Multiple Target Localization Using Compressive Sensing

Chen Feng[1,2], Shahrokh Valaee[1], Zhenhui Tan[2]
[1] Department of Electrical and Computer Engineering, University of Toronto
[2] State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University
Email: {chenfeng, valaee}@comm.utoronto.ca, zhhtan@center.njtu.edu.cn

***Abstract*— In this paper, a novel multiple target localization approach is proposed by exploiting the compressive sensing theory, which indicates that sparse or compressible signals can be recovered from far fewer samples than that needed by the Nyquist sampling theorem. We formulate the multiple target locations as a sparse matrix in the discrete spatial domain. The proposed algorithm uses the received signal strengths (RSSs) to find the location of targets. Instead of recording all RSSs over the spatial grid to construct a radio map from targets, far fewer numbers of RSS measurements are collected, and a data pre-processing procedure is introduced. Then, the target locations can be recovered from these noisy measurements, only through an ℓ_1-minimization program. The proposed approach reduces the number of measurements in a logarithmic sense, while achieves a high level of localization accuracy. Analytical studies and simulations are provided to show the performance of the proposed approach on localization accuracy.**

I. Introduction

Accurate localization of multiple targets is one of the fundamental and challenging problems in signal processing [1]. In various applications, including indoor location-based services for mobile users, equipment monitoring in wireless sensor networks, and radio frequency identification (RFID)-based tracking, accurate and timely location information plays a prime role. Current literature shows a growing interest in using localization techniques based on the received signal strength (RSS), so that they can be applied to almost any radio device. A traditional wisdom in designing RSS-based localization techniques treats the location finding as a distance estimation problem based on RSS measurements directly. However, due to the complexity of the radio channel, it always fails to provide satisfactory accuracy in most applications. Thus, one of the key challenges arises: *how to estimate target locations accurately while conducting only a small number of RSS measurements*.

This work was supported by the State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, under project No.863 (2007AA01Z277).

In this paper, we use *compressive sensing* (CS) for target localization. Compressive sensing provides a novel framework for recovering signals that are sparse or compressible under a certain basis, with far fewer noisy measurements than the traditional methods [2]–[5]. It then uses an ℓ_1-minimization program to acquire the sparse signal or its unique sparse representation, which can be effectively solved in polynomial time. In the localization problem, since the location of a target is unique in the discrete spatial domain at a certain time, it can be modeled as an ideal 1-sparse vector. Hence, this paper focuses on investigating an accurate localization approach for multiple targets from only a small number of noisy RSS measurements using the CS theory.

Other approaches to target localization have also been proposed in the literature. In [6]–[8], the localization problem was formulated from pair-wise measurements as a dimensionality reduction problem on a Riemann manifold. This method improved the accuracy in dense wireless sensor networks. However, when the networks are sparse, the accuracy decreases sharply. Meanwhile, it assumed continuous communication between each sensor node and the centre node to transmit the pair-wise measurement data, which implied an extremely high communication cost in terms of bandwidth and energy consumption.

From the viewpoint of device complexity and cost, indoor localization via RSS fingerprinting of Wireless LAN (WLAN) infrastructure was proposed in [9]. The fundamental insight was that a radio map with the RSS fingerprinting over spatial discrete grid points was generated during an off-line phase, and the location of a target was estimated by comparing its online measurements with the radio map. Some improved models were proposed based on the fingerprinting method [10]. However, an accurate localization scheme requires a large grid size, while on each grid point an RSS measurement is needed. Since this method is highly dependent on the environment, any significant change to the topology implies a costly new re-calibration.

In [11][12], it has been realized that the localization problem can be formulated as a distributed sparse approximation problem, by which inter-sensor communication costs can be reduced significantly. However, a localization dictionary has to be locally estimated at each sensor node, which induces estimation error. Meanwhile, communication requirements are demanded among sensor nodes, which also leads to poor result in sparse networks.

In this paper, we propose a novel multiple target localization approach by using the CS theory. The target locations are formulated as a sparse matrix in the discrete spatial domain. By measuring only a small number of signal strengths from targets, the target locations can be fully recovered through an ℓ_1-minimization program. To apply the CS theory, appropriate data processing is needed. A pre-processing procedure on the original measured data is introduced to induce incoherence needed in the CS theory; and a post-processing procedure to compensate for the spatial discretization caused by grid assumption. In this paper, we use Basis Pursuit (BP) [13], Basis Pursuit Denoising (BPDN) [14], and Dantzig Selector (DS) [15] for ℓ_1-minimization programs, and compare their performance for location estimation.

The proposed localization method can be applied in a number of applications. It is frequently desirable to detect the location of wireless nodes inside a building. Fig. 1 illustrates an example with K target nodes. These nodes can be wireless access points or RFID tags transmitting radio frequency signals. The strength of the signal transmitted by each node is measured at M points, possibly by a receiver traveling in the coverage area and registering the RSS received from each transmitter at the sampling points. If the transmitters are RFID tags, then the receiver will be an RFID reader. If the wireless nodes are access points, the receivers are laptops, PDAs, or smart phones with WiFi capability. The sample points may also be multiple fixed readers measuring the RSS of targeted tags.

The remainder of this paper is organized as follows. In Section II, we describe the multiple target localization approach, presenting and proving the idea of locating targets using the CS theory via spatial sparsity. The performance of the proposed approach is studied through simulations in Section III. Finally, Section IV concludes the paper.

II. Target Localization Using Compressive Sensing

Consider a case, where K targets are located in an isotropic area, which is divided into a discrete grid with N points, while their positions are unknown. Wireless nodes take RSS measurements from these targets at M arbitrary reference points (RPs) over the grid. The goal is to determine the locations of these targets simultaneously

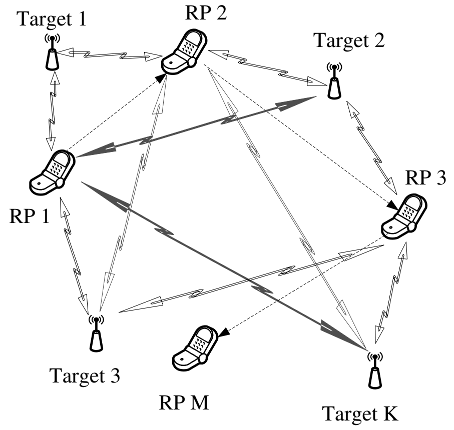

Fig. 1. The scenario of multiple target localization.

and accurately, using only a small number of noisy RSS measurements and simple operations. It is noticed that the problem has a sparse nature, that is $K \ll N$. Furthermore, the number of measurements is much smaller than the grid size, $M \ll N$.

Compressive sensing provides a novel framework for recovering signals, which are sparse or compressible under a certain basis, with far fewer noisy measurements than the traditional methods. It exploits an ℓ_1-minimization program to acquire the sparse signal or its unique sparse representation. To make this possible, two basic components should be held in CS: *sparsity* and *incoherence*. In the localization problem, since the location of a target is unique in the discrete spatial domain at a certain time, it can be modeled as an ideal 1-sparse vector. Meanwhile, the incoherence requirement from the CS theory can be achieved with the same effect through an appropriate data pre-processing. Thus, the localization problem can be well formulated as a sparse matrix recovery problem in the discrete spatial domain.

Assume that the location of the targets over the grid is denoted by $\Theta_{N\times K}$, which is shown as the $N \times K$ matrix,

$$\Theta = [\theta_1, ..., \theta_k, ..., \theta_K] \tag{1}$$

where, each θ_k is an $N \times 1$ vector with all elements equal to zero except $\theta_k(n) = 1$, where n is the index of the grid point at which the kth target is located.

According to the CS theory, rather than measuring the K-sparse signal or its sparse representation Θ directly, compressive noisy RSS measurements in an M-dimensional space are conducted. The compressive measurements are obtained by multiplying a random matrix on the original signal,

$$y = \Phi\Psi\Theta + \varepsilon \tag{2}$$

where,

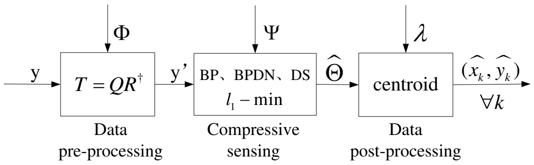

Fig. 2. The flow chart of multiple target localization approach based on the CS theory.

1) $\Psi_{N\times N}$ is the sparsity basis, under which the measured signals have sparse coefficients Θ, as defined in (1). Assume that the transmission power for each target is P_t (dBm). The RSS dictionary indicates the radio propagation channel model, which follows [16]:

$$RSS(d) = P_t + K_e - 10\eta log_{10}(\frac{d}{d_0}) + \alpha + \beta \quad (3)$$

where K_e is a unitless constant that depends on the environment; d is the real transmitter-receiver distance, and d_0 is a reference distance for the antenna at far field; η is the path loss coefficient; α accounts for the fast fading effect, which is zero in our case; β denotes the random attenuation due to shadowing. Typically, shadowing is almost constant over long time periods. Thus, $[\Psi]_{ij} = RSS(d_{ij})$, records the RSS reading on grid point i from the target located at grid point j, for all $1 \leq i \leq N,\ 1 \leq j \leq N$.

2) $\Phi_{M\times N}$ is the measurement matrix. Instead of measuring all the RSS readings on the overall grid, only a small number of measurements are collected on several arbitrary grid points, which are referred to as RPs. Thus, each row of Φ represents the location of each RP, with an element of 1 to indicate the grid point at which the RP is located.

3) ε is the measurement noise.

Based on the above formulations, the $M \times K$ matrix y, is the compressive noisy RSS measurements from K targets on M RPs, with each row vector indicating one measurement value. The number of measurements obeys $M = O(K \log(N/K))$, with $M \ll N$. The overall localization approach based on the CS theory is illustrated in Fig. 2.

Since the sparsity basis Ψ and the measurement matrix Φ are coherent in spatial domain, the CS theory cannot be directly applied. To solve this problem, a data pre-processing on measurement matrix y is introduced. It is proved that this procedure has the same effect as orthogonalizing the two matrices.

Proposition 1: Assume that y is a compressive noisy measurement matrix, with size $M \times K$. Let $y = \Phi\Psi\Theta + \varepsilon$, where Θ is a K-sparse matrix in a N-dimensional space, and $M = O(K \log(N/K))$. Let T be a pre-processing operation on y, *i.e.*, $y' = Ty$. Let

$$T = QR^{\dagger} \quad (4)$$

where $R = \Phi\Psi$, and $Q = orth(R^T)^T$, where $orth(A)$ is an orthogonal basis for the range of A, and A^T returns the transpose of matrix A. Then, Θ can be well recovered from y' via an ℓ_1-minimization program.

Proof: Note that y' can be written as

$$y' = QR^{\dagger}y = QR^{\dagger}R\Theta + QR^{\dagger}\varepsilon = Q\Theta + \varepsilon'. \quad (5)$$

Since Q is an orthogonal matrix, Θ can be well recovered from y' via an ℓ_1-minimization program based on the CS theory. ■

Under the conditions of sparsity and incoherence stated above, the CS theory indicates that the original sparse coefficients Θ can be well recovered given the compressive noisy measurements y', only via an ℓ_1-minimization program, which can be effectively solved in polynomial time.

In this paper, three formulations are employed and compared for the recovery problem from compressive noisy measurements, *i.e.*, Basis Pursuit (BP) [13], Basis Pursuit Denoising (BPDN) [14], and Dantzig Selector (DS) [15]:

- **BP** formulates the problem with equality constraints, and solves the problem by a primal-dual interior point method,

$$\hat{\theta} = \arg\min_{\theta \in R^N} ||\theta||_1, \quad s.t.\ y' = Q\theta. \quad (6)$$

- **BPDN** formulates the problem with quadratic constraints, and reformulates it as a second-order cone program, which can be solved by a log-barrier algorithm,

$$\hat{\theta} = \arg\min_{\theta \in R^N} ||\theta||_1, \quad s.t.\ ||y' - Q\theta||_2 \leq \epsilon. \quad (7)$$

- **DS** formulates the problem with minimal residual correlation (the Dantzig selector), and recasts the problem as a linear program, which also can be solved by the primal-dual interior point method:

$$\hat{\theta} = \arg\min_{\theta \in R^N} ||\theta||_1, \quad s.t.\ ||Q^T(y' - Q\theta)||_\infty \leq \mu \quad (8)$$

where μ is a constraint relaxation parameter.

For each $\hat{\theta_k}$, the output of the three ℓ_1 programs turns out to be a $N \times 1$ vector with all elements equal to zero except one element equal to one, which exactly indicates the grid point at which the target is located. This means that if the targets are exactly located at the grid points, then the recovery can be precise.

However, the targets may not necessary exactly located at these grid points. In such cases, the recovered location $\hat{\theta_k}$ does not turn out to be an exact 1-sparse vector, but with a few non-zero coefficients. In order to compensate for the error induced by the grid assumption, a post-processing procedure is conducted. We choose the dominant coefficients in $\hat{\theta}_k$ whose values are above a certain threshold λ, and take the centroid of these grid points as the location indicator. Let $\mathcal{S}_k$ be the set of all indexes of the elements of $\hat{\theta}_k$ such that

$$\mathcal{S}_k = \{n|\hat{\theta}_k(n) > \lambda\} \tag{9}$$

These are potential candidate points for the estimate of the location of the kth source. Each $n \in \mathcal{S}$ represents a point in the two dimensional space (x_n, y_n). The location of source k can be estimated by finding the centroid of the candidate points, that is

$$(\hat{x}_k, \hat{y}_k) = centroid\{(x_n, y_n)| \text{ for } n \in \mathcal{S}_k\} \tag{10}$$

III. Simulation Results and Analysis

The effectiveness and properties of the proposed localization approach based on the CS theory are studied and analyzed through simulations. In order to examine the performance of position recovery from compressive noisy measurements, three ℓ_1-minimization programs (*i.e.*, BP, BPDN, and DS) are employed and compared. Meanwhile, the localization error with respect to the number of measurements needed, as well as the number of targets whose positions can be recovered are derived, under three programs. Furthermore, both measurement noise and channel noise are considered to demonstrate the reliability and robustness of the proposed approach. Finally, we compare our CS approach with some traditional location estimation schemes on the localization accuracy.

In our simulations, the RSS dictionary Ψ is obtained by employing the indoor empirical model defined by the IEEE 802.15.4 standard [17].

$$RSS(d) = \begin{cases} P_t - 40.2 - 20\log d, & d \le 8 \\ P_t - 58.5 - 33\log d, & d > 8. \end{cases} \tag{11}$$

In the first simulation, the effectiveness of position recovery from compressive noisy measurements via three different ℓ_1-minimization programs is studied. An $100m^2$ area is divided into a 23×23 grid. There are 4 targets randomly located on the grid, whose positions are unknown. RSS measurements from these 4 targets are collected at 12 arbitrary RPs. Assume signal-to-noise ratio (SNR) is equal to $25dB$, which is the ratio of the transmit power to the noise power at the receiver. The result in Fig. 3 shows that all of the three ℓ_1-minimization programs can achieve a similar high level of accuracy on the sparse signal recovery, as long as the number of measurements M conforms with the CS theory. A maximum localization error of 0.4m (4%) is observed using the DS program over 100 simulations. The error in the other two techniques is smaller.

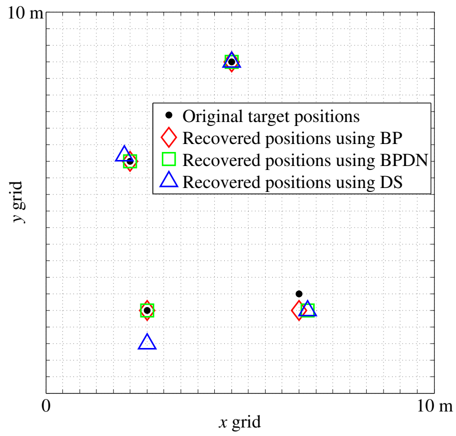

Fig. 3. The performance of position recovery under 4 targets with only 12 measurements via BP, BPDN, DS, respecitvely.

Next, we increase the number of targets to 20 to see if the above performance is still achieved. The effectiveness of position recovery shown in Fig. 4 is derived under the same grid size and the same SNR, while the number of measurements M is equal to 30, which still approximately obeys $M = O(K\log(N/K)) \approx 28.4$. A maximum localization error of 0.11m (1.1%) is observed using the DS program over 100 simulations.

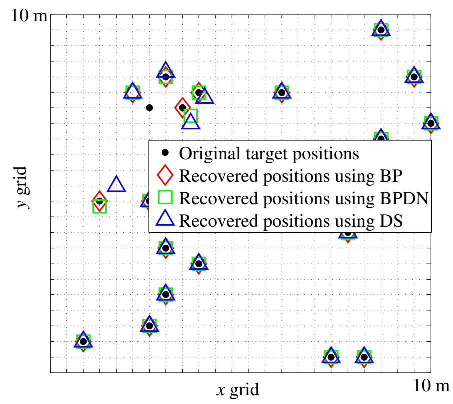

Fig. 4. The performance of position recovery under 20 targets with only 30 measurements via BP, BPDN, DS, respecitvely.

In the second simulation, the localization error versus the number of measurements needed using the CS approach via the above three recovery programs is studied. The number of targets is fixed at 4, and the number of measurements varies between 2 and 24. Assume SNR=$25dB$. The localization error is defined as the average Euclidean distance between the true positions and the recovered positions of the 4 targets, that is

$$P_e = \frac{1}{K}\sum_{k=1}^{K}\sqrt{(x_k-\hat{x_k})^2+(y_k-\hat{y_k})^2}. \quad (12)$$

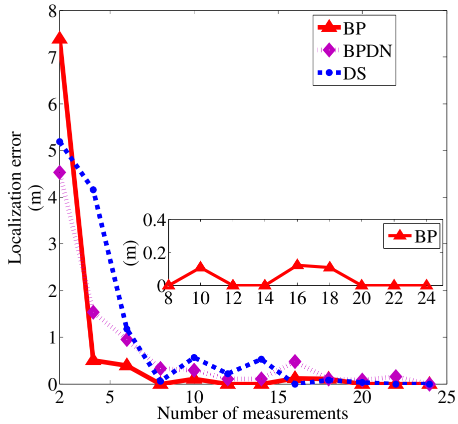

Fig. 5. The localization error versus the number of measurements using the CS approach via BP, BPDN, DS, respecitvely.

Fig. 5 shows the localization error versus the number of measurements needed in the CS approach. As seen, in all three techniques, the localization error decreases sharply and becomes very small as the number of measurements increases. Interestingly, the number of measurements for which the localization error becomes negligible is $M = 8$, which is approximately equal to $K\log(N/K) \approx 8.5$. In the BP program, with sufficient number of measurements $M = 20$, the positions of these targets are recovered precisely in most cases, with an accuracy of almost 100%.

Third simulation investigates the performance of the CS technique as a function of the number of targets. The number of targets changes from 1 to 10, and the number of measurements is fixed at 18. The simulation is conducted under the same grid size and the same SNR. Fig. 6 illustrates the sparsity level that can be achieved. The maximum number of targets whose positions can be recovered under $M = 18$ measurements within 0.7m localization error is approximately equal to 10. It is noticed that for the localization of 10 targets, the number of measurements is $M = 18$, which is approximately equal to $K\log(N/K) \approx 17.2$, as the CS theory indicates. As illustrated in the figure, BP achieves better accuracy than other techniques, and BPDN achieves better accuracy than DS under the same parameter settings. Thus, we would use BP recovery program in the following analysis.

In the fourth simulation, the localization error with respect to the measurement noise using the CS approach via the BP program is studied. The number of targets is fixed at 4, and the number of measurements varies

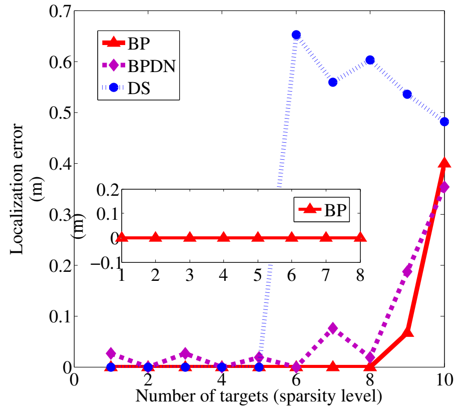

Fig. 6. The localization error with respect to the number of targets using the CS approach via BP, BPDN, DS, respecitvely.

from 2 to 24. $N = 23 \times 23$, and SNR changes from $5dB$ to $30dB$. Fig. 7 shows that the CS approach can tolerate a certain level of measurement noise. When the number of measurements conforms with the CS theory, the localization error is below 0.58m (5.8%) for SNR above $5dB$, and below 0.19m (1.9%) for SNR above 20dB.

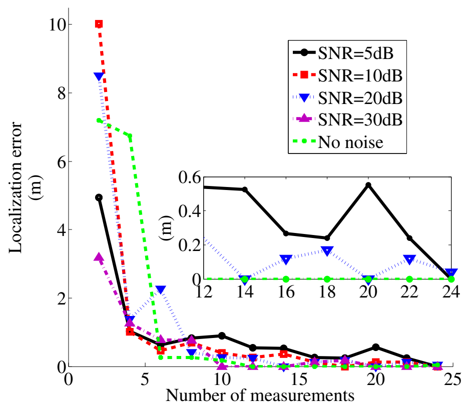

Fig. 7. The localization error with respect to the measurement noise.

In the fifth simulation, the localization error with respect to the channel noise using the CS approach via the BP program is studied. The number of targets is fixed at 4, and the number of measurements varies from 2 to 42. $N = 23 \times 23$, and SNR=$25dB$. A random noise matrix Δ is added on the channel Ψ. We define the perturbation of channel by $\eta = \frac{||\Delta||}{||\Psi||}$, which is set to be $31.6\%, 10.0\%, 5.0\%, 1.0\%, 0.1\%, 0.0\%$ respectively in our simulations. Fig. 8 shows that the CS approach can tolerate a certain level of channel perturbation. With sufficient number of measurements that conform with the CS theory, the localization error is less than 1m when the channel perturbation is below 31.6%. In addition, the

CS approach is quite stable, with an error of less than 0.2m (2%) if the channel perturbation is below 10%.

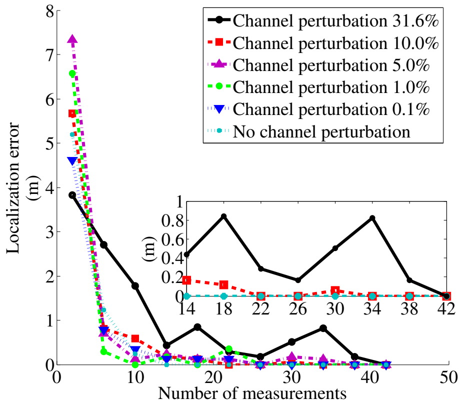

Fig. 8. The localization error with respect to the channel perturbation.

Finally, we compare our CS approach with some traditional localization estimation schemes, based on the prior work in [10]. Fig. 9 depicts the cumulative average error distribution. A percentage of localization error is used as a criterion, in which the absolute error in meter (defined by (12)) is divided by the overall localization area. Although the *kernel method* with spatial filtering and access point selection leads to improvements of accuracy over the K Nearest Neighbours (KNN) and the histogram methods, RSS measurements on each grid point are needed. The number of measurements M increases linearly with the grid size N. While in the CS approach, only a small M is needed, which is logarithmic to the grid size N. Furthermore, the CS approach outperforms the other three methods by $30\% \sim 50\%$ for the localization error about 5%.

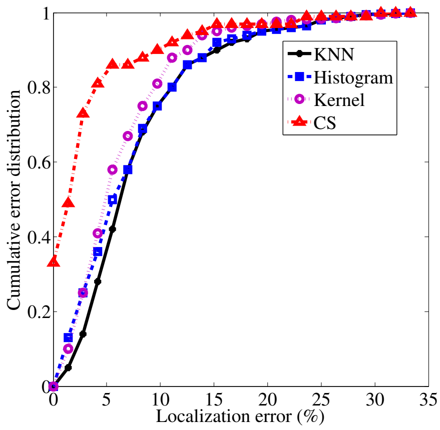

Fig. 9. Cumulative average error for KNN, histogram, kernel, and the CS approach.

IV. Conclusion

In this paper, we have proposed a multiple target localization scheme based on compressive sensing. The intuition behind this technique is that location finding is a sparse problem and that in the CS theory, the locations can be well recovered from only a small number of noisy measurements through an ℓ_1-minimization program. We have used pre-processing to induce incoherence needed in the CS theory, and post-processing to compensate for the spatial discretization caused by grid assumption. Simulation results demonstrate that the proposed CS method outperforms the earlier algorithms that RSS for wireless node localization.

References

[1] N. Patwari, J. N. Ash, and S. Kyperountas, "Locating the nodes: cooperative localization in wireless sensor networks," *IEEE Signal Processing Magazine*, pp. 54-69, July 2005.

[2] J. C. Emmanuel, and B. W. Michael, "An introduction to compressive sampling," *IEEE Signal Processing Magazine*, pp. 21-30, March 2008.

[3] J. Romberg, "Imaging via compressive sampling," *IEEE Signal Processing Magazine*, pp. 14-20, March 2008.

[4] D. L. Donoho, "Compressive sensing," *IEEE Trans. Info. Theory*, vol.52, no. 4, pp. 1289-1306, September 2006.

[5] R. G. Baraniuk, M. Davenport, R. Devore, and M. B. Wakin, "A simple proof of the restricted isomatry property for random matrices," *Constructive Approximation*, 2008.

[6] X. Ji, and H. Zha, "Sensor positioning in wireless ad-hoc sensor networks with multidimensional scaling," *IEEE INFOCOM*, pp. 2652-2661, 2004.

[7] N. Patwari, and A. O. Hero III, "Manifold learning algorithms for localization in wireless sensor networks," *IEEE Signal Processing Magazine*, pp. 54-69, July 2005.

[8] J. A. Costa, N. Patwari, and A. O. Hero III, "Distributed weighted-multidimensional scaling for node localization in sensor networks," *ACM Transactions on Sensor Networks*, Vol.2, pp. 39-64, February 2006.

[9] U. Grossmann, M. Schauch, and S. Hakobyan, "RSSI based wlan indoor positoning with personal digital assistants," *IEEE International Workshop on Intelligent Data Acquisition and Advanced Computing Systems: Technology and Applications*, September 2007.

[10] A. Kushki, K. N. Plataniotis, and A. N. Venetsanopoulos, "Kernel-based Positioning in Wireless Local Area Networks," *IEEE Transactions on Mobile Computing*, 6(6), pp.689-705, 2007.

[11] V. Cevher, P. Boufounos, R. G. Baraniuk, A. C. Gilbert, and M. J. Strauss,"Near-optimal bayesian localization via incoherence and sparsity,"*IPSN 2009, San Francisco, CA*, 13-16 April 2009.

[12] V. Cevher, M. F. Duarte, and R. G. Baraniuk, "Distributed target localization via spatial sparsity,"*EUSIPCO 2008, Lausanne, Switzerland*, 25-29 August 2008.

[13] S. S. Chen, D. L. Donoho, and M. A. Saunders,"Atomic decomposition by basis pursuit," *SIAM Journal on Scientific Computing*, 20(1), pp. 33-61, 1998.

[14] H. Krim, D. Tucker, S. Mallat, and D.L. Donoho, "On denoising and best signal representation," *IEEE Transactions on Information Theory*, 45(7), pp. 2225-2238, 1999.

[15] E. Candes, and T. Tao, "The dantzig selector: statistical estimation when p is much larger than n," *Technical report*, UCLA, May 2005.

[16] G. Zanca, F. Zorzi, and A. Zanella, "Experimental comparison of RSSI-based localization algorithms for indoor wireless sensor networks,"*European Conference on Computer System Proceedings of the Workshop on Real-world Wireless Sensor Networks, Glasgow, Scotland*, 1-5, 2008.

[17] *IEEE* standard online resource provided by *IEEE* 802.15 WPAN, Februry 2009 http://www.ieee802.org/15/pub/TG4.html