Switched and Sectored Beamforming [1]

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1 Introduction

Having investigated the use of antenna arrays for direction of arrival estimation we now turn to the use of arrays for data processing. Remember the goal of this course was to investigate the use of arrays to support more users, reduce the rate of dropped calls and increase the efficiency of the wireless infrastructure. Here we will look at how *fixed beamforming*, i.e., *non-adaptive* signal processing at a base station antenna array can significantly improve on all these figures of merit. This non-adaptive processing is the traditional approach to array processing. Our analysis will be based, as well, on some traditional figures of merit.

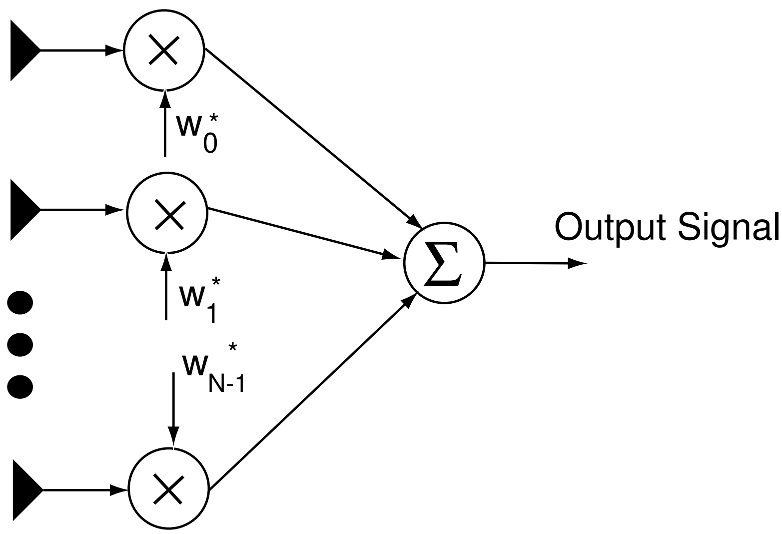

Figure 1: Generalized beamforming on receive

Any beamformer adds the signals at the N antenna elements weighted by scalars. Consider the array structure shown in Fig. 1. The signal at each element of the array is multiplied with a complex weight (the conjugate of the weights). The output signal is given by

$$y = \sum_{n=0}^{N-1} w_n^* x_n = \mathbf{w}^H \mathbf{x}, \tag{1}$$

where $\mathbf{w}$ is the length-N vector of weights. In a fixed beamformer, the weights used are fixed, i.e., the weight vector $\mathbf{w}$ is fixed.

The structure shown there is the general formulation of how we will process received signals - this is beamforming on receive. Note that due to reciprocity, if on transmit the excitation on

[1] Most of the material in this review is from Janaswamy [1]

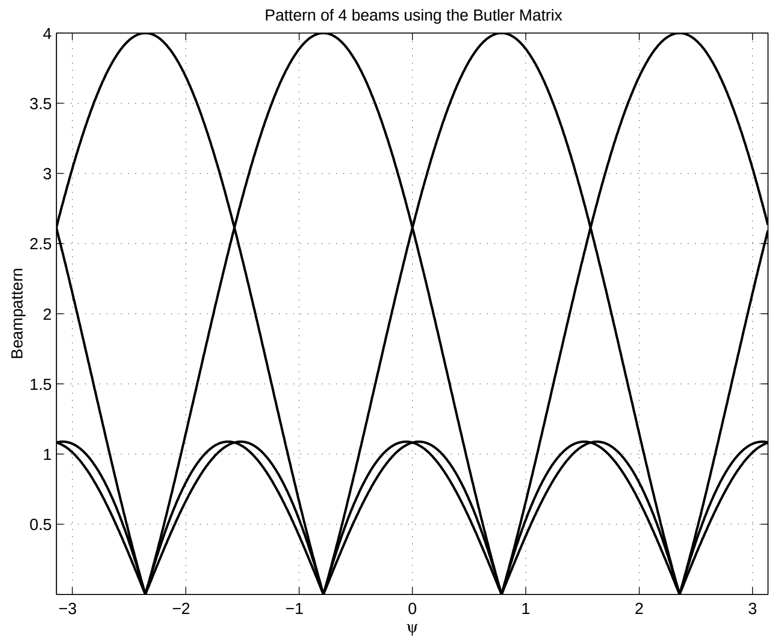

Figure 2: The four output beams of the Butler matrix

the n^{th} antenna were the weights w_n, the transmit beampattern would be the same as the receive beampattern. So, most of what we will say about receive is valid on transmit too.

2 The Butler Matrix

Each weight *vectors* corresponds to a particular beam pattern (beam). From our earlier discussion on array theory, we know that choosing $w_n = e^{jn\psi} = e^{jnkd\cos\phi}$ has a mainlobe in the direction ϕ. A simple choice of the set of weights, then, is to choose N sets of $\mathbf{w}$ such that each of the resulting beams is equally spaced in the ψ dimension. There is no theoretical limit on how many beams can be formed, however, choosing N beams makes the forming of the beams equivalent to a FFT of the received signals.

In practice, at the radio frequencies involved, this FFT can be implemented using a Butler matrix as shown in Fig. 3. The four outputs in the figure correspond to the four beampatterns shown in Fig. 2). The beams are plotted in a linear, not the usual dB scale. Note that the four beams are equally spaced in ψ space. Furthermore, the locations of each mainbeam is at the nulls of the other three beams. This is also a characteristic of the FFT operation. This is important for communication applications, because if the desired signal arrives from a direction at the peak of one beam, it does not leak into any other beam. We will see that this is the ideal situation for a *switched beam* system. Of course, in a more practical situation, the signal would not be perfectly at the mainbeam location. There would be an associated signal loss and leaking into multiple beams.

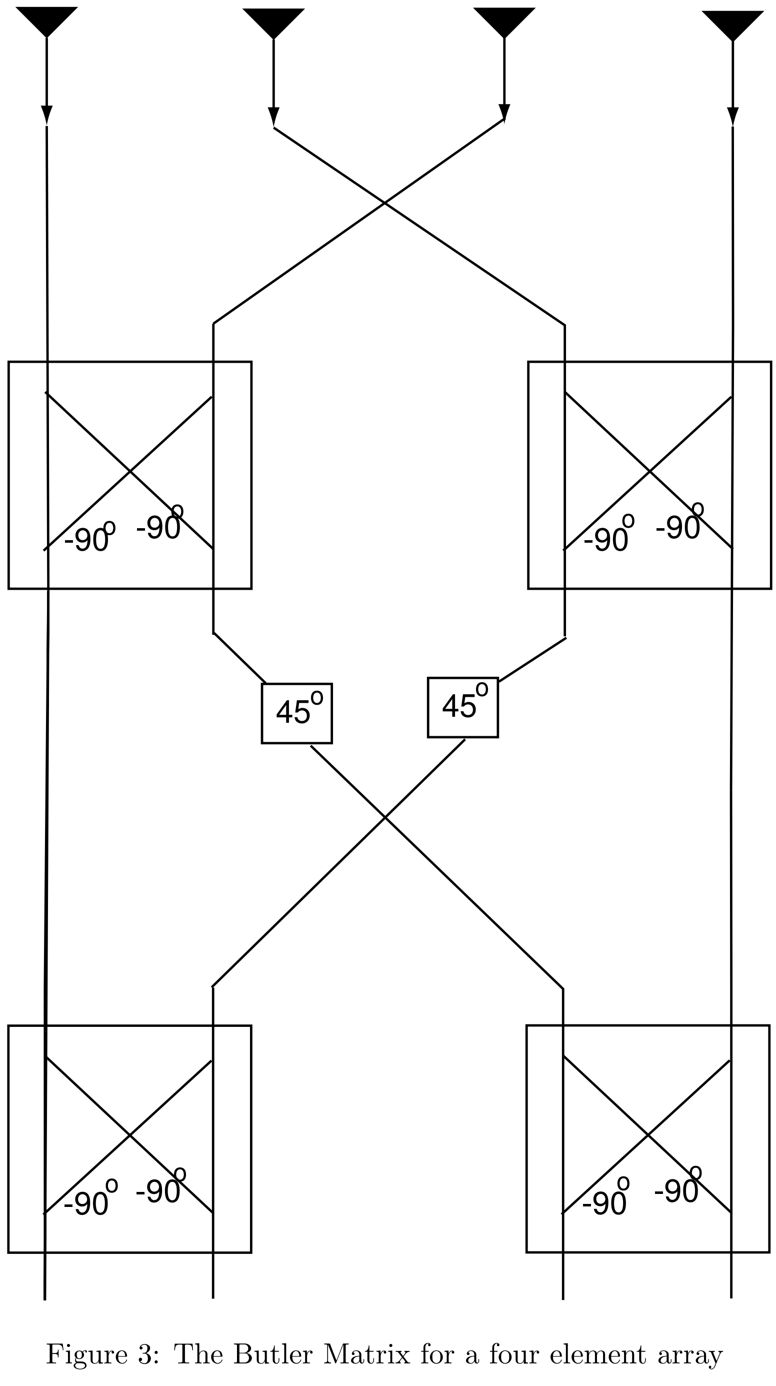

Figure 3: The Butler Matrix for a four element array

The signal loss is known as the *scalloping loss* [2].

2.1 Switched and Sectored Approaches

In a communication setting, fixed beamforming has been exploited in two ways: switched or sectored beamforming. Figure 4 presents the general scheme of a switched beam system. The signals from the N antenna elements are the input to a *beamforming* network with P beams. In general, $P \geq N$. The Butler matrix is a beamforming network that forms $P = N$ beams. For each user, the signal-to-noise ratio (SNR) of each beam is measured and the maximum is determined. The beam associated with the *largest SNR* is chosen for further processing. At any given time, all channels assigned to the cell that this array is serving are available to all users. Therefore, a particular beam, at certain times, may serve several users.

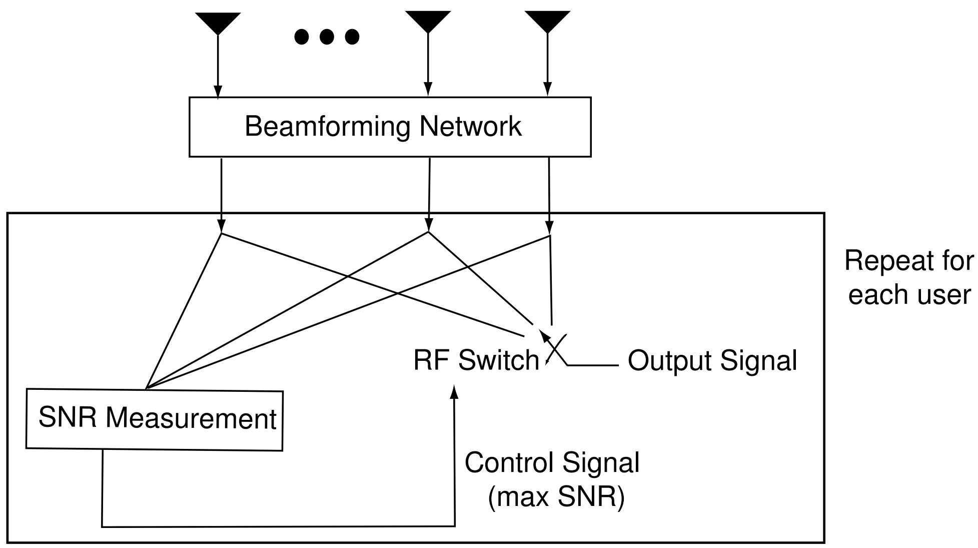

Figure 4: The general scheme of a switched beam system

In a sectored approach, on the other hand, the channels available in the cell are equally divided between the P beams. A mobile that transmits within that beam is assigned a channel (if available) from that beam only. If the mobile moves into the area covered by a different beam, a *handoff* must occur. A sectored approach, therefore, inherently suffers from greater numbers of handoffs.

Clearly a switched approach is more efficient. All channels are available to all beams. A particular beam would be able to cover a hot spot, an area with many users. A switched beamformer could serve up to N_k users (where N_k is the number of channels available to the base station). However, the trade off is that a switched beam approach is significantly more complex to implement - implementing such a system requires the ability to measure the SNR, determine the maximum and a RF switch that choose the appropriate beam. This process must be repeated for each user.

The sectored approach, on the other hand, is equivalent to P new sub-cells within the cell served by the base station. A sectored approach could only serve N_k/P users within any beam.

3 Figures of Merit

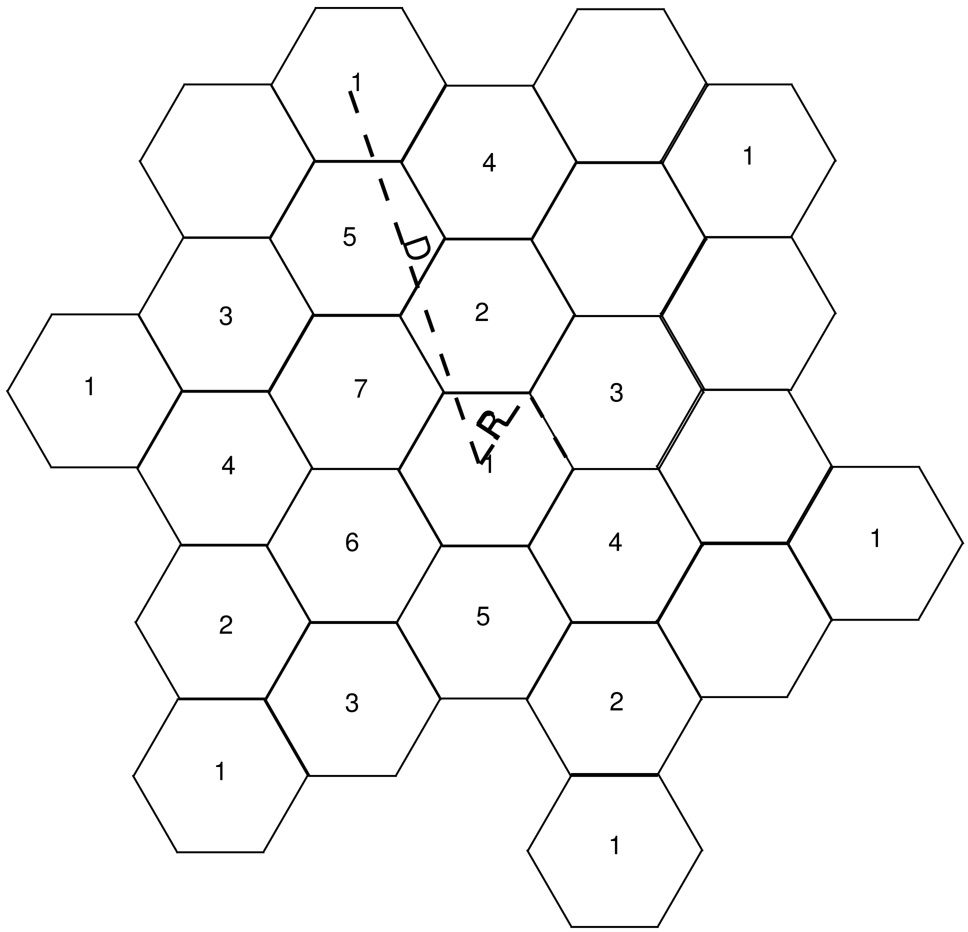

Figure 5: Channel distribution in a cellular system

To illustrate the improvements in performance due to beamforming, we use three figures of merit: spectral efficiency, number of users that may be serviced and probably of outage. The analysis will be performed in context of the cellular system shown in Fig. 5. The geographical area to be covered is divided into *clusters*, each of N_c *cells*. The available channels are divided amongst the cells such that no adjacent cells use the same set of channels. In the figure, 7 cells (marked 1 through 7) form the cluster. The central cell acts as the "desired cell". It is assumed that the situation is homogeneous, i.e., each cell is like every other cell.

If the total bandwidth available is B_T Hz and each channel occupies B_c Hz, the total channels available and channels per cell are simply

$$\text{Total Channels} = N_T = \frac{B_T}{B_c}, \tag{2}$$

$$\text{Channels Per Cell} = N_k = \frac{N_T}{N_c}. \tag{3}$$

3.1 Spectral Efficiency

The spectral efficiency of a system is given by

$$\eta_s = \frac{\#\text{ of channels per Hz}}{\text{Area covered}} = \frac{B_T/B_c}{B_T \times \text{Area}} = \frac{B_T/B_c}{B_T \times N_c A}, \tag{4}$$

where A is the area of each cell. Note that N_c, the cluster size is in the denominator, implying that we could improve spectral efficiency by reducing the cluster size (or the effective cluster size).

3.2 Trunking Efficiency...

... or equivalently, the number of paying users per cell. This is not the number of active users, but the total number of users that can be supported.

Before determining the number of users per cell in a switched or sectored beamforming system, we must define some terms from communication theory. The overall and per user *traffic intensity* are defined to be

$$E = \frac{(\text{avg. \# calls in time T}) \times (\text{avg. duration of call})}{T}, \tag{5}$$

$$E_u = \frac{(\text{avg. \# calls made by an individual user in time T}) \times (\text{avg. duration of call})}{T}, \tag{6}$$

respectively. Both are measured in terms of *Erlangs*. A traffic load of one Erlang would use one channel all the time. If a system can offer E Erlangs of traffic intensity, the number of users that can be supported is E/E_u. In a cell with N_k channels, with probability of blocked call set at 0.01, one could support K users, where

$$K = \frac{N_k}{E_u}\left[0.855\tanh(0.07N_k) - 1.41\times 10^{-3}N_k^2\exp(-0.07N_k)\right]. \tag{7}$$

Note that the number of users per cell N_k is determined by the cluster size as in Eqn. (3), i.e., it is inversely proportional to the cluster size. Beamforming allows us to reduce the effective cluster size and so increase the number of channels.

Consider the cellular system in Fig. 5. The central area, marked "1", is the primary cell under analysis. The other areas marked "1" use the same sets of frequencies - signals from these areas would interfere with each other. Each cell has radius R. Assuming all base stations transmit at the same power level, the worst case carrier power is

$$C \propto \frac{1}{R^n}, \tag{8}$$

where n is the path loss exponent. The interference power is given by

$$\begin{aligned} \text{Interference per interfering cell} &\propto \frac{1}{D^n}, \\ \Rightarrow \text{Total Interference} = I &\propto N_I \frac{1}{D^n}, \end{aligned} \tag{9}$$

$$\Rightarrow \text{Carrier to Interference Ratio} = C/I = \frac{1}{N_I}\left(\frac{D}{R}\right)^n, \tag{10}$$

where N_I is the number of interfering cells (co-channel cells) using the same set of frequencies as the cell under consideration ($N_I = 6$ in the example scenario of Fig. 5). In general, it is can be safely assumed that the interference arrives from only the first layer of interfering cells surrounding the cell under consideration.

For the geometry shown, $D = \sqrt{R^2 + (4R)^2 - 2(R)(4R)\cos(120^o)} = R\sqrt{21}$. In general,

$$D = R\sqrt{3N_c}, \tag{11}$$

$$\Rightarrow C/I = \frac{1}{N_I}(3N_c)^{n/2}. \tag{12}$$

Reversing Eqn. (12), if a certain (C/I) is required to maintain a good communication link, the minimum required cluster size is

$$N_{c_{\text{reqd}}} = \frac{1}{3}\left[N_I (C/I)_{\text{reqd}}\right]^{2/n}. \tag{13}$$

3.3 Outage Probability

The outage probability is the probability that an on-going call is dropped due the carrier power falling below a threshold (usually defined in terms of interference power). We shall consider a call to be dropped if the carrier power (C) is below a margin (q) over the interference power (I), i.e., $C/I < q$. Not that we would usually model the *envelope* of the signal (e.g. log-normal, Rayleigh, etc.) and not the power level.

Interference occurs due a user in a co-channel cell is using the same channel as the user in the cell under consideration. Since we can safely assume the signals from interfering cells are independent, if there are i co-channel cells interfering, then the interference power is given by

$$I = \sum_{k=1}^{i} I_k,$$

where I_k is the interference power of the k^{th} interfering cell.

The probability that a particular channel is in use in a given co-channel cell (p_c) is the traffic intensity divided by the total number of channels,

$$p_c = \frac{E_k}{N_k}.$$

In point of fact, this probability is reduced by a factor $(1 - p_b)$ where p_b is the probability that a call is blocked. The probability that i cells (of a total possible N_I co-channels) use the same channel is therefore

$$P_i = \left(\begin{array}{c} N_I \\ i \end{array} \right) p_c^i (1 - p_c)^{(N_I - i)}.$$

Outage occurs if

$$\begin{aligned} P_{\text{out}} &= P(C/I < q) = \sum_{i=1}^{N_I} P(C/I < q/i) P_i \\ &= \sum_{i=1}^{N_I} P(C/I < q/i) \left(\begin{array}{c} N_I \\ i \end{array} \right) p_c^i (1 - p_c)^{(N_I - i)}, \end{aligned}$$

where $P(C/I < q/i)$ is the probability of outage, $P(C/I < q)$, *given* there are i interfering co-channels. Note that $P(C/I < q/i)$ is highly dependent on the fading associated with the channel. In general, as we are looking at relatively large distances, the model used to determine $P(C/I < q/i)$ would assume large scale fading (such as log-normal fading).

4 Performance Improvements

Having determined our figures of merit, what performance gains does a sectored antenna or switched beamforming achieve? The simplest (and ideal) beamformer has a flat response over an angular section. Given an N element array, such a beam would cover $1/N$ of the total angular region, such as shown in Fig. 6. Assuming that the interference is distributed uniformly in angle, on average, such a beamformer would receive only $(1/N)^{th}$ of the interference and (C/I) would increase by a factor of N. More rigorously, the interference power is given by

$$\begin{aligned} I_{\text{omni}} &\propto \int_0^{2\pi} P_0 d\phi, \\ I_{\text{s.b.}} &\propto \int_0^{2\pi} |AF|^2 P_0 d\,\phi, \end{aligned}$$

where P_0 is the average angular interference power density. The subscripts "omni" and "s.b." refer to an omnidirectional antenna (single element/no directivity) and switched beamforming respectively. In terms of interference power, the gain in using a directive array is therefore

$$\frac{I_{\text{omni}}}{I_{\text{s.b.}}} = \frac{2\pi}{\int_0^{2\pi} |AF|^2 P_0\, d\phi} = D_{\text{max}} \tag{14}$$

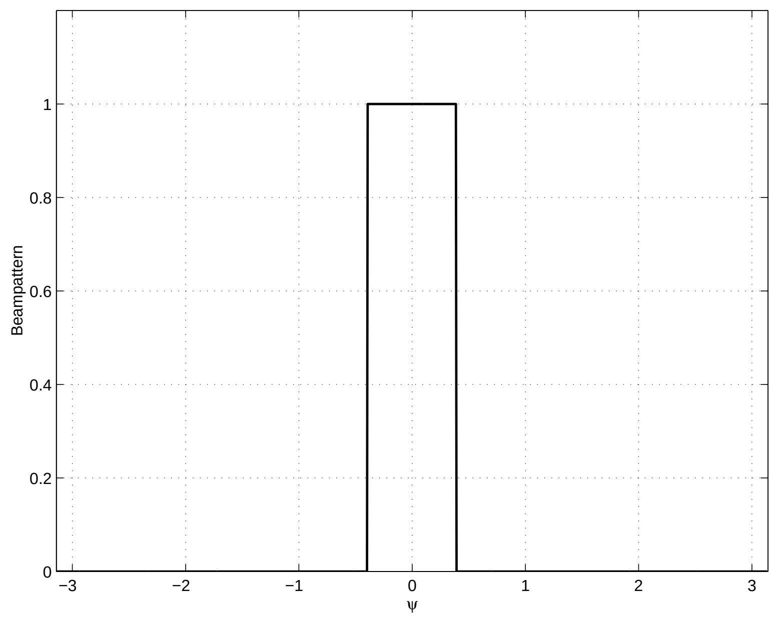

Figure 6: An ideal sectored beam

where $D_{\max}$ is the directivity of the array (more rigorously, the maximum directivity of the array). If the mainbeam of the array is pointed in direction of the user, this is also the gain in terms of (C/I). However, if there is a mismatch between mainbeam direction and the mobile location, this gain would be reduced by the *scalloping loss*. **Note:** This gain is the same for a sectored antenna system using the same beam pattern as our switched beam system.

Due to the gain in the carrier to interference ratio, using Eqn. (13), the required cluster size reduces (with attendant increase in number of channels per cell), i.e.

$$\begin{aligned} N_{c_{\text{reqd (s.b.)}}} &= \frac{1}{3}\left[\frac{N_I}{D_{\max}}(C/I)_{\text{reqd}}\right]^{2/n} \\ &= N_{c_{\text{reqd (omni)}}} D_{\max}^{(-2/n)} \end{aligned} \tag{15}$$

$$\Rightarrow N_{k_{\text{(s.b.)}}} = \frac{N_T}{N_{c_{\text{reqd (s.b.)}}}} = N_{k_{\text{(omni)}}} D_{\max}^{(2/n)}. \tag{16}$$

The gain in terms of cluster size and number of channels per cell is therefore $D_{\max}^{(2/n)}$. Unfortunately, n, the path loss exponent is usually greater than 2 and the gain is less than the maximum directivity.

4.1 Spectral Efficiency

Equation (4) shows that the spectral efficiency is inversely proportional to the cluster size, implying an increased spectral efficiency for switched beam systems. The spectral efficiency of a switched

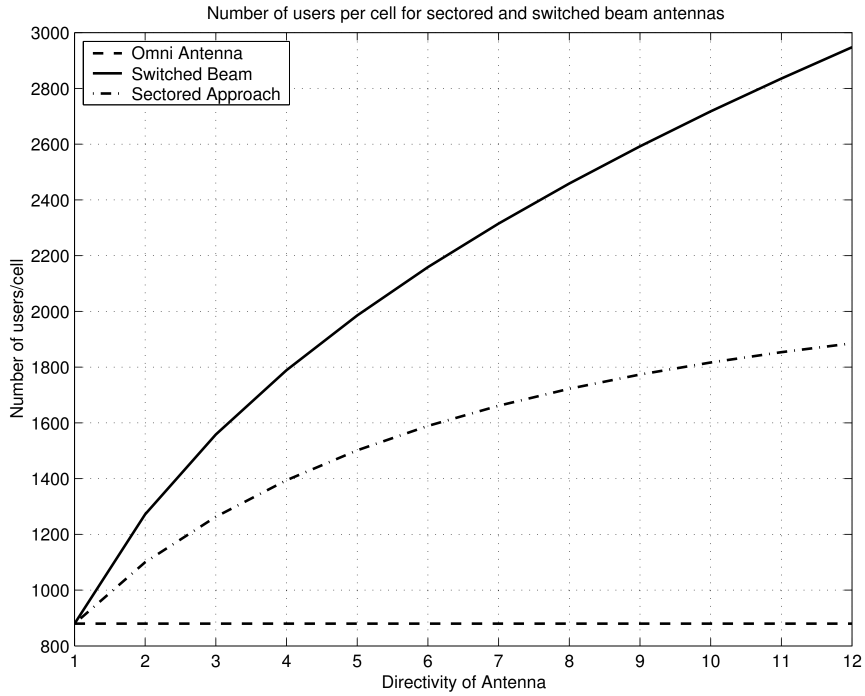

Figure 7: Number of users in sectored and switched beam systems

beam system is

$$\eta_{s_{(\text{s.b.})}} = \frac{\#\text{ of channels per Hz}}{\text{Area covered}} = \frac{B_T/B_c}{B_T \times \text{Area}} = \frac{B_T/B_c}{B_T \times N_{\text{creqd (s.b.)}} A} \tag{17}$$

$$= \eta_{s_{(\text{omni})}} D_{\max}^{(2/n)}. \tag{18}$$

Since the improvement in cluster size is the same for switched beam and sectored systems, so is the improvement in spectral efficiency.

4.2 Trunking Efficiency ...

... or the number of paying users per cell.

To determine the number of users per cell that a switched beamforming system can support, we use the number of channels from Eqn. (16) in Eqn. (7). Due to the increased number of available channels, the number of possible users increases substantially.

$$K_{\text{s.b.}} = \frac{N_{k_{(\text{s.b.})}}}{E_u}\left[0.855\tanh\left(0.07N_{k_{(\text{s.b.})}}\right) - 1.41\times 10^{-3}\left(N_{k_{(\text{s.b.})}}\right)^2 e^{-0.07N_{k_{(\text{s.b.})}}}\right]. \tag{19}$$

For a sectored system, this increase in number of channels is spread over the number of beams

(P). The number of channels per beam and total number of users that can be supported are,

$$N_{k_{(\text{sect})}} = \frac{N_{k_{(\text{s.b.})}}}{P}, \tag{20}$$

$$\Rightarrow K_{\text{sect}} = P\frac{N_{k_{(\text{s.b.})}}}{P \times E_u}\left[0.855\tanh\left(0.07\frac{N_{k_{(\text{s.b.})}}}{P}\right) - 1.41\times 10^{-3}\left(\frac{N_{k_{(\text{s.b.})}}}{P}\right)^2 e^{-0.07N_{k_{(\text{s.b.})}}/P}\right]. \tag{21}$$

It is the non-linear relationship between number of users supported and the number of channels available that gives switched beam systems the advantage over sectored approaches. Figure 7 plots the number of users per cell in three possible approaches - the omnidirectional antenna system of Eqn. (7), switched beam and sectored beam systems. As is clear, the sectored beam system results in a very large increase in the number of users that can be supported (over the omnidirectional antenna without any directivity).

Switched beamforming is a much more complicated (and costly) system - unlike in a sectored system, a SNR measurement, decision mechanism and RF switch is required for each user in service. However, as is shown in Fig. 7, switched beam systems can support many more users than a sectored beam system. This is the primary justification for the added cost of a switched beam system.

4.3 Outage Probability

In a sectored or switched beam system, the individual components of the interference envelope are weighted by the directivity, i.e.,

$$I = \sum_{k=1}^{i} I_k d_k,$$

where d_k is the directivity of the array in direction of the interference source. As the directivity is implementation dependent, we can only look at specific examples. Here we pick the easiest example - one in which the beam pattern is the ideal beam pattern of Fig. 6, i.e. $D_{\max} = N$. In this case, the probability that any co-channel cell is using the same channel as in the primary cell is weighted by the probability that that individual cell is in within the beam selected, i.e.

$$p_{c_{(\text{s.b.})}} = \frac{p_{c_{(\text{omni})}}}{N} = \frac{p_c}{N} \tag{22}$$

$$\Rightarrow P_{\text{out}} = \sum_{i=1}^{N_I} P(C/I < q/i)\begin{pmatrix} N_I \\ i \end{pmatrix} p^i_{c_{(\text{s.b.})}}(1 - p_{c_{(\text{s.b.})}})^{(N_I - i)}, \tag{23}$$

$$= \sum_{i=1}^{N_I} P(C/I < q/i)\begin{pmatrix} N_I \\ i \end{pmatrix}\left(\frac{p_c}{N}\right)^i (1 - \frac{p_c}{N})^{(N_I - i)}. \tag{24}$$

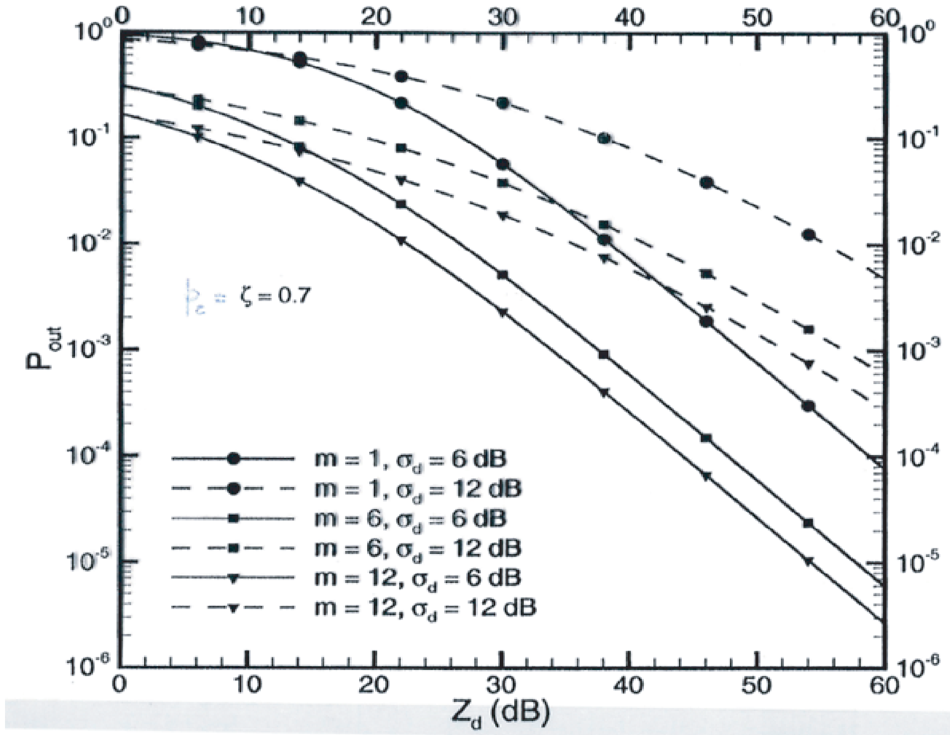

Figure 8: Outage probability in a switched beam system

Figure 8 plots the outage probability of a switched beam system in comparison to a omnidirectional antenna. The plot is Fig. 8.11 of Janaswamy [1]. The fading is modelled as log-normal with two different standard deviations. The number m is the number of elements (our N) while ζ is the probability a particular co-channel cell is interfering with the primary cell (our p_c). Z_d is the SNR. As is seen, switched beamforming results in a significantly lower probability of outage at a chosen SNR.

5 Summary

This section has analyzed the impact of non-adaptive processing on traditional measures of performance of a wireless communication system. All measures of performance improve and the improvement is dependent on the directivity of the antenna array. The array processing used here is fixed, i.e., is non-adaptive. The impact of using an array was analyzed for switched and sectored beam systems.

Switched beamforming antennas provide for a significant increase in spectral efficiency and large increases in number of users that can be supported. This increase is over and above increases due to directivity (as exploited by a sectored beam system). The primary motivation for switched beamforming is the large increase in possible number of users.

A note to remember: We have analyzed switched and sectored beamforming in extremely simple scenarios. The results presented in the figures here are dependent on system implementations and only provide a guideline to the motivations. In a real system, one would have to include the parameters, fading and the modulation seen by the individual system. For example, a CDMA system may have a different interference pattern than a FDMA system.

References

[1] R. Janaswamy, *Radiowave Propagation and Smart Antennas for Wireless Communications.* Kluwer Academic Publishers, 2000.

[2] F. Harris, "On the use of windows for harmonic analysis with the Discrete Fourier Transform," *Proceedings of the IEEE*, vol. 66, No. 1, pp. 51–83, Jan 1978.