 1 - \text{Pr}_{\text{dec}} \end{cases}. \tag{6}$$

The number of successful decoding sensors can therefore be expressed as $K = \sum_{j=1, j \neq i}^{n} I_{dec}^j$. We make use of the Chernoff bound [13] to find the number of successful decoding relays with good precision. Using the lower bound $\text{Pr}[K < (1-\delta)\text{Pr}_{dec} n] < e^{-\frac{n\text{Pr}_{dec}\delta^2}{2}}$, we have

$$\lim_{n \to \infty} \text{Pr}_{dec} \to \frac{1}{e} \Rightarrow \lim_{n \to \infty} \text{Pr}(K < (1-\delta)\frac{n}{e}) < e^{-\frac{\frac{n}{e}\delta^2}{2}}. \tag{7}$$

As $\delta \to 0$ it suffices to have $n > \frac{M}{\delta^2}$, where $M \to \infty$, to guarantee that $\lim_{n \to \infty, \delta \to 0} \text{Pr}(K < \frac{n}{e}) = 0$. By the same token, the upper bound in the Chernoff bound results in $\lim_{n\to\infty,\delta\to 0}\Pr(K > \frac{n}{e}) = 0$. Therefore, $\lim_{n\to\infty}\Pr(K = \lfloor\frac{n}{e}\rfloor) = 1$.

For the constant power case, the expected throughput can be expressed as

$$\mathbb{E}_{H,\Phi}[\mu(\Pr_s, \Pr_{rel}, n)] =$$
$$\frac{1}{2}\big(n(1-\Pr_s)^{n-1}\int_{h_s}^{\infty} R(\frac{P_1}{\Pr_s}h)f_H(h)dh + \mathbb{E}_\Phi[R_C^{\text{CP}}(\Phi)]\big),$$

where the first term represents the direct transmission throughput conditioned on the event that one sensor transmits and the $n-1$ remaining sensors are silent and the second term represents the expected throughput of cooperative transmission. The expected value of the direct transmission throughput is straightforward to obtain, since a sensor transmits only if its channel state $H_{id} > h_s$. We next elaborate on the derivation of the cooperative throughput expression $R_C^{\text{CP}}(\Phi)$.

Since the transmission is slotted, the set of successful decoding relays forward the same message in Phase B synchronously, and the cooperative phase throughput $R_C^{\text{CP}}(\Phi)$ is similar to that of a multiple input single output channel, which will be written as

$$R_C^{\text{CP}}(\Phi) = R(\sum_{k=1}^{K} I[G_{kd} > g_{rel}]P_{t2}G_{kd}), \tag{8}$$

where $I[G_{kd} > g_{rel}]$ is the indicator function representing the event that the relay-destination channel state for relay k is above a determined threshold, and R is a rate function which satisfies the asymptotic elasticity property and $P_{t2} = \frac{\bar{P}_2}{\Pr_{rel}}$.

In order to find the expected value of the cooperative phase throughput we express the throughput conditioned on the number of forwarding relays as follows.

$$\mathbb{E}_\Phi[R_C^{\text{CP}}(\Phi)] =$$
$$\sum_{l=1}^{K}\binom{K}{l}(1-\Pr_{rel})^{K-l}\mathbb{E}\big[R(\sum_{i=1}^{l} P(G_{s_id})G_{s_id}|\mathbb{F} = \mathbf{s})\big]$$
$$= \sum_{l=1}^{K}\binom{K}{l}(1-\Pr_{rel})^{K-l}\underbrace{\int_{g_{rel}}^{\infty}\cdots\int_{g_{rel}}^{\infty}}_{l \quad \text{integrations}}$$
$$R(\frac{\bar{P}_2}{\Pr_{rel}}(g_{s_1d}+\ldots+g_{s_ld}))f_G(g_{s_1})\ldots f_G(g_{s_l})dg_{s_1}\ldots dg_{s_l}, \tag{9}$$

where $\mathbf{s} = \{s_1, \ldots, s_l\}$, represents a subset of potential relays with cardinality l chosen from K relays successful in decoding, and $\mathbb{F}$ is the set of forwarding relays during Phase B. The term $(1-\Pr_{rel})^{K-l}$ in (9) reflects the probability that $K-l$ relays remain silent and do not forward. The second term is the expected cooperative throughput conditioned on the event that l relays are forwarding the message to the sink. This result is also a special case of Theorem 1 in [3], where we have used a binary type transmission control by using the notion of g_{rel} as the relaying threshold instead of general transmission control, which is a function of the channel state.

We further evaluate the scaling behavior of the above throughput expression in the following proposition.

Proposition 1: As $n \to \infty$ the integral term

$$\int_{g_{rel}}^{\infty}\cdots\int_{g_{rel}}^{\infty} R(\frac{\bar{P}_2}{\Pr_{rel}}\sum_{i=1}^{l} g_{s_id})f_G(g_1)\ldots f_G(g_l)dg_1\ldots dg_l,$$

in the limit approaches $\Pr_{rel}^l R(\frac{\bar{P}_2}{\Pr_{rel}}lg_{rel})$.

Proof: Refer to Appendix I. ∎

By replacing the result of Proposition 1 in (9) in the limit of large n we have

$$\lim_{n\to\infty}\mathbb{E}_\Phi[R_C^{\text{CP}}(\Phi)] =$$
$$\sum_{l=1}^{\lfloor\frac{n}{e}\rfloor}\binom{\lfloor\frac{n}{e}\rfloor}{l}(1-\Pr_{rel})^{\lfloor\frac{n}{e}\rfloor-l}\Pr_{rel}^l R(\frac{\bar{P}_2}{\Pr_{rel}}lg_{rel}), \tag{10}$$

where we have replaced K by $\lfloor\frac{n}{e}\rfloor$ in the limit as we showed by use of the Chernoff bound. The next proposition quantifies the optimal relaying probability as $n \to \infty$.

Proposition 2: The optimal relaying probability scales as $\Pr_{rel}^* = \frac{\beta(n)}{n}$ as $n \to \infty$, where $\beta(n)$ is a constant.

Proof: Refer to Appendix II for the proof. ∎

We are now ready to compute the overall scaling behavior of the throughput $\mu(\Pr_s^*, \Pr_{rel}^*, n)$. As a special case of Proposition 1 with $l = 1$ it can be shown that

$$\lim_{n\to\infty}\int_{h_s}^{\infty} R(\frac{P_1}{\Pr_s}h)f_H(h)dh = \Pr_s R(\frac{\bar{P}_1}{\Pr_s}h_s). \tag{11}$$

The overall throughput can, therefore, be written as

$$\lim_{n\to\infty}\mathbb{E}[\mu(\Pr_s, \Pr_{rel}, n)] =$$
$$\frac{1}{2}\big(\lim_{n\to\infty} n\Pr_s(1-\Pr_s)^{n-1}R(\frac{\bar{P}_1}{\Pr_s}h_s) + \mathbb{E}_\Phi[R_C^{\text{CP}}(\Phi)]\big). \tag{12}$$

Corollary 1: The direct term throughput is maximized by choosing $\Pr_s = \frac{\alpha(n)}{n}$, where $\lim_{n\to\infty}\alpha(n) = 1$.

Proof: This is a direct result of Proposition 4 of [4] and considering the fact that $n\Pr_s(1-\Pr_s)^{n-1}$ attains its maximum for $\Pr_s = \frac{1}{n}$. The rate term is a decreasing function of $\Pr_s$. Therefore $0 < \alpha < 1$. As n increases the scaling of the rate becomes independent of α, therefore, $\mathrm{P}_s = \frac{1}{n}$ is the optimal value. ∎

Replacing the optimal probabilities in (12) we have

$$\lim_{n\to\infty}\mathbb{E}[\mu(\text{Pr}_s,\text{Pr}_{rel},n)]=\frac{1}{2}\lim_{n\to\infty}[\frac{1}{e}R(\bar{P}_1 n\log n)$$
$$+\sum_{l=1}^{\lfloor\frac{n}{e}\rfloor}\binom{\lfloor\frac{n}{e}\rfloor}{l}(1-\frac{\beta}{\lfloor\frac{n}{e}\rfloor})^{n-l}(\frac{\beta}{\lfloor\frac{n}{e}\rfloor})^{l}R(\bar{P}_2\frac{\lfloor\frac{n}{e}\rfloor}{\beta}\log\frac{\lfloor\frac{n}{e}\rfloor}{\beta})], \tag{13}$$

where we have assumed Rayleigh fading to derive $g_{rel}=\bar{F}_G^{-1}(\frac{\beta e}{n})=\log\frac{n}{\beta e}$. Based on our reasoning in Appendix II the scaling behavior of rate term R does not depend on β and we can assign $\beta=l$ and take the rate term out of the summation. Therefore, the above expression scales as $\frac{1}{2}(1+\frac{1}{e})R(n\log n)$.

B. CAAC-FR

For applications with the requirement of a fixed data rate the sensors can exploit the channel state information and perform power control to attain a constant rate, while the channel state is changing. In this section we will quantify the throughput when $n\to\infty$. While the scaling behavior of the throughput has been addressed for direct transmission in [4], the question that arises is "Can throughput increases be obtained by using cooperation compared to the direct transmission in this setting?"

For the case of direct transmission each sensor performs channel inversion to maintain a constant bit-rate. Therefore, the expected throughput constraint can be written as $\int_{h_s}^{\infty}P_{t1}f_H(h)dh\le\bar{P}_1$, where $P_{t1}=\frac{P_r}{h}$, and P_r is the received constant power at the destination. This translates to the requirement $P_r\le\frac{\bar{P}_1}{\int_{h_s}^{\infty}\frac{1}{h}f_H(h)dh}$. We now evaluate the network throughput performance under this setting and show that cooperation in this case *deteriorates* the performance.

The overall throughput for the two phase transmission is

$$\mathbb{E}[\mu(\text{Pr}_s,\text{Pr}_{rel},n)]=$$
$$\frac{1}{2}\big(n\text{Pr}_s(1-\text{Pr}_s)^{n-1}R(P_r)+\mathbb{E}_\Phi[R_C^{\text{FR}}(\Phi)]\big),$$

where $\mathbb{E}_\Phi(R_C^{\text{FR}}(\Phi))$ represents the cooperative throughput for the fixed rate setting. During Phase A sensor i sends its message towards the destination and performs power control to maintain a constant bit rate $R(P_r)$ as a function of the required received power at the destination. In this phase the maximum possible transmitted power is $P_{t1}=\frac{Pr}{H_{id}}\le\frac{\bar{P}_1}{H_{id}\int_{h_s}^{\infty}\frac{1}{h}f_H(h)dh}$. We can write the correct decoding probability at sensor j, when only sensor i is transmitting as

$$\text{Pr}_{dec}=\text{Pr}(\mathbb{NC}(j))\text{Pr}(P_{t1}H_{ij}>\gamma|\mathbb{NC}(j))=$$
$$\text{Pr}(\mathbb{NC}(j))\text{Pr}(\frac{\bar{P}_1}{H_{id}\int_{h_s}^{\infty}\frac{1}{h}f_H(h)dh}H_{ij}>\gamma). \tag{14}$$

We consider the special case of Rayleigh fading channel with mean h_0 to obtain a closed form expression for the decoding probability. In this case we have

$$\text{Pr}_{dec}=\text{Pr}(\mathbb{NC}(j))\text{Pr}(H_{ij}\ge\frac{H_{id}}{A})=$$
$$\text{Pr}(\mathbb{NC}(j))\int_{h_s}^{\infty}\bar{F}_H(\frac{h_{id}}{A})f_H(h_{id})dh_{id}=$$
$$n\text{Pr}_s(1-\text{Pr}_s)^{n-1}\int_{h_s}^{\infty}e^{-\frac{h_{id}}{h_0A}}\frac{e^{-\frac{h_{id}}{h_0}}}{h_0}dh_{id}=\frac{A}{A+1}e^{-h_s(\frac{A+1}{h_0A})},$$

where $A=\frac{\bar{P}_1}{\gamma\int_{h_s}^{\infty}\frac{f_H(h)}{h}dh}$.

The direct term throughput in this case is $n\text{Pr}_s(1-\text{Pr}_s)^{n-1}R(P_r)$. It is maximized by $\text{Pr}_{s,\text{opt}}=\frac{1}{n}$ as $n\to\infty$ and approaches $\frac{1}{e}R(P_r)$. Since this value of transmission probability also minimizes the collision probability at the relays, it is indeed the optimal value. We are interested in evaluating the scaling behavior of the throughput, so we evaluate the decoding probability as $n\to\infty$. To this end we need to find $\lim_{n\to\infty}A$. For $h>h_s$, we can write $\frac{f_H(h)}{h}<\frac{f_H(h)}{h_s}$. In Appendix I of [5] it is proved that for well behaved densities $\int_{h_s}^{\infty}\frac{f_H(h)}{h_s}dh\to\frac{\bar{F}_H(h_s)}{h_s}$, as $n\to\infty$ and h_s increases. Therefore, we have $\lim_{n\to\infty}A=\frac{\bar{P}_1}{\gamma\int_{h_s}^{\infty}\frac{f_h(h)}{h_s}dh}=\frac{\bar{P}_1h_s}{\gamma\text{Pr}_s}$. Replacing the optimal transmission probability, since $h_s=\bar{F}_H^{-1}(\text{Pr}_s)=h_0\log(\frac{1}{\text{Pr}_s})$, for the case of Rayleigh fading, we deduce that as $n\to\infty$, $A\to\infty$. Therefore, the decoding probability in the limit scales as

$$\lim_{n\to\infty}\text{Pr}_{dec}=\frac{1}{e}\lim_{A\to\infty}\frac{A}{A+1}e^{-h_s(\frac{A+1}{h_0A})}=\frac{1}{e}e^{-\frac{h_s}{h_0}}=\frac{1}{e}\text{Pr}_s. \tag{15}$$

Therefore, the decoding probability Pr_{dec} scales as $\frac{1}{n}$. Similar to CAAC-CP, the number of successful decoding relays, K, is a binomial random variable. However, in this case the Chernoff bound is no longer tight since $n\text{Pr}_{dec}\to 1$.

The number of successful decoding relays is

$$\text{Pr}(K=c)=\binom{n-1}{c}(\frac{1}{ne})^c(1-\frac{1}{ne})^{n-1-c}.$$

Hence, for the fixed rate allocation setting, the pmf of K decreases rapidly. In fact, the expected value of K is $\frac{1}{e}$, showing that on average less than 1 relay is successful in decoding!. Therefore, the number of successful decoding relays does not scale with the number of the sensors in the network and only the first few terms of $\text{Pr}(K=c)$ affect the cooperative phase throughput.

We now evaluate the cooperative phase throughput $\mathbb{E}_\Phi[R_C^{\text{FR}}(\Phi)]$ conditioned on the event that c relays have decoded the message of Phase A successfully. The number of sensors that relay the message is a binomial random variable Q, with the distribution $\text{Pr}(Q=q)=$

$\binom{c}{q}\Pr_{rel}^q(1-\Pr_{rel})^{c-q}$. For q relays sending the message to the destination the power of the received signal is

$$P_r^{\mathrm{C}} = \sum_{i=1}^{q} P_{r,s_i} = \sum_{i=1}^{q} P_{t2}(G_{s_i d})G_{s_i d}, \quad (16)$$

where **s** is again a subset of relays with cardinality q, P_{r,s_i} represents the power received from relay s_i at the destination and P_r^{C} is the total received power of the cooperative phase at the destination. The relays employ channel inversion to maintain a fixed bit rate. Therefore, the average power constraint for each relay follows the same format as the power constraint for the source in the direct transmission phase and we have $P_{r,s_i} \leq \frac{\bar{P}_2}{\int_{g_{rel}}^{\infty}\frac{1}{g}f_G(g)dg}$, $\forall \;\; 1 < i < q$. The overall received power during the cooperative phase is constraint as $P_r^{\mathrm{C}} \leq \frac{q\bar{P}_2}{\int_{g_{rel}}^{\infty}\frac{1}{g}f_G(g)dg}$. Based on the assumption of distributed antennas sending the same message we can again express $R(\cdot)$ as a function of the received power at the destination and write its expected value as

$$\mathbb{E}_\Phi(R_C^{\mathrm{FR}}(\Phi)) = \sum_{c=1}^{n-1}\Pr(K=c)\sum_{q=1}^{c}\Pr(Q=q)R(\frac{q\bar{P}_2}{\int_{g_{rel}}^{\infty}\frac{1}{g}f_G(g)dg}). \quad (17)$$

Obtaining a closed form expression for $\Pr_{rel}$ in this case follows from setting the derivative of (17) equal to zero and is intractable. Instead, we will evaluate the throughput performance in our numerical results. Intuitively, since only very few sensors on average decode the message correctly, cooperation does not increase the spatial diversity. The maximum decoding probability occurs when we have one successful relay and equals $\Pr(K=1) \simeq 0.25$, which represents the throughput loss compared to a direct transmission. We next address the throughput performance under the optimal power allocation assumption.

C. CAAC-OVR

Under this setting we assumed that each sensor performs optimal power allocation to maximize the sum-rate, under the long term power constraint. To find the decoding probability in this case, we first need to find the sensor transmission power as a function of its channel state. This issue is addressed in [5] and the optimization problem, which has a "Water-filling" [12] power allocation, is expressed as $\max \int_{\mathrm{h}_s}^{\infty}\log(1+P(h_{id})h_{id})f_H(h_{id})dh_{id}$, subject to $\int_{\mathrm{h}_s}^{\infty}P(h_{id})f_H(h_{id})dh_{id} = \bar{P}_1$. We will only use the solution of this problem $P_{t1}(h_{id}) = (\frac{1}{\lambda_p}-\frac{1}{h_{id}})^+$, where $\lambda_p = \dfrac{\Pr_s}{\bar{P}_1+\int_{h_s}^{\infty}\frac{f_H(h)}{h}dh}$. Then the successful decoding probability for the message sent to sensor j from sensor i in Phase A can be written as

$$\begin{aligned}&\Pr_{dec} = \Pr(P_{t1}(H_{id})H_{ij} > \gamma | H_{id} > H_{rel})\Pr(\mathbb{NC}(j)) = \\ &\Pr\big((\frac{\bar{P}_1+\int_{h_s}^{\infty}\frac{f_H(h)}{h}dh}{\Pr_s}-\frac{1}{H_{id}})H_{ij} > \gamma | H_{id} > h_s\big)\times \\ &\Pr(\mathbb{NC}(j)).\end{aligned} \quad (18)$$

As $n \to \infty$, since $\Pr_s \to 0$ and $\lim_{n\to\infty} h_s = \lim_{n\to\infty}\bar{F}_H^{-1}(\Pr_s) = \infty$, we have $\int_{h_s}^{\infty}\frac{f_H(h)}{h}dh \leq \frac{\int_{h_s}^{\infty}f_H(h)dh}{h_{rel}} = \frac{\Pr_s}{h_s} \to 0$. Since the channel between the source and the destination $H_{id} > h_s$, it tends to ∞. In contrast, the source-relay channel H_{ij} is an unconditional random variable, and since $\lim_{h'\to\infty}F_{H'}(h') = 1$, it is limited with probability 1. Hence, we conclude that $\lim_{n\to\infty}\frac{H_{ij}}{H_{id}} = 0$ with probability 1, and the decoding probability has the form

$$\lim_{n\to\infty}\Pr_{dec} = \Pr\big(\frac{\bar{P}_1}{\Pr_s}H_{ij} > \gamma\big)\Pr(\mathbb{NC}(j)). \quad (19)$$

Interestingly, this expression is the same as the decoding probability for the case of *constant power* addressed in Section III-A. Along the same lines of reasoning, the decoding probability tends to $\frac{1}{e}$ in this case, and by using the Chernoff bound we conclude that the number of decoding relays $\lim_{n\to\infty}K = \lfloor\frac{n}{e}\rfloor$.

We can use (8) by replacing the constant power term by the optimal allocation. Then the instantaneous cooperative throughput term is

$$R_C^{\mathrm{OVR}}(\Phi) = R(\sum_{k=1}^{K} I[G_{kd} > g_{rel}]P_{t2}(G_{kd})G_{kd}). \quad (20)$$

Since each relay employs water-filling independently from the other relays, the power allocation for relay k has the same solution as water-filling for the source and can be written as $P_{t2}(G_{kd}) = \dfrac{\bar{P}_2+\int_{g_{rel}}^{\infty}\frac{f_G(g)}{g}dg}{\Pr_{rel}} - \frac{1}{G_{kd}}$. Therefore, conditioned on the event that l relays of the subset $\mathbf{s} = \{s_1,\ldots,s_l\}$ are currently transmitting, the rate delivered obeys

$$R_C^{\mathrm{OVR}}(\Phi) = R(\frac{\bar{P}_2+\int_{g_{rel}}^{\infty}\frac{f_G(g)}{g}dg}{\Pr_{rel}}\sum_{i=1}^{l}G_{s_i d} - l). \quad (21)$$

Following the same lines as (9), and replacing the result of (21) for the cooperative throughput expression, the expected value of cooperative throughput can be written as

$$\begin{aligned}&\mathbb{E}_\Phi(R_C^{\mathrm{OVR}}(\Phi)) = \sum_{l=1}^{K}\binom{K}{l}(1-\Pr_{rel})^{K-l}\underbrace{\int_{g_{rel}}^{\infty}\cdots\int_{g_{rel}}^{\infty}}_{l \text{ integrations}} \\ &R(\frac{\bar{P}_2+\int_{g_{rel}}^{\infty}\frac{f_G(g)}{g}dg}{\Pr_{rel}}\sum_{i=1}^{l}g_{s_i d} - l)\prod_{i=1}^{l}(f_G(g_{s_i})dg_{s_i}).\end{aligned} \quad (22)$$

Using the same first order expansion that we used in the proof of Appendix I, the integral part can be approximated in the limit of large n as $\mathrm{Pr}_{rel}^{l} R(\frac{\bar{P}_2 + \int_{g_s}^{\infty} \frac{f_G(g)}{g} dg}{\mathrm{Pr}_{rel}} l g_{rel} - l)$. Since the proof follows along the lines of Proposition 1, we omit it due to the space limit.

By the assumption of a well behaved distribution for the channel gain, we have $\lim_{n \to \infty} \int_{g_{rel}}^{\infty} \frac{f_G(g)}{g} dg \leq \frac{\mathrm{Pr}_{rel}}{g_{rel}}$ [5]. Replacing this result in (22) and considering that the performance of optimal power allocation is at least as good as the constant power case, we have the same cooperative throughput as (10).

Hence, we conclude that the optimal power allocation results in the same scaling performance as that of the constant power allocation and does not give extra benefits in the limit of large number of sensors. However, for smaller values of n the relaying threshold will decrease and the optimal power allocation can help increase the throughput since the power is allocated optimally for the poor (small) channel states. The partial throughput increase of CAAC-OVR compared with CAAC-CP will be validated in our simulation results.

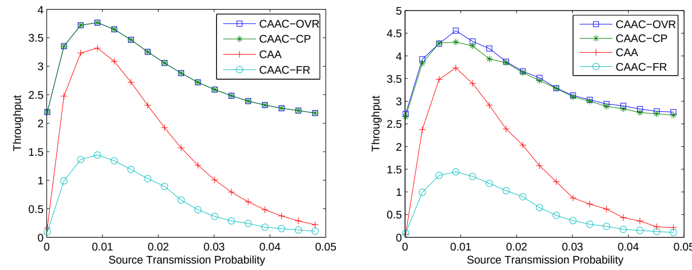
IV. Numerical Examples and Simulation

In this section numerical comparison between different power settings of the decentralized cooperative scheme is presented. We also compare the results with *Channel Aware Aloha* without cooperation and a simple *Aloha-C* cooperative scheme in which the sensors transmit independently from their channel state.

A. Simulation

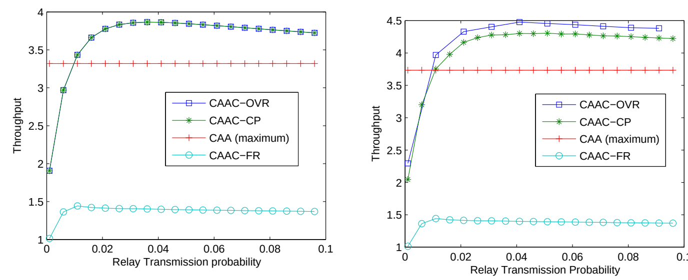
We consider a network of 100 sensors with the power constraint $\bar{P}_1 = 1$ for the transmission phase and $\bar{P}_2 = 0.1$ for the cooperation phase. The decoding threshold is set to $\gamma = 1$, and the sensors are assumed to undergo normalized Rayleigh fading during each time slot.

For different power settings of CAAC, Fig. 2 presents the throughput performance for different source transmission probabilities Pr_s, while we have considered the relaying probability to be fixed and equal to its optimal value. As we have shown in Section III, the scaling behavior of CP and OVR is the same, and we have used the same analytical results in Fig. 2 (a). Fig. 2 (b) presents the simulation results. These results are close to the analytical expressions. The partial throughput increase of OVR compared to CP is due to the fact that the number of sensors is limited for the simulations. Therefore, in this case the transmission threshold, which is of the order of $\log n$ is small. This results in the sensors



(a) Analysis. (b) Simulation.

Fig. 2. Network throughput vs. source transmission probability for 100 sensors.



(a) Analysis. (b) Simulation.

Fig. 3. Network throughput vs. relaying probability for 100 sensors.

with smaller channel states to be allowed to transmit. Hence optimal power allocation can partially benefit these sensors with poor channel conditions, improving the overall throughput compared to CP.

Fig. 3 demonstrates the throughput vs. the relaying probability, while the source transmission probability is fixed and equal to its optimal value. It can be seen that the optimal relaying probability follows the result of Proposition 2. However, in this case as we move further from the optimal value, the throughput does not decrease dramatically. This can be explained by cooperation between the sensors. The higher the relaying probability is, the more cooperative sensors are probable to relay. Although the multiuser diversity effect as a result of the choice of optimal threshold decreases, having more potential relays to some extent compensates for this shortage. In Fig. 3 (b) the partial increase in the the throughput obtained in OVR scheme compared to CP in the simulation results can again be seen. The slight difference between the analysis and simulation results stems from our approximations in proving Propositions 1 and 2, which become precise as $n \to \infty$.

B. Comparison with CAA and Aloha-C

It is clear from Fig. 2 and 3 that the CAAC-CP and CAAC-OVR schemes outperform CAA due to the spatial reuse gain obtained by cooperation. This is further confirmed in Fig. 4(a), where we plot the throughput vs. the number of sensors. Note, however, that CAAC-FR results in deteriorated performance. This is due to the lack of cooperation. As explained in Section III-B, the decoding probability in this case decreases as $\frac{1}{ne}$. Therefore, the cooperative phase throughput is less than CAA, and hence the overall throughput is below that of CAA.

For further comparison, we also consider a naive cooperative scheme with pure Aloha, termed Aloha-C, in which the sensors do not base their transmission probability on the state of the channel. Sensors transmit their message with probability $\frac{1}{n}$ during Phase A and the successful decoding relays transmit with power $\bar{P}_2$ in Phase B. The cooperative phase throughput can be expressed as $R_C(\Phi) = \log_2(1+\bar{P}_2\sum_{k=1}^{K} G_{kd})$, where K is the number of successful decoding relays in the cooperative phase. For this setting, the correct decoding probability is $\Pr(\mathbb{NC}(j))\Pr(\bar{P}_1 H_{ij} > \gamma) = \frac{1}{e}\bar{F}_{H'}^{-1}(\frac{\gamma}{\bar{P}_1})$. Using Jensen's inequality we have $\mathbf{E}_{\mathbf{G}}[R_C(\Phi)] = \mathbf{E}_{\mathbf{G}}[\log_2(1+\bar{P}_2\sum_{k=1}^{K} G_{kd})] \le \log_2(1+\bar{P}_2 E[K]E[G_{kd}]) = \log_2(1+\bar{P}_2 E[K])$, where $E[K] = \frac{n}{e}\bar{F}_{H'}^{-1}(\frac{\gamma}{\bar{P}_1})$. As $n \to \infty$ the overall throughput of this scheme is upper-bounded by $\frac{1}{2e}(\log_2(1+P_1) + \log_2(1+\bar{P}_2\frac{n}{e}\bar{F}_{H'}^{-1}(\frac{\gamma}{\bar{P}_1})))$. This bound has been plotted in Fig. 4(a). The significant throughput increase obtained by CAAC-CP and CAAC-OVR can be seen.

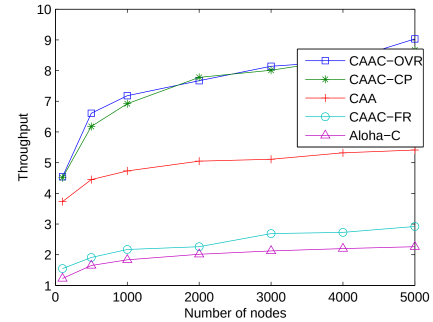
C. Scaling Behavior

Fig. 4 (a) further illustrate that increasing the number of sensors results in a higher level of diversity and leads to further increase in the network throughput. This increase is justified by (13), which predicts that the throughput scales as $\frac{1}{2}(1+\frac{1}{e})R(n\log n)$ for large values of n. Furthermore, in [4] it has been shown that the throughput of CAA scales as $\frac{1}{e}R(n\log n)$. Fig. 4 (a) confirms the $\frac{1+e}{2}$ asymptotic performance gain of CAAC over CAA.

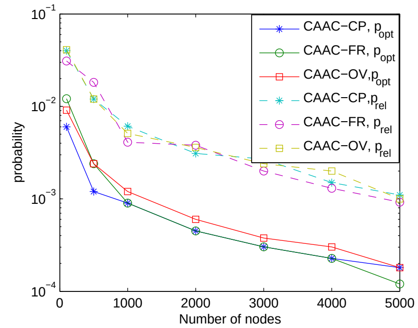
The scaling behavior of the optimal transmission probability and optimal relaying probability is depicted in Fig. 4 (b). The log-scale plot of the probabilities confirms that the optimal source transmission and relaying probabilities are decreasing with a scaling behavior of $\frac{1}{n}$ while we increase the number of sensors as explained in Section III.

V. Conclusion

We have studied the decentralized union of cooperative relaying and multiuser diversity in the context of sensor networks, using the proposed Channel Aware Aloha with Cooperation scheme. We propose an analytical performance evaluation framework considering Rayleigh fading and a collision-based reception model. We studied three power allocation schemes, Constant Power, Fixed Rate, and Optimal Variable Rate, and derived their asymptotic performance. Both analytical and simulation results demonstrate the throughput improvement obtained by CAAC in comparison with CAA without cooperation, or cooperative relaying without considering multiuser diversity. Furthermore, we show that CAAC with constant power allocation is asymptotically optimal, which suggests a low complexity means for achieving significant throughput increases even with simple sensors.



(a)



(b)

Fig. 4. a) Throughput vs. number of sensors. b) Relaying and source transmission probabilities vs. the number of sensors.

Appendix I
Proof of Proposition I

It suffices to prove that

$$\lim_{n\to\infty} \frac{\int_{g_{rel}}^{\infty}\cdots\int_{g_{rel}}^{\infty} R(\frac{\bar{P}_2}{\Pr_{rel}}(\sum_{i=1}^{l} g_{s_i d}))\prod_{i=1}^{l} f_G(g_{s_{id}})dg_{s_{id}}}{\Pr_{rel}^{l} R(\frac{\bar{P}_2}{\Pr_{rel}} l g_{rel})} = 1$$

To evaluate an upper-bound on the integral in the numerator at each step, we only consider integration over one of the variables $g_{s_i d}$ and assume the other variables to be fixed. Since R is assumed to be a concave function, using the first order expansion of $R(.)$, as a function of $g_{s_i d}$ and assuming $G = \sum_{j=1, j\neq i}^{l} g_{s_j d}$ we have $R(\frac{\bar{P}_2}{\mathrm{Pr}_{rel}}(G + g_{s_i d})) \leq R(\frac{\bar{P}_2}{\mathrm{Pr}_{rel}}(G + g_{rel})) + \frac{\bar{P}_2}{\mathrm{Pr}_{rel}} R'(\frac{\bar{P}_2}{\mathrm{Pr}_{rel}}(G + g_{rel}))(g_{s_i d} - g_{rel})$. Using an approach which is similar to the one used in [5] for integration with respect to each variable, we can write

$$\lim_{n\to\infty} \frac{\int_{g_{rel}}^{\infty} R(\frac{\bar{P}_2}{\mathrm{Pr}_{rel}}(G + g_{s_i d})) f_G(g_{s_i d}) dg_{s_i d}}{\mathrm{Pr}_{rel} R(\frac{\bar{P}_2}{\mathrm{Pr}_{rel}}(G + g_{rel}))} \leq$$

$$\lim_{n\to\infty} \frac{\int_{g_{rel}}^{\infty} [R(\frac{\bar{P}_2}{\mathrm{Pr}_{rel}}(G + g_{rel})) f_G(g_{s_i d}) dg_{s_i d}}{\mathrm{Pr}_{rel} R(\frac{\bar{P}_2}{\mathrm{Pr}_{rel}}(G + g_{rel}))} +$$

$$\frac{\int_{g_{rel}}^{\infty} \frac{\bar{P}_2}{\mathrm{Pr}_{rel}} R'(\frac{\bar{P}_2}{\mathrm{Pr}_{rel}}(G + g_{rel}))(g_{s_i d} - g_{rel}) f_G(g_{s_i d}) dg_{s_i d}]}{\mathrm{Pr}_{rel} R(\frac{\bar{P}_2}{\mathrm{Pr}_{rel}}(G + g_{rel}))}$$

$$= 1 + \lim_{n\to\infty} \frac{\frac{\bar{P}_2}{\mathrm{Pr}_{rel}} R'(\frac{\bar{P}_2}{\mathrm{Pr}_{rel}}(G + g_{rel}))}{R(\frac{\bar{P}_2}{\mathrm{Pr}_{rel}}(G + g_{rel}))} \times$$

$$\frac{\left(\int_{g_{rel}}^{\infty} g_{s_i d} f_G(g_{s_i d}) dg_{s_i d} - g_{rel}\mathrm{Pr}_{rel}\right)}{\mathrm{Pr}_{rel}}.$$

We multiply the first term by $G + g_{rel}$ and divide the second term by the same value. Pr_{rel}^{*} tends to 0 as n increases as we show in Proposition 2. Therefore, $\lim_{n\to\infty} g_{rel} = \bar{F}^{-1}(\mathrm{Pr}_{rel}) = \infty$. We can then use the asymptotic elasticity property of R to deduce that $\lim_{n\to\infty} \frac{\frac{\bar{P}_2}{\mathrm{Pr}_{rel}}(G+g_{rel}) R'(\frac{\bar{P}_2}{\mathrm{Pr}_{rel}}(G+g_{rel}))}{R(\frac{\bar{P}_2}{\mathrm{Pr}_{rel}}(G+g_{rel}))} = 0$. The second term $\frac{\int_{g_{rel}}^{\infty} g_{s_i d} f_G(g_{s_i d}) dg_{s_i d}}{(G+g_{rel})\bar{F}_G(g_{s_i d})} - \frac{g_{rel}}{g_{rel}+G}$ tends to 0, which can be shown by using Hoptial's rule (both the numerator and denominator tend to 0) similar to [5]. Therefore, when we perform the integration with respect to another variable $g_{s_j d}$, we can use the denominator as an upper-bound for the numerator and take the integral with respect to $g_{s_j d}$. Using the same reasoning l times, and in each step defining G as the sum of variables over which the integration is not performed yet, the integral in the numerator of equation can be shown to be upper-bounded by $\mathrm{Pr}_{rel}^{l} R(\frac{\bar{P}_2}{\mathrm{Pr}_{rel}} l g_{rel})$. It is clear that the denominator is also a lower-bound for the numerator. Therefore, the result holds.

APPENDIX II
PROOF OF PROPOSITION II

We first show that the optimal probability has to decrease inversely proportional to n and then evaluate β. The optimal relay probability decreases as a function of the number of sensors (due to the multiuser diversity effect) and can be written as $\mathrm{Pr}_{rel} = \frac{\beta(K)}{K^d}$ for $0 < d < \infty$. Taking the derivative of each term in $\binom{K}{l}\mathrm{Pr}_{rel}^{l}(1-\mathrm{Pr}_{rel})^{K-l}$, we see that its maximum occurs at $\mathrm{Pr}_{rel} = \frac{l}{K}$. For the lth component of the summation the throughput term $R(.)$ scales as $R(K\log(\frac{K}{l}))$, where we have used the fact that for a channel with normalized Rayleigh fading $\mathrm{Pr}_{rel} = e^{-\frac{g_{rel}}{g_0}}$, $g_{rel} = g_0 \log(\frac{1}{\mathrm{Pr}_{rel}})$. For any $\mathrm{Pr}_{rel} = \frac{1}{K^d}$, if $d < 1$, the first term is smaller than the value found by replacing the optimal $\mathrm{Pr}_{rel} = \frac{l}{K}$, and also the rate term scales as $R(K^d \log(\frac{K^d}{l}))$, which is smaller than $R(K\log(\frac{K}{l}))$. Therefore d cannot be smaller than 1. For $d > 1$, $\binom{K}{l}\frac{1}{K^{dl}}$ approaches 0, and scales like $\frac{1}{K^{(d-1)l}}$, while the rate term $R(.)$ has increased by a multiplicative factor d compared to the case where $d = 1$. Therefore, the maximum is attained for $d = 1$. So far, we have derived the optimal probability which attains the maximum value for each term. Since $R(K\log(\frac{K}{l}))$ can be written as $R(K) + R(\log(\frac{K}{l}))$, assuming that R has a logarithmic form, it is clear that the first term is dominant which is not dependent on l. Therefore, it can be taken out of the summation and the choice of the constant β will not lead to a change in terms of the scaling behavior.

REFERENCES

[1] R. Knopp and P. Humblet, "Information capacity and power control in single cell multiuser communications," *IEEE ICC, Seattle, WA*, June 1995.

[2] S. Shamai and E. Telatar, "Some information theoretic aspects of decentralized power control in multiple access fading channels," *1999 Information Theory and Networking Workshop*, June 1999.

[3] S. Adireddy and L. Tong, "Exploiting decentralized channel state information for random access," *IEEE Transactions on Information Theory*, vol. 51, pp. 537–561, February 2005.

[4] X. Qin and R. Berry, "Exploiting multiuser diversity for medium access control in wireless networks," *IEEE Infocom*, vol. 2, pp. 084–1094, April 2003.

[5] X. Qin and R. Berry, "Distributed approaches for exploiting multiuser diversity in wireless networks," *IEEE Transactions on Information Theory*, vol. 52, pp. 392–413, February 2006.

[6] J. N. Laneman, D. N. Tse, and G. W. Wornell, "Cooprative diversity in wireless networks: Efficient protocols and outage behaviour," *IEEE Transactions on Information Theory*, vol. 50, pp. 3062–3080, December 2004.

[7] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Transactions on Information Theory*, vol. 49, pp. 2415–2425, October 2003.

[8] A. Sendonaris, E. Erkip, and B. Azhang, "User cooperation diversitypart I: System description," *IEEE Transactions on Wireless Communications*, vol. 51, pp. 1927–1938, November 2003.

[9] R. Lin and A. P. Petropulu, "A new wireless network medium access protocol based on cooperation," *IEEE Transactions on Signal Processing*, vol. 53, pp. 4675–4684, December 2005.

[10] S. Vakil and B. Liang, "Balancing cooperation and interference in wireless sensor netowrks," *IEEE SECON*, September 2006.

[11] D. Gunduz and E. Erkip, "Opportunistic cooperation by dynamic resource allocation," *Accepted for publication in IEEE Transactions on Wireless Communications*, 2006.

[12] R. G. Gallager, *Information Theory and Reliable Communication*. John Wily and Sons, 1968.

[13] R. Motwani and P. Raghavan, *Randomized Algorithms*. Cambridge, UK: Cambridge University Press, 1 st ed., 1995.