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Joint Reduction of Peak to Average Power Ratio and Symbol Loss Rate in Multicarrier Systems

Amin Alamdar Yazdi, Sameh Sorour, Shahrokh Valaee
ECE Department
University of Toronto
Toronto, Ontario, Canada M5S 3G4
Email: {ayazdi, samehsorour, valaee}@comm.utoronto.ca

Ronny Yongho Kim
Mobile Communications Technology Research Lab.
LG Electronics, Inc., 533 Hogye-1dong,
Dongan-gu, Anyang-shi, Kyungki-Do, 431-749, Korea
Email: ronnykim@lge.com

***Abstract*—Peak to average power ratio (PAPR) and symbol loss rate (SLR) are two challenges of multicarrier based communications that have recently drawn much attention. High SLR renders the system unreliable and high PAPR is associated with power inefficiency and nonlinearity of the system. There are rich literatures studying these two issues separately but, unfortunately, only a few works have studied simultaneous reductions of PAPR and SLR. This paper studies the problem of reducing the PAPR while keeping the SLR at minimum.**

In [1], we derived the conditions for the minimum SLR in On-Off channels. The algorithm proposed in this paper simultaneously satisfies the conditions derived in [1] and reduces the PAPR substantially. This paper differs from previous techniques in the sense that none of the previously proposed techniques are capable of reducing PAPR substantially while achieving the minimum symbol loss rate.

We compare our algorithm with the optimum selected mapping PAPR reduction method, which is well known in the literature for having a strong reduction capability. The comparison is done in terms of Complementary Cumulative Distribution Function (CCDF) of the PAPR of the multicarrier signal. The simulation results show that our algorithm can achieve a stronger PAPR reduction while maintaining the minimum SLR.

***Index Terms*—Multicarrier Based Communication Systems, PAPR Reduction, Communication through On-off Channel**

I. INTRODUCTION

Multi-carrier modulation (MCM), known as orthogonal frequency division multiplexing (OFDM) in wireless and discrete multi-tone (DMT) in wired communication, is a well-known transmission scheme for reliable communication. MCM is mostly implemented over frequency selective fading channels as well as On-Off channels and is adopted in several international standards such as Digital Audio Broadcasting (DAB) [2], Digital Video Broadcasting (DVB-T) [3], Asymmetric Digital Subscriber Lines (ADSL) [4] and more recently in the IEEE 802.11a [5], 802.11g [6] and 802.16m standards and also 3GPP-LTE pre-standards [7].

MCM is a technique for transmitting data by splitting it into several components, and sending each component separately over separate carriers. We refer to the sequence of components by a MCM symbol. Like any other technique, MCM also has some challenges two of which are symbol loss rate (SLR) and the peak-to-average power ratio (PAPR) of the transmitted signal. High PAPR will require a large power back off in the transmitting amplifier, which translates to low power efficiency. This is a critical issue in portable wireless devices where power is at a premium.

There have been different PAPR reduction approaches proposed, each one having some advantages and disadvantages over the others. One can partition the proposed techniques into two categories: distortion and distortionless techniques. As the names indicate, the former technique distorts the transmit signal while the latter one does not. Some examples of distortion techniques are clipping [8] and companding [9]. Distortionless techniques include systematic coding [10], selective coding [11], partial transmit sequence [12], tone injection/reservation [13] and active constellation extension [14].

Recently, a distortionless technique called selected mapping (SLM) has attracted much attention because of its effectiveness, strong PAPR reduction capabilities and low implementation complexities [15]–[17].

In this paper, we propose an algorithm which reduces PAPR substantially and simultaneously keeps the system symbol loss rate at minimum. In [1], we derived the conditions required for keeping the symbol loss rate at minimum in On-Off channels. These conditions is recalled in Theorem 1 of this paper.

Our work differs from the previous ones since it reduces PAPR significantly while keeping the symbol loss rate of the system at minimum. This is while none of the previously proposed techniques has this capability. The strong PAPR reduction capability of the proposed technique comes from two facts:

- The proposed technique combines the SLM and coding techniques.
- The generated MCM symbols (from which the symbol for transmission is chosen) do not have high correlation.

The rest of the paper is organized as follows: In Section II, the modeling used in this paper is explained in details and peak to average power ratio of the MCM signal is formulated. The proposed algorithm and its merits are discussed in Section III. Simulation results and their discussion are given in Section IV. Finally, we conclude this paper in Section V.

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II. SYSTEM MODEL AND PEAK TO AVERAGE RATIO FORMULATION

Consider a system in which F carriers, denoted by $f_0, \cdots, f_{(F-1)}$, are allocated to each user for transmission and also each user requires to keep its throughput above a threshold R_{th}. We define the throughput R_{th} as the number of *information bits* sent in each *transmission slot* through the F available carriers. To satisfy the condition on throughput, $U = \left\lceil \frac{R_{th}}{\log_2(M)} \right\rceil$ symbols $(u_1, \cdots, u_U)$ have to be transmitted in each *transmission slot* through the available F carriers, where M is the modulation level. Obviously, each symbol is made up of $\log_2(M)$ information bits.

The communication channel is modeled as an On-Off channel with the symbol drop probability ϵ. In other words, each symbol is either dropped with probability ϵ or is correctly received with probability $1-\epsilon$. Due to the On-Off nature of the channels, the number of allocated carriers is more than the number of symbols to employ symbol diversity and increase the reliability of the system. In other words, $F = dU$ where $d > 1$. Let us denote the symbol transmitted in carrier f_j by s_j, $1 \leq j \leq F$. In general, the relation between u_i, $1 \leq i \leq U$, and s_j, $1 \leq j \leq F$, can be modeled as follows:

$$\underline{s} = \underline{u} \times \mathbf{A} \tag{1}$$

where $\underline{s} = [s_1, \cdots, s_F]$ is hereafter referred to as *transmit block*, $\underline{u} = [u_1, \cdots, u_U]$, $\mathbf{A}$ is a matrix of dimension $U \times F$ whose elements are chosen from a Galois field of proper size q (F_q), and "$\times''$ is a finite field matrix multiplication. Matrix A is required at the receiver side for recovering $\underline{u}$ and therefore has to be transmitted to the receiver side through an error protected channel.

The complex base band representation of the transmit signal can be formulated as:

$$x(t) = \sum_{n=0}^{F-1} s_n . \exp\left(j2\pi f_n t\right), 0 \leq t \leq T \tag{2}$$

where $j = \sqrt{-1}$ and T is the useful transmit block period. The PAPR of $x(t)$ is defined as:

$$PAPR = \frac{\max_{0 \leq t \leq T} |x(t)|^2}{\frac{1}{T}\int_0^T |x(t)|^2 dt} \tag{3}$$

It is well known that, if the correlation between s_i, $1 \leq i \leq F$, is high, the PAPR will also be high. Based on this observation, [18] introduces a parameter for predicting the PAPR behavior of the signal, and [19] introduces a PAPR reduction algorithm.

III. PROPOSED TECHNIQUE

Our algorithm has to minimize SLR and reduces the PAPR substantially.

We adopt characteristic matrix from [20]:

$$C = \mathcal{X}(\mathbf{A}) \quad : \quad \begin{cases} c_{ij} = 1 & \text{if} \quad \alpha_{ij} \neq 0 \\ c_{ij} = 0 & \text{if} \quad \alpha_{ij} = 0 \end{cases}$$

where α_{ij} and c_{ij} are respectively the entries of matrices $\mathbf{A}$ and C.

To minimize the SLR, we use the following theorem which we proved in [1].

Theorem 1. *A system in the above set-up has the minimum symbol loss rate and consequently maximum reliability iff* $C = \mathcal{X}(\mathbf{A})$ *has the following property:*

$$C = [I_{U\times U} \; E_{U\times(F-U)}] \tag{4}$$

where $I_{U\times U}$ *is an identity matrix and all of the entries of* $E_{U\times(F-U)}$ *are 1.*

To force matrix $\mathbf{A}$ to satisfy the conditions required for minimizing the SLR as derived in Theorem 1, we use the following theorem from [21]:

Theorem 2. *Matrix* A *defined as follows satisfies the conditions of Theorem 1 iff* v_i, w_j *are* F *distinct elements.*

$$A = [I_U \; V(\underline{v}, U, U)^{-1} \times V(\underline{w}, U, F-U)]$$

where $V(\underline{v}, x, y)$ *is the Vandermonde matrix of dimension* $x \times y$ *built up of vector* $\underline{v}$.

$V(\underline{v}, x, y)$ is defined as:

$$V = \begin{pmatrix} 1 & v_1 & v_1^2 & \ldots & v_1^{y-1} \\ 1 & v_2 & v_2^2 & \ldots & v_2^{y-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & v_x & v_x^2 & \ldots & v_x^{y-1} \end{pmatrix}$$

We also need the following lemma from finite field literature:

Lemma 1. *If* F_q *is a finite field with* q *elements, where* q *is a prime number, then there exists a non-zero element* x *in* F_q*, called "a generator of the field" such that*

$$F_q = \{0, 1, x, x^2, ..., x^{q-2}\}$$

Furthermore, If we fix a generator, then for any non-zero element a *in* F_q*, there is a unique integer* n *with* $0 \leq n \leq q-2$ *such that* $a = x^n$.

From Lemma 1 and by choosing x to be "the generator of the field", we have the following property:

if $v_1 \neq v_2$ are neither 0 nor 1, then $x^{v_1} \neq x^{v_2}$ and therefore there is a one-to-one relation between the elements of $\underline{v}$ and $x^{\underline{v}}$.

Therefore, from Theorem 2, any U columns of

$$V_x = \left[I_U V\left(x^{\underline{v}}(1:U), U, U\right)^{-1} \times V\left(x^{\underline{v}}(U+1:F), U, F-U\right) \right] \tag{5}$$

are independent and furthermore, none of the entries of V_x is zero. Therefore, from Theorem 1, this new matrix has the minimum symbol loss rate.

Note that $\underline{v}$ is a vector and $x^{\underline{v}}$ is a new vector whose i^{th} elements is $x^{i^{th}\text{element of}\underline{v}}$. Furthermore, $x^{\underline{v}}(a:b)$ is defined to be a vector composed of a^{th}, , $(a+1)^{th}$, , ..., b^{th} elements of $x^{\underline{v}}$.

Assuming that the number of bits reserved for transmission of side information is $\log_2(M)$, we propose the following algorithm to reduce PAPR:

1) *Fix x to be a generator of F_q where q is the largest prime number less than M.*
2) *Pick different values excluding 0 and 1 for entries of vector $\underline{v}$.* The values of x and $\underline{v}$ should be fixed and known to both the transmitter and receiver before the transmission starts.
3) For $r = 1$ to $r = M$, repeat the following steps:
   - Construct V_x as shown in (5);
   - Form $\underline{s}^{(r)} = \underline{u} \times V_x$ and its corresponding base band representation signal from (2) and PAPR from (3), call it $PAPR_r$;
   - Substitute v by $x^{\underline{v}}$;
   - Increase r by 1.
4) Transmit the signal which has the minimum PAPR among $PAPR_r$, $r = 1, \ldots, M$.
5) Transmit r as the side information, $1 \leq r \leq M$.

At the receiver side, the recovery phase is as follows:

Having r, the receiver applies the reverse process of $x^{\underline{v}}$ element-wise on the received vector. Note that x is chosen to be the generator of the field and therefore $x^{\underline{v}}$ is a one-to-one relation and thus the reverse process exists. In other words, x has to be the generator of the field for the recovery phase to be possible.

As simulation results illustrate, this algorithm can achieve the maximum PAPR reduction gained using Optimum SLM (O-SLM) while simultaneously achieving the minimum symbol loss rate.

Regarding the merits of this algorithm:

- This algorithm minimizes the symbol loss rate and simultaneously reduces the PAPR significantly. As an alternative to our technique, one can use exhaustive search to check all the possible matrices which reduce the PAPR and keep the symbol loss rate minimum. But, exhaustive search is not practical since its complexity is extremely high and it causes a lot of overhead required for transmitting the values of the encoding matrix. In our technique, the encoding matrix has a specific structure and can be generated at the receiver side by having the index r which can be transmitted with only $\log_2(M)$ bits.
- The strong capability of our algorithm comes from two facts:
  - This algorithm combines the SLM and coding PAPR reduction techniques. The combination of these two techniques has not been proposed before.
  - Our technique uses function $f(\underline{v}) = x^{\underline{v}}$ to generate $\underline{s}^{(r)}$. One of the properties of function $f(\underline{v})$ is that the correlation between the elements of vector $\underline{v}$ is very much different from that of the elements of $f(\underline{v})$. Therefore, $PAPR_r$ takes different values for $1 \leq r \leq M$ and so one of them by a high probability has a very low PAPR.

IV. Simulation Results

A. The Complementary Cumulative Distribution Function (CCDF) of PAPR

The CCDF of the PAPR denotes the probability that the PAPR of a transmit block exceeds a given threshold. We

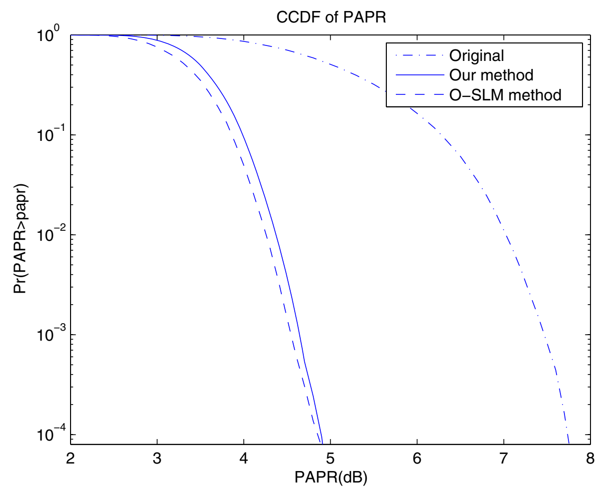


Fig. 1. CCDF of PAPR, no. of iterations=16, $\frac{F}{U} = 5$.

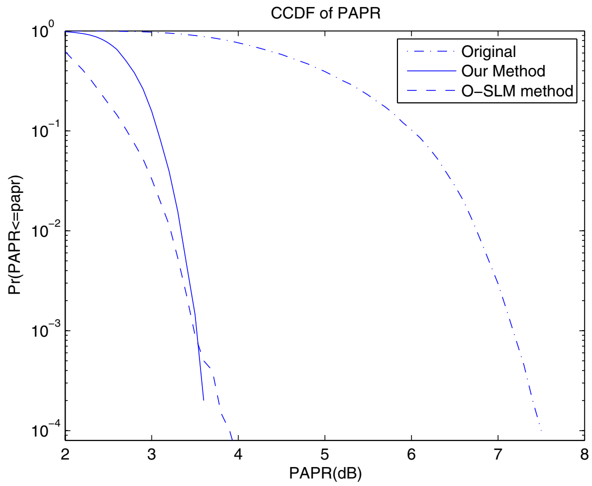
ran a simulation for the case of $\frac{F}{U} = 5$ whose results are depicted in Figure 1. The number of iterations (M) in this simulation has been taken to be 16. As we clearly see from this figure, the proposed technique has a strong PAPR reduction capability. Crossing the probability line at 10^{-3}, the proposed technique achieves 2.9 dB gain in PAPR. Moreover, the proposed technique closely follows the CCDF curve of O-SLM technique.

B. The Effect of $\frac{F}{U}$ on CCDF

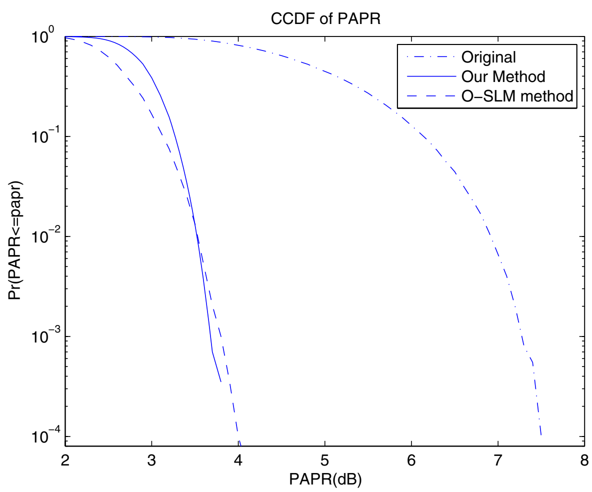
The number of retransmissions has an obvious and notable effect on the performance of the proposed technique. As Figure 2 indicates, by increasing the number of transmissions, the reduction in PAPR increases and the capability of the proposed technique in some cases is stronger than that of O-SLM. Crossing a line at probability 10^{-3}, when the retransmission is performed 3 times, the PAPR reduction for both techniques are almost 3.4 dB. In the case of $\frac{F}{U} = 5$, the difference between the performances of the two techniques enlarges and clearly our technique outperforms O-SLM. So, in summary, by increasing the number of retransmission, our technique gives more gain compared to O-SLM.

C. The Effect of the Number of Iterations on CCDF

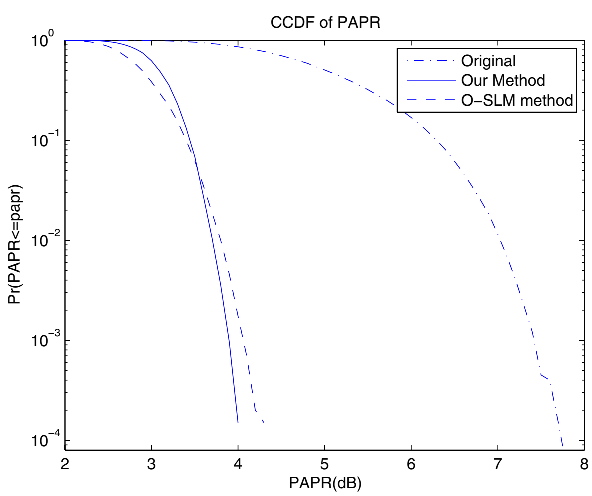
As we see in Figure 3, the performance of our technique improves by increasing the number of iterations M. As the number of iterations increases, our technique considerably outperforms O-SLM technique. In Figure 3(c), the probability of PAPR being more than 4 dB using our technique is 10^{-4} while the one using O-SLM is almost 10^{-3} which is different by an order of magnitude.



(a) $\frac{F}{U} = 3$

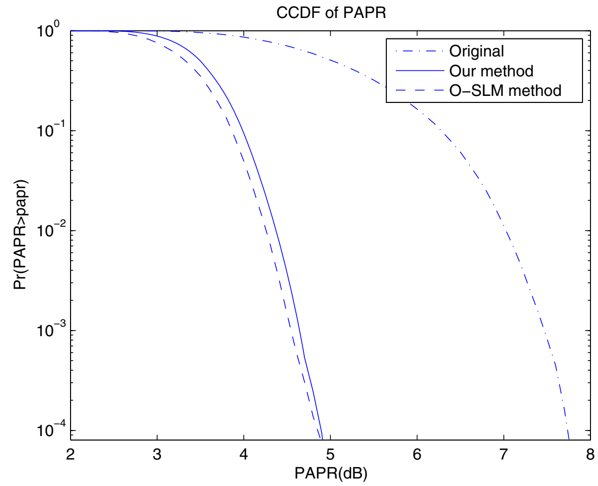


(b) $\frac{F}{U} = 4$

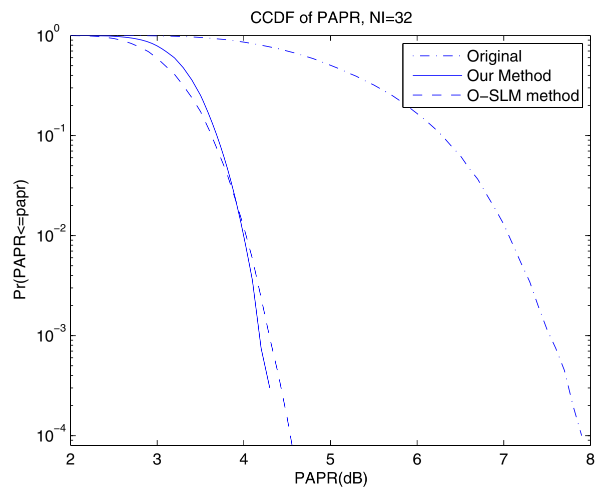


(c) $\frac{F}{U} = 5$.

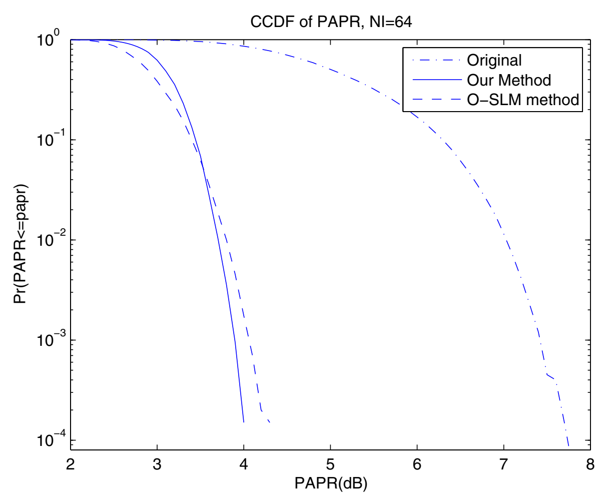
Fig. 2. The effect of $\frac{F}{U}$ on CCDF of PAPR.



(a) no. of iterations=16



(b) no. of iterations=32



(c) no. of iterations=64

Fig. 3. The effect of the number of iterations on CCDF of PAPR

V. Conclusion

Peak to average power ratio (PAPR) and symbol loss rate (SLR) are two challenges of the multicarrier based communication and are studied in this paper. In this paper, we studied the problem of reducing the PAPR while keeping the SLR at minimum. The algorithm proposed in this paper satisfies the conditions required for having minimum symbol loss rate and simultaneously reduces PAPR substantially. This paper differs from previous techniques since none of the previously proposed techniques has the capability to reduce PAPR substantially and to keep the symbol loss rate at minimum. We have compared our algorithm to the optimum selected mapping PAPR reduction technique which is well known in literature to have a strong reduction capability. The simulation results showed that our proposed technique has a strong capability.

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