Minimum Norm Mutual Coupling Compensation With Applications in Direction of Arrival Estimation

C. K. Edwin Lau, Raviraj S. Adve, *Senior Member, IEEE*, and Tapan K. Sarkar, *Fellow, IEEE*

***Abstract*—This paper introduces a new mutual coupling compensation method based on the minimum norm solution to an underdetermined system of equations. The crucial advantage over previous techniques is that the formulation is valid independent of the type of antenna element and provides good results in situations where signal strengths vary considerably. In using the matrix pencil algorithm to estimate the directions of arrival, the examples show that the proposed method results in significantly lower bias than the traditional open circuit method. The analysis of mutual coupling is also applied in the context of a Code Division Multiple Access communication system.**

***Index Terms*—Code division multiaccess, direction of arrival estimation, matrix pencil, MUSIC, mutual coupling compensation.**

I. Introduction

Direction of arrival (DOA) estimation is an important feature of smart antenna arrays. It could serve as a fundamental building block for applications such as space division multiple access (SDMA) and Enhanced 911 (E911), the proposed wireless emergency service [1]. Several algorithms have been proposed for DOA estimation, including the popular MUSIC-type techniques, ESPRIT [1] and matrix pencil (MP) [2]–[4]. These signal processing algorithms have been shown to provide accurate estimates, even in moderate signal to noise (SNR) conditions.

The problem is that these signal processing algorithms generally ignore the electromagnetic behavior of the receiving antenna. The receiver is assumed to be an ideal, equispaced, linear array of isotropic point sensors. In this case, the array samples, but does not reradiate the incident signals. Each signal can be associated with a linear phase front, the slope of which is directly related to the DOA. Most signal processing techniques rely heavily on this assumption. In practice, this ideal situation cannot be met. The elements of the array must be of some nonzero size. The elements sample and reradiate the incident fields, causing mutual coupling. Mutual coupling distorts the linear phase front of the incoming signal, significantly degrading performance [5]–[7]. Only in the case of a single incoming signal is the phase front somewhat retained. However, for arrays with strong mutual coupling, the phase front is significantly corrupted and the DOA estimate is inaccurate. Any practical implementation of DOA estimation therefore requires compensation for mutual coupling.

Research into compensating for the mutual coupling has been based mainly on the idea of using open circuit voltages, first proposed by Gupta and Ksienski [5]. The authors argue that due to the lack of a terminal current, the open circuit voltages are free of mutual coupling. However, as shown in [7], this only reduces the effects of mutual coupling. The technique presented there is more effective in suppressing mutual coupling effects [7], [8].

A big drawback with the approaches of [5] and [7], is that they are valid for only linear dipoles. The work of [5] is valid only for a linear array of half wavelength dipoles spaced apart by half a wavelength. The work of [7] is restricted to linear arrays of linear dipoles, though of arbitrary length and spacing. In this paper we introduce the use of a *minimum norm* technique, based on the technique in [7], for general arrays with arbitrary elements. As an aside we also extend the open circuit technique of [5] to arbitrary arrays. The method of moments (MoM) is used to accurately model the interactions between antenna elements. In the minimum norm approach, the MoM admittance matrix is used to estimate the incident fields, with minimum energy, that would generate the received voltages. Unlike in [9], this technique does not require the solution to the entire MoM problem. The compensation matrix depends only on the MoM admittance matrix and can be calculated *a priori* to reduce computation load.

In this paper, we use the MP [2] and the popular MUSIC [1], [10] DOA estimation algorithms to compare various compensation methods. Section II presents the model for mutual coupling using antenna analysis based on the MoM. This eventually leads to the formulation of minimum norm mutual coupling compensation method. Section III presents examples illustrating the performance of the open circuit and the minimum norm methods in case of a equispaced, linear array of dipoles. Section IV ends with some conclusions and a summary of the contributions presented here.

II. Mutual Coupling and Compensation

Most DOA estimation algorithms including MP and MUSIC assume an ideal, linear array of isotropic sensors. Unfortunately, such an ideal sensor is clearly not realizable. A practical antenna array comprises elements of some physical size. Such elements

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sample and reradiate incident fields that interact with other elements, i.e., the elements are mutually coupled. Mutual coupling severely degrades the accuracy of the DOA estimator [6]. Any implementation of DOA estimation must account for the mutual coupling between elements.

In a practical antenna array, the received signals are the voltages measured across the load at the port of each element. To deal with mutual coupling, researchers originally proposed processing these measured voltages to obtain the open circuit voltages, the voltages if all the ports were open circuited [5], [6]. Open circuiting the ports reduces the currents on the elements, consequently the reradiated fields and therefore the mutual coupling. However, as shown in [7], this methodology is valid only when all signals have similar strengths. In [7], we use a MoM analysis to compensate for mutual coupling. That technique is very effective, but is valid only for a linear array of parallel dipoles.

We present here a technique that is theoretically valid for all kinds of arrays. Based on a minimum norm solution to an undetermined system of MoM equations, the technique makes no assumptions regarding the type of antenna, or the spacing between elements. However, for simplicity, this methodology is presented here for a linear array of dipoles. We begin with a brief review of the analysis technique, as the MoM analysis for dipole arrays is well known [7], [11]. The review included here sets the stage for the minimum norm solution.

A. System Model

We begin with the general formulation of the MoM based on subdomain basis functions for a receiving antenna array of N-elements. The central assumption is that only a single basis function contributes to the current at the port of each element in the array. The incident electric field is related to the currents on the antenna through a linear operator L [12]

$$\mathbf{E}^{\mathrm{inc}} = \mathrm{L}(\mathbf{J}). \tag{1}$$

The current $\mathbf{J}$ is approximated by a set of subdomain basis functions, $\mathbf{g}_n$, with P basis functions per element, i.e.

$$\mathbf{J} = \sum_{n=1}^{NP} I_n \mathbf{g}_n \tag{2}$$

where I_n is the nth current coefficient. Using a set of NP testing functions, $\mathbf{w}_m$, and a convenient definition of inner product, (1) can be reduced to a matrix equation

$$\mathbf{V}_{\mathrm{MoM}} = \mathbf{ZI} = [\mathbf{Z}_{\mathrm{MoM}} + \mathbf{Z}_{\mathrm{load}}]\,\mathbf{I} \tag{3}$$
$$\Rightarrow \mathbf{I} = \mathbf{Z}^{-1}\mathbf{V}_{\mathrm{MoM}} = \mathbf{Y}\mathbf{V}_{\mathrm{MoM}} \tag{4}$$

where the mth element of $\mathbf{V}_{\mathrm{MoM}}$ and the (m,n)th element of the $NP \times NP$ matrix $\mathbf{Z}_{\mathrm{MoM}}$ are

$$\mathbf{V}_{\mathrm{MoM},m} = \langle \mathbf{w}_m, E^{\mathrm{inc}} \rangle \tag{5}$$
$$\mathbf{Z}_{\mathrm{MoM},m,n} = \langle \mathbf{w}_m, \mathrm{L}(\mathbf{g}_n) \rangle. \tag{6}$$

The matrix $\mathbf{Z}_{\mathrm{load}}$ is the $NP \times NP$ matrix with zero entries other than the N diagonal entries, where the (n,n)th entry corresponds to the port. This (n,n)th entry is the load impedance of the corresponding element. The matrix $\mathbf{Z}_{\mathrm{MoM}}$ is the MoM impedance matrix. Assuming a single basis function corresponds to the current at a port of the array, from (4), the measured voltages, affected by mutual coupling are given by

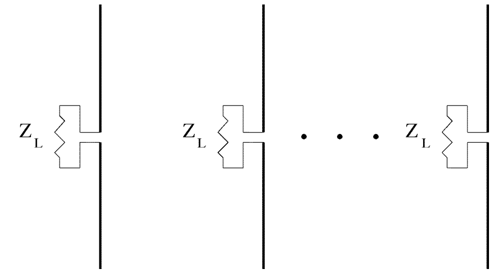

Fig. 1. Linear array of wire dipoles terminated in loads Z_L.

$$\mathbf{V}_{\mathrm{mea}} = \mathbf{Z}_L \mathbf{I}_{\mathrm{port}} = \mathbf{Z}_L \mathbf{Y}_{port} \mathbf{V}_{\mathrm{MoM}} = \mathbf{C}\mathbf{V}_{\mathrm{MoM}}. \tag{7}$$

The matrix $\mathbf{Y}_{\mathrm{port}}$ is the $N \times NP$ submatrix of $\mathbf{Y}_{\mathrm{MoM}}$ corresponding to the N ports. $\mathbf{Z}_L$, a compressed version of $\mathbf{Z}_{\mathrm{load}}$, is the $N \times N$ diagonal matrix of port impedances. $\mathbf{C}$ is a $N \times NP$ matrix of dimensionless entries. Note that the entries of $\mathbf{V}_{\mathrm{MoM}}$ are directly related to the incident fields and are free of mutual coupling.

In this paper, this general formulation is applied to a linear array of dipoles. It must be emphasized that this choice is not fundamental to the theoretical development here and is made only for purposes of illustration. Consider a wire dipole antenna array of N z-directed elements as shown in Fig. 1. Each element has a centrally located port terminated in impedance Z_L. To analyze this array we use sinusoidal basis functions. Each element is divided into $P+1$ segments of equal length. To satisfy the requirement that only a single basis function corresponds to the current on the array P is chosen to be odd. Based on a Galerkin formulation, the weighting and testing functions are the same. The entries for the MoM voltage and impedance matrices are available in [7], [11].

B. Open Circuit Voltages

The principal idea of [5] is to use the open circuit voltages instead of the measured voltages for further signal processing. However, the theory is valid only for half wavelength dipoles with half wavelength spacing. In the more general case, one can use the MoM analysis in conjunction with the Thevenin and Norton equivalent circuits to obtain the open circuit voltages. Define the MoM admittance matrix $\mathbf{Y}_{\mathrm{MoM}}$ to be the inverse of the impedance matrix $\mathbf{Z}_{\mathrm{MoM}}$. Note that this is not the same matrix, $\mathbf{Y}$, in (4). Also define a new $N \times N$ matrix $\mathbf{Y}_{pp}$ whose entries are those rows and columns of the MoM admittance matrix $\mathbf{Y}_{\mathrm{MoM}}$ that correspond to ports, i.e.

$$\mathbf{Y}_{pp}(p,q) = \mathbf{Y}_{\mathrm{MoM}}(i,l), \qquad 1 \le p,q \le N \tag{8}$$
$$i = (p-1)\,P + \frac{(P+1)}{2} \tag{9}$$
$$l = (q-1)\,P + \frac{(P+1)}{2}. \tag{10}$$

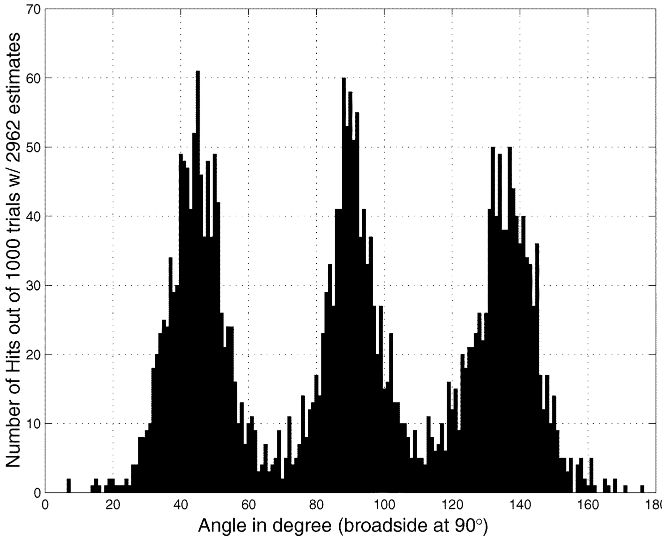

Fig. 2. Example 3.1. MP, using uncompensated voltages.

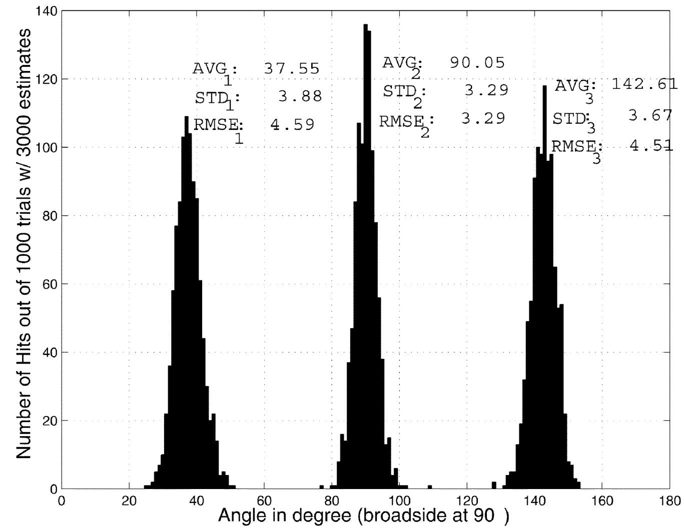

Fig. 3. Example 3.1. MP, using open circuit voltages.

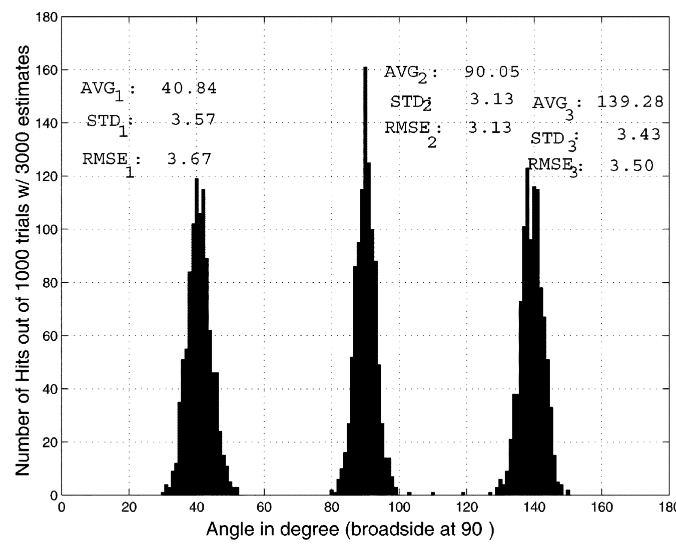

Fig. 4. Example 3.1. MP, using minimum norm compensation.

The open circuit voltages are then related to the short circuit currents as

$$\mathbf{V}_{\rm oc} = \mathbf{Y}_{pp}^{-1}\mathbf{I}_{sc} \tag{11}$$

and the measured voltages to the short circuit currents as

$$\mathbf{V}_{\rm mea} = \left[\mathbf{Y}_{pp} + \mathbf{Z}_L^{-1}\right]^{-1}\mathbf{I}_{sc}. \tag{12}$$

Eliminating the short circuit currents from (11) and (12) yields the open circuit voltages

$$\mathbf{V}_{\rm oc} = \mathbf{Y}_{pp}^{-1}\left[\mathbf{Y}_{pp} + \mathbf{Z}_L^{-1}\right]\mathbf{V}_{\rm mea}. \tag{13}$$

In the following sections the open circuit voltages we refer to are obtained from the measured voltages using (13).

C. Minimum Norm Compensation Formulation

As shown in [7], using the open circuit voltages only somewhat reduces the effects of mutual coupling. In [7], we reconstruct a part of the MoM voltage vector under the assumption of a linear dipole array. The motivation comes from the fact that, from (5), the MoM voltages are directly related to the incident fields and so are free of mutual coupling.

In the general case, from (7), the equation relating the measured and MoM voltages is underdetermined and the $\mathbf{V}_{\rm MoM}$ cannot be reconstructed exactly. However, one can find the *minimum norm solution* to this equation. This solution provides the vector with the minimum two-norm (minimum energy) that would result in the received voltages. The resulting vector is an estimate of the MoM voltage vector. Using (7), the minimum norm solution to the MoM voltage vector is

$$\tilde{\mathbf{V}}_{\rm MoM} = \mathbf{C}^H\left[\mathbf{C}\mathbf{C}^H\right]^{-1}\mathbf{V}_{\rm mea} \tag{14}$$

where $\mathbf{C}^H$ is the conjugate transpose (Hermitian) of matrix $\mathbf{C}$. Entries in $\tilde{\mathbf{V}}_{\rm MoM}$ corresponding to the ports may be used for further signal processing.

Physically, the compensation procedure may be interpreted as finding the signal with minimum energy that results in the measured voltages. Since the MoM analysis and so the matrix $\mathbf{C}$ may be obtained *a priori*, the computation load to use (14) is no greater than finding the open circuit voltages or using the technique of [7]. In the following section, we compare the performance of the two compensation methods in various settings.

III. Numerical Examples

In this section, we present numerical examples to illustrate the workings of the two compensation techniques, the open circuit and minimum norm methods. The application here is DOA estimation. The first two examples deal with the DOA estimation of multiple signals and demonstrates the impact of mutual coupling and compares the two compensation techniques. The third example deals with the impact of mutual coupling on code division multiple access (CDMA) communications in particular, and the effectiveness of mutual coupling compensation in this

TABLE I
COMPARING OPEN CIRCUIT AND MINIMUM NORM TECHNIQUES. EQUAL SIGNAL STRENGTHS

	Open Circuit				Minimum Norm			
	Mean	Bias	Std. Dev.	RMSE	Mean	Bias	Std. Dev.	RMSE
Signal 1	37.55°	2.45°	3.88°	4.59°	40.84°	0.84°	3.57°	3.67°
Signal 2	90.05°	0.05°	3.29°	3.29°	90.05°	0.05°	3.13°	3.13°
Signal 3	142.61°	2.61°	3.67°	4.51°	139.28°	0.72°	3.43°	3.50°

case. Due to mutual coupling, the signal level at each element may be different. The SNR is defined here as the average SNR at all ports of the array, i.e., in adding white, complex Gaussian noise at each element, the power level is chosen to set an average SNR. In all examples using MP, the pencil parameter is set to $(N+1)/2$.

A. Three Signals of Equal Strength

This example uses a seven element array with interelement spacing of 0.3λ. The MoM analysis uses 7 unknowns per element, i.e., a total of 49 unknowns are used. The array receives three signals from 40°, 90° and 140°. Each signal has a SNR of 1 dB. The MP algorithm uses only a single snapshot. The plots shown here use the results of 1000 independent trials.

Fig. 2 shows a histogram of the results of using MP without any compensation for mutual coupling. 38 times, the estimation procedure fails completely by resulting in imaginary angles. This happens because MP estimates the complex phase $z = e^{jkd\cos\phi}$ before estimating the direction ϕ. In 38 instances, the argument to the $\cos^{-1}()$ function becomes greater than 1. As is clearly seen in the figure, the DOA estimation is very poor with very large errors.

Figs. 3 and 4 plots the performance after compensation for mutual coupling. Fig. 3 plots the use of open circuit voltages while Fig. 4 plots the results of using the minimum norm technique. In both figures, the hugely improved performance over the uncompensated case is very clear. Neither technique results in any imaginary angles. Note that because of the accurate performance, we can estimate a standard deviation, which for all cases is approximately 3.5°.

As Table I shows, the crucial difference between the two compensation techniques is in the bias. The bias resulting from using the minimum norm compensation approach is significantly smaller than using the open circuit voltages. This is because using the open circuit voltages only implies the lack of a terminal current. Physically, there is still a nonzero current on the dipole arms. These currents reradiate, resulting in some residual mutual coupling.

Fig. 5 explains the improved performance of the minimum norm technique over the open circuit approach. The figure plots of the phase front of the three incoming signals in the various scenarios of this example. It plots the phase at each element in the ideal case, in the case of no mutual coupling compensation, using the open circuit approach and using the minimum norm solution. Without compensation, the phase information is significantly corrupted, explaining the erroneous results. Both compensation techniques correct this somewhat. However, clearly the minimum norm solution is better than

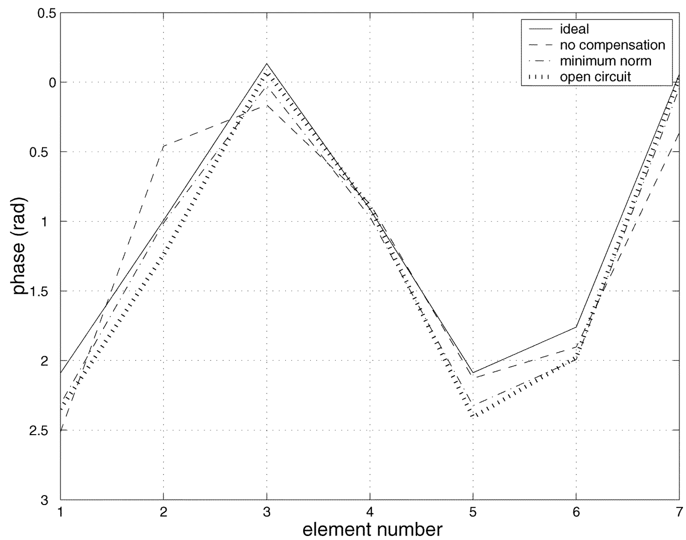

Fig. 5. Example 3.1. Phase front of three incoming signals.

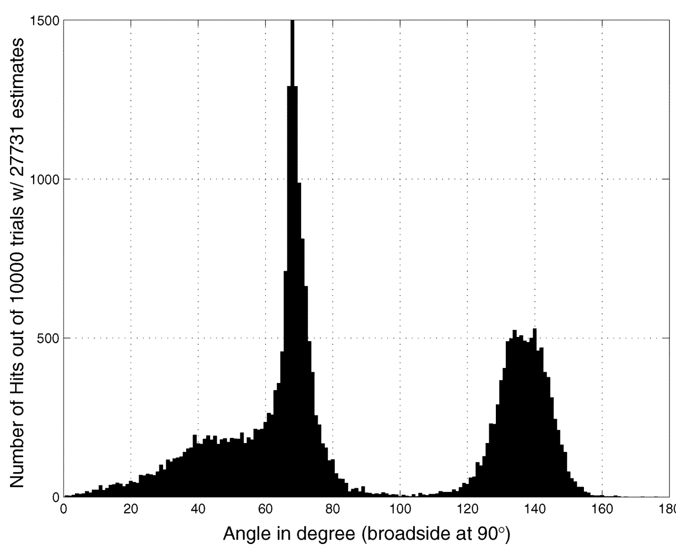

Fig. 6. Example 3.2. MP, using uncompensated voltages.

using the open circuit voltages. This explains, from the phase point of view, why the two compensation methods work and why the proposed approach is better than the traditional open circuit approach.

B. Three Signals of Unequal Strength

In this example we use the same array as in the first example with the the signal bearings at 40°, 70°, and 140°, with SNR's of 7, 15, and 5 dB respectively. We use 10 000 independent trials.

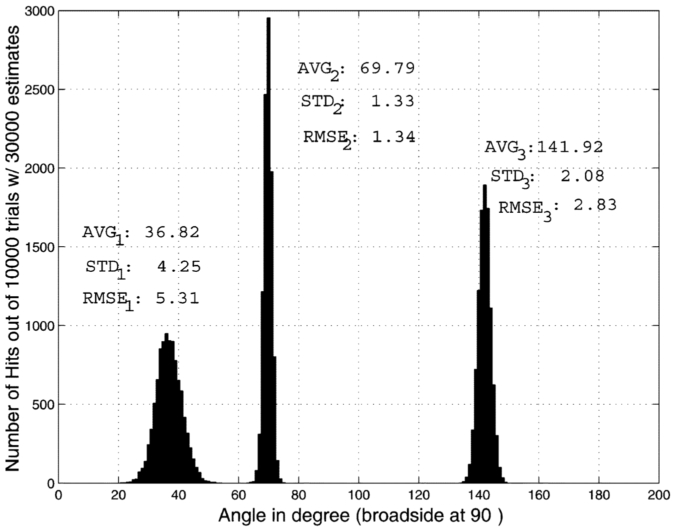

Fig. 7. Example 3.2. MP, using open circuit voltages.

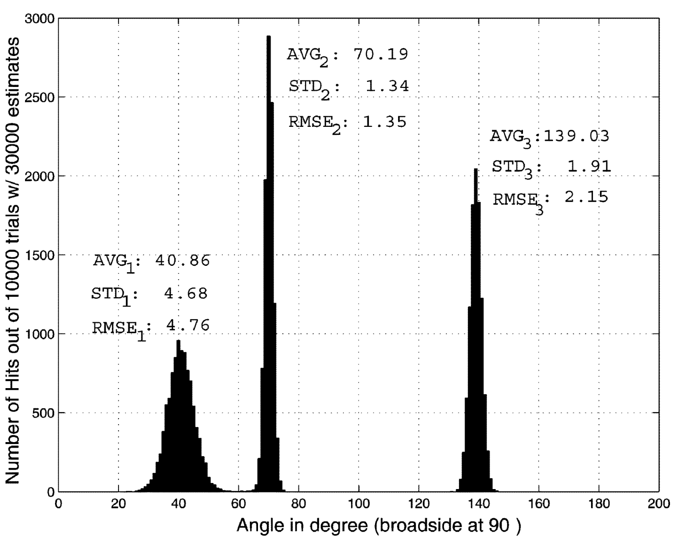

Fig. 8. Example 3.2. MP, using minimum norm compensation.

Fig. 6 shows a histogram of MP estimate without any mutual coupling compensation. 2269 estimates result in imaginary angles. Clearly the remaining estimates are not of any practical use.

Figs. 7 and 8 are results of using MP compensated with the open circuit and minimum norm approaches respectively. Both compensation methods improve the estimation dramatically. All the imaginary angles are recovered. Similar to the previous example, the open circuit method exhibits a larger bias than the minimum norm approach. The bias is even stronger in this example than the last one as the signal at 70° is relatively strong and closer to the 40° signal.

Fig. 9 shows the pseudo-spectrum generated by MUSIC without compensation, using the open circuit voltages and with minimum norm compensation. In all cases, 15 time samples are used to estimate the covariance matrix. As can be seen, with either compensation technique, the resolution improves and the bias is reduced. Again, the bias in the estimation is less with minimum norm method than that with open circuit method. This is in agreement with the examples presented

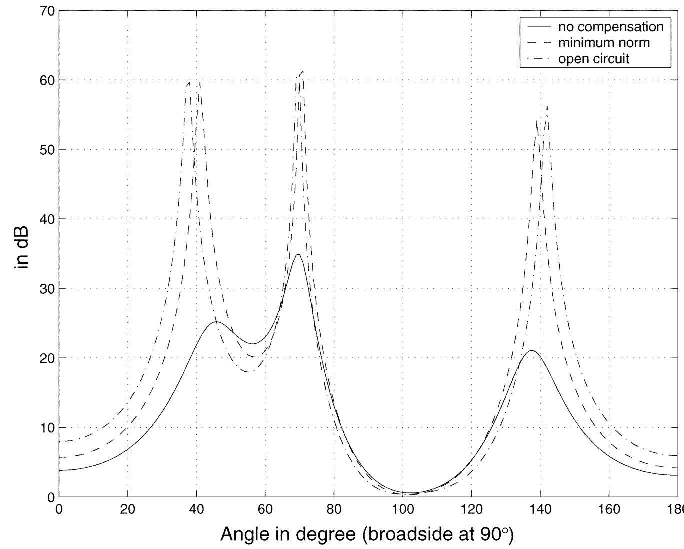

Fig. 9. Example 3.2. MUSIC.

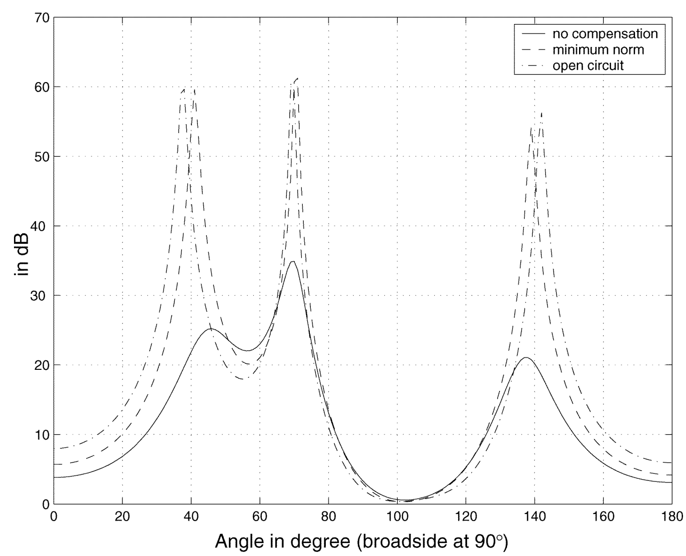

Fig. 10. Example 3.2. MUSIC. The middle signal is at 60°.

for the MP algorithm. If the signal at 70° is moved to 60°, as shown in Fig. 10, the results are even more dramatic. If no compensation is used, the signals at 40° and 60° merge. But after the compensation, the two spikes are recovered. Again, using the open circuit voltages results in a greater bias than the minimum norm method.

Figs. 11 and 12 show the results if the strength of the signal at 70° is increased to 25 dB. The results are shown in . We can see from the figures that the bias in not significant when using the minimum norm method in Fig. 12. When using the open circuit method, the bias in the weaker signals is 2° and 3.7°. Table II summarizes our statistical findings of this example.

C. Mutual Coupling Compensation in CDMA Communications

One motivation for this research is position location in wireless communication systems. Here we focus on a CDMA system. In applying the MP technique to a practical array in a CDMA based communication setting, a curious fact emerges.

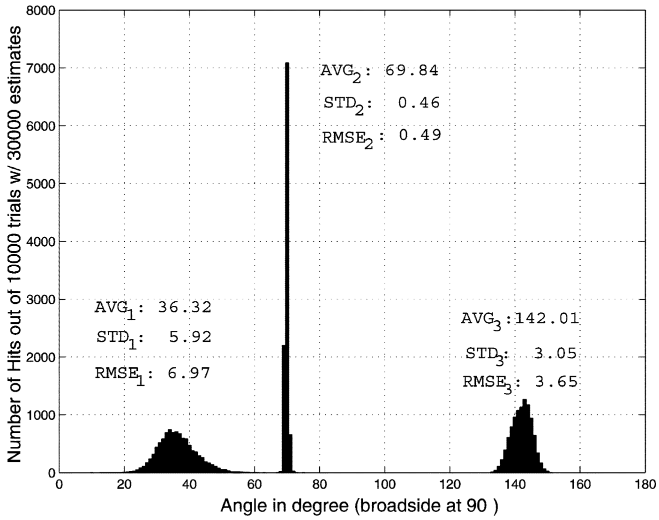

Fig. 11. Example 3.2. MP, using open circuit voltages. Signal at 70° is 25 dB.

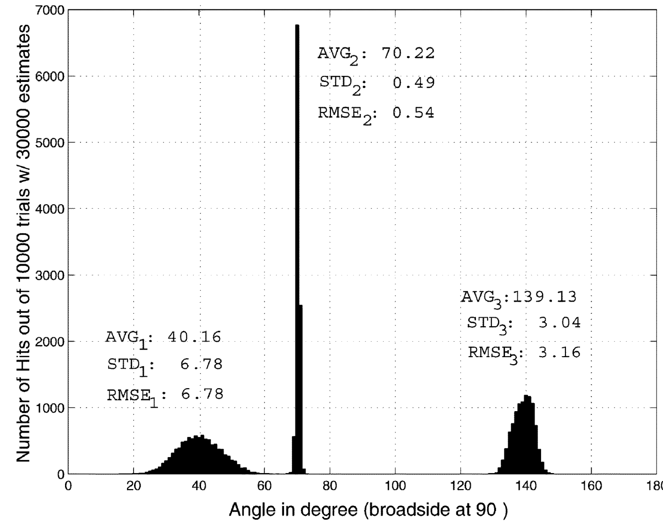

Fig. 12. Example 3.2. MP, using minimum norm compensation. Signal at 70° is 25 dB.

The CDMA processing gain provides some resistance to mutual coupling. This is because, after the matched filter, there is effectively only one signal plus relatively weak residual interference. With only one signal impinging on the array, the linear phase front is not fatally corrupted and it is possible to estimate the DOA. This is true particularly of arrays with moderate mutual coupling.

To illustrate this effect, we use the same example as in Section III-A. However, each signal is spread with a spreading gain of 128. We use four signal samples per chip. For a fair comparison, the power of each signal is reduced by the spreading gain. Using the filter matched to the first signal, two of three signals are suppressed. Note that in using MP to estimate the DOA of this signal after the matched filter, we set the number of signals to one, i.e., $M = 1$. This also eliminates a drawback associated with MP, the restriction on the number of signals that can be estimated simultaneously [13]. MP is applied *without compensating for mutual coupling*.

Fig. 13 plots the histogram of the resulting estimates. In comparison to Fig. 2 the accuracy is dramatically improved. No estimate results in imaginary angles. In fact, the accuracy is comparable to using the open circuit voltages as in Fig. 3.

It must be emphasized that this resistance to mutual coupling is only an approximation. Depending on the accuracy required, compensation for mutual coupling can still play an important role. Fig. 14 plots the results of using the minimum norm approach. The performance is improved with significantly reduced bias.

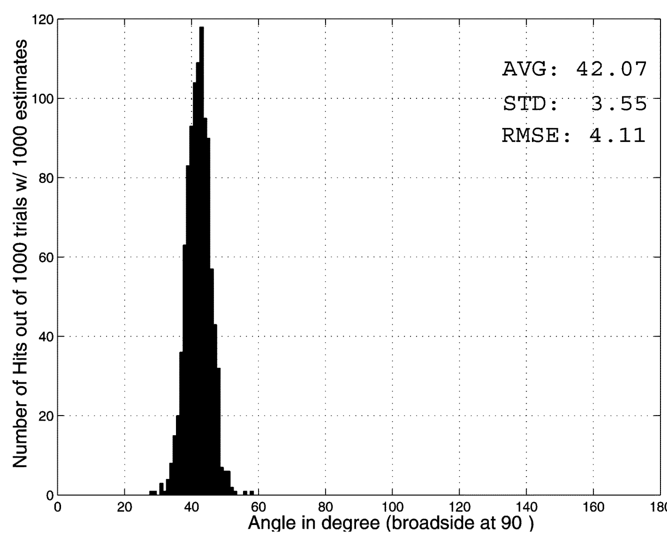

Fig. 13. Example 3.3. CDMA/MP, using uncompensated voltages.

IV. Conclusion

Practical implementations of DOA estimation must deal with the problem of mutual coupling between antenna elements. The work of [7] introduced the concept of reconstructing a part of the MoM voltage vector. We extend this concept here and develop a very effective technique based on the minimum norm solution to an underdetermined system of equations. The approach is to find the signals, with minimum energy, that would create the mutually coupled measured signals. The overhead associated with the compensation procedure is limited to a matrix multiplication.

In testing the proposed approach, the technique proves to be more accurate than the classical open circuit approach. The minimum norm technique reduces the bias in the estimates because the phase response is reconstructed more accurately than when using open circuit voltages.

In applying DOA estimation specifically to CDMA communications a curious fact emerges. If DOA estimation is applied after the matched filter, the CDMA spreading gain results in the desired signal plus residual interference. The phase front of a single signal is not significantly corrupted and so the resulting DOA estimation, *without compensation* is fairly accurate. However, one cannot conclude that mutual coupling compensation is not required. Applying compensation further improves performance. Since the additional cost is restricted to a matrix multiplication, the resulting performance gains would probably outweigh the cost of implementing mutual coupling compensation.

TABLE II
COMPARING OPEN CIRCUIT AND MINIMUM NORM TECHNIQUES. UNEQUAL SIGNAL STRENGTHS.

SNR	Open Circuit				Minimum Norm			
	Mean	Bias	Std. Dev.	RMSE	Mean	Bias	Std. Dev.	RMSE
1. 7dB	36.82°	3.18°	4.25°	5.31°	40.86°	0.86°	4.68°	4.76°
2. 15dB	69.79°	0.21°	1.33°	1.34°	70.19°	0.19°	1.34°	1.35°
3. 5dB	141.92°	1.92°	2.08°	2.83°	139.03°	0.97°	1.91°	2.15°
	Mean	Bias	Std. Dev.	RMSE	Mean	Bias	Std. Dev.	RMSE
1. 7dB	36.32°	3.68°	5.92°	6.97°	40.16°	0.16°	6.78°	6.78°
2. 25dB	69.84°	0.16°	0.46°	0.49°	70.22°	0.22°	0.49°	0.54°
3. 5dB	142.01°	2.01°	3.05°	3.65°	139.13°	0.87°	3.04°	3.16°

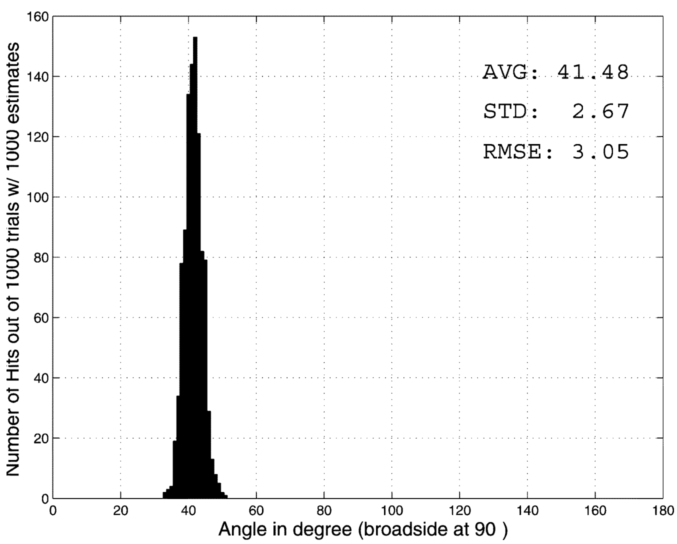

Fig. 14. Example 3.3. CDMA/MP, using minimum norm compensation.

In summary, we have presented a practical and accurate minimum norm mutual coupling compensation method. The new approach proves to more accurate than the traditional open circuit approach. This method can theoretically also be applied to arrays of arbitrary elements.

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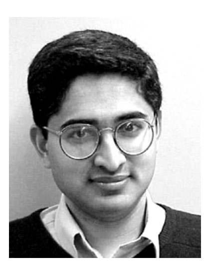

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