

COMPRESSIVE DETECTION FOR WIDE-BAND SPECTRUM SENSING

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ABSTRACT

In this paper we propose a novel wide-band spectrum sensing scheme using compressive sensing. The wide-band signal is fed into a number of wide-band filters and the outputs of the filters are used to reconstruct the vector of channel energies through the compressive sensing's ℓ_1 norm minimization. An energy detection is then performed by comparing the obtained vector to a vector of energy thresholds to decide about the occupancy of each channel.

Performance of the proposed approach is compared to the current wide-band spectrum sensing algorithms as well as the conventional channel-by-channel scanning method.

Index Terms— compressive detection, compressive sensing, wide-band spectrum sensing

1. INTRODUCTION

With the rapid growth in wireless applications, spectrum resource becomes scarce. Although the current static spectrum management avoids interference effectively, this comes with the price of very low spectrum utilization. While some frequency bands are overcrowded, other bands are rarely used. Cognitive radio (CR) promises to increase the utilization of frequency bands that are under-utilized by providing opportunistic spectrum access. This is done through authorizing the unlicensed users (secondary users) to access the band assigned to the licensed user (primary user) when it is unoccupied. spectrum sensing plays an essential role in cognitive radio since secondary users need to detect primary signals in order to make decisions about the occupancy of the spectrum bands.

When a wide-band spectrum is assigned to a number of primary users, secondary users can search for unoccupied channels (spectrum holes) within the wide-band spectrum and communicate in that band. The traditional way for detecting holes in a wide-band spectrum is channel-by-channel scanning.

In order to implement this, an RF front-end with a bank of tunable and narrow bandpass filters is needed. The occupancy of each channel can be determined by measuring the energy of the signal at the output of each filter. The high complexity encountered by such approach is a major challenge as numerous RF components are required for the implementation. Furthermore, such method introduces large latency to the spectrum sensing process.

Alternative techniques have been proposed in the area of wide-band spectrum sensing. A wide-band spectrum sensing technique called multi-band joint detection has been proposed in [1]. In this method, signal energy levels over multiple channels are jointly detected. The problem is formulated as a class of optimization problems in which the opportunistic aggregate throughput is maximized subject to practical constraints. It has been shown that multi-band

joint detection outperforms the approach that searches for threshold levels that maximize the aggregate throughput.

In [2], maximum likelihood estimation of the signal and noise power has been used to detect the primary signals. An iterative asymptotic ML estimate has been proposed that can be simplified to obtain an efficient least squares estimator. The performance of this approach has been studied through simulations for different number of channels and different SNRs.

Compressive sensing theory has also been considered in wide-band spectrum sensing techniques [3,4]. In [3], an analog-to-digital converter has been used to transform the analog received signal into a digital signal by sampling at the Nyquist rate. Next, compressive sampling is applied to the sampled vector to compress it into a smaller vector and then the spectrum is reconstructed by solving an ℓ_1 norm minimization problem. In [4] the received analog signal is sampled at the information rate of the signal using an analog-to-information-converter (AIC). Here, the compressive sensing is embedded in the AIC. The same ℓ_1 norm minimization method is used to estimate the original spectrum. In both [3,4], wavelet edge detector has been used to detect the channel borders in the estimated spectrum and the detection's performance has been evaluated in terms of mean square error through simulations. It has been shown that MSE performance of [3] outperforms that of [4] for all compression rates, but their detection performances are comparable. In both [3,4], the signal needs to be sampled at the Nyquist rate and then compressed based on its sparsity.

In this paper, we propose a novel method of compressive detection for wide-band spectrum sensing. In the proposed method, the signal is fed into a number of filters, much less than the number of channels within the wide-band spectrum. The energies of the filter outputs are used as the compressed measurement to reconstruct the signal energy in each channel. The energy vector is then compared with a threshold vector to detect the spectrum holes.

The effect of noise on the received signal is investigated through simulations. Numerical results suggest that the compressive sensing method enhances the detection performance of the receiver by suppressing the noise energy in the unoccupied bands.

2. COMPRESSIVE DETECTION, THE PROPOSED APPROACH

In this section, we introduce the novel method of wide-band compressive channel occupancy detection. First, compressive sensing basics are briefly introduced. Second, the design and the approach of each CR to obtain the channel occupancy estimate is discussed. Third, the compressive detection algorithm is introduced. In this algorithm, CRs estimate the signal energy of all channels compressively and decide on the occupancy of the channels. Application of

the proposed method in ad-hoc networks and the advantages of the approach over current wide-band spectrum sensing algorithms are also discussed.

2.1. Compressive Sensing Basics

Compressive sensing is a method to recover signals from far fewer measurements than needed for traditional sampling. Assume that an $N \times 1$ vector \mathbf{x} is to be measured. Also suppose that there is a basis Ψ in which \mathbf{x} is sparse. Mathematically, \mathbf{x} can be written as

$$\mathbf{x} = \Psi \mathbf{s} \quad (1)$$

where the $N \times 1$ vector \mathbf{s} is the representation of \mathbf{x} in the basis Ψ and has just $L_s \ll N$ non zero elements.

Compressive sensing theory states that \mathbf{x} can be accurately recovered from $K \ll N$ measurements of the signal. Assume that we use a set of K linear combinations of the signal as the measurement vector \mathbf{y}

$$\mathbf{y} = \Phi \mathbf{x}. \quad (2)$$

where Φ is the sensing matrix. Then by properly choosing K and Φ , and based on sparsity of the representation of \mathbf{x} in the Ψ basis, \mathbf{x} can be recovered from \mathbf{y} . The value of K depends on N , L_s and a measure of coherence (correlation) between the sensing matrix Φ and the basis matrix Ψ . As the basis matrix is determined by the nature of the problem, choosing a sensing matrix having a low coherence with Ψ will lead to a smaller K . This suggests choosing Φ to be a totally random matrix [5].

If the above conditions apply, then the sparse vector \mathbf{s} can be recovered from the measurement vector \mathbf{y} through an ℓ_1 norm minimization

$$\begin{aligned} & \min_{\mathbf{s}} \|\mathbf{s}\|_1 \\ & \text{Subject to } \mathbf{y} = \Phi \Psi \mathbf{s} \end{aligned} \quad (3)$$

2.2. System Model and Problem Statement

Suppose that a total spectrum of W Hz is considered to be shared among a number of primary and secondary users. This can be either an ad-hoc network sharing a total of W Hz spectrum among its nodes or a secondary network of cognitive radios trying to use the licensed spectrum opportunistically for secondary communication.

Assume that each node in the ad-hoc network in the first scenario or each cognitive radio in the second scenario needs a bandwidth of B Hz for the communication. Define $N \triangleq \frac{W}{B}$ to be the number of available channels and denote by f_i the center frequency of the i th channel. Also assume that each node is using a wide-band antenna listening to the whole spectrum and providing the node with the wide-band time domain signal $x(t)$.

Each node is also provided with a filter bank $\{H_k(f)\}_{k=1}^K$ consisting of $K \ll N$ wide-band filters with a bandwidth equal to the total spectrum W . Alternatively, a $K \times N$ totally random complex matrix Φ can be assigned to the nodes. This matrix is used to design the K filters so that the frequency response of the k th filter at the i th channel is $[\Phi]_{ki}$, or equivalently

$$H_k(f_i) = [\Phi]_{ki}, \quad k = 1, 2, \dots, K, \quad i = 1, 2, \dots, N. \quad (4)$$

Here, $H_k(f)$ represents the transfer function of the k th filter. The Φ matrices can be generated once and stored in the nodes (equivalently the filters can be generated and stored). Assume that, the wide-band

signal at the input of the node, $x(t)$, is sampled to obtain the time sequence vector \mathbf{x}_t . The node then feeds the wide-band signal into the filters and the output at the k th filter is

$$\mathbf{z}_k = \text{Conv}(\mathbf{x}_t, \mathbf{h}_k) \quad (5)$$

where $\text{Conv}(\cdot, \cdot)$ denotes the convolution operation and \mathbf{h}_k is the impulse response sequence of the k th filter. The energy of the output signal of each filter is then measured to get the $K \times 1$ energy vector \mathbf{y}

$$y_k = \mathbf{z}_k^H \mathbf{z}_k, \quad k = 1, 2, \dots, K \quad (6)$$

$$\mathbf{y} = [y_1, y_2, \dots, y_K]^T \quad (7)$$

where $(\cdot)^T$ and $(\cdot)^H$ represent transpose and complex transpose of a matrix respectively. Lets denote the portion of the received signal's energy in the i th channel by E_i . Mathematically,

$$E_i = \int_{f_i - B/2}^{f_i + B/2} \mathcal{F}x(t) df. \quad (8)$$

Here \mathcal{F} denotes the continuous Fourier transform of a signal. Suppose that the frequency response of each filter is approximately constant throughout each channel and equals $H_k(f_i) = \Phi_{ki}$ for the k th filter and the i th channel. Hence the energy at the output of the k th filter can be represented as

$$y_k = \sum_{i=1}^N |H_k(f_i)|^2 E_i, \quad k = 1, 2, \dots, K$$

In the vector form, we can write the above set of equations as

$$\mathbf{y} = \bar{\Phi} \mathbf{e} \quad (9)$$

where

$$\bar{\Phi} = \begin{bmatrix} |H_1(f_1)|^2 & |H_1(f_2)|^2 & \dots & |H_1(f_N)|^2 \\ |H_2(f_1)|^2 & |H_2(f_2)|^2 & \dots & |H_2(f_N)|^2 \\ \vdots & \vdots & \ddots & \vdots \\ |H_K(f_1)|^2 & |H_K(f_2)|^2 & \dots & |H_K(f_N)|^2 \end{bmatrix}.$$

Here $\bar{\Phi}$ is a matrix whose elements are square absolute values of the elements of the random matrix Φ and $\mathbf{e} = [E_1, E_2, \dots, E_N]^T$ is the vector of energies of the received signal in different channels. *The goal of the node is to estimate the length N vector \mathbf{e} using the length K measurements vector \mathbf{y} .*

2.3. Compressive Detection

It is now straightforward to establish the correspondence between our filter-based node design and the compressive sensing theory. We assume that at each node and at each instance of time, only a small portion of the channels are occupied. This is equivalent to assuming that the energy vector \mathbf{e} is sparse. Therefore, by properly choosing the number of the filters, K , based on the compressive sensing theory, the channel energy vector \mathbf{e} can be recovered from the measurement vector \mathbf{y} as

$$\mathbf{e} = \arg \min_{\mathbf{e}} \|\mathbf{e}\|_1$$

$$\text{Subject to } \mathbf{y} = \bar{\Phi} \mathbf{e}$$

This is the base for the compressive detection receiver. Each node reconstructs the energy vector \mathbf{e} from the vector of measurements \mathbf{y} . Next, a threshold is adopted and the values of \mathbf{e} are compared with the threshold to decide on the occupancy of the channels.

In the cognitive radio scenario, the threshold adopted in each channel depends on the maximum level of interference allowed by the primary user. Assume that the distance from the primary transmitter to the primary receiver is denoted by R . If the guaranteed signal to interference ratio (SIR) for the primary communication is γ , the interference range of the primary receiver, D , can be determined by

$$\frac{P_p L(R)}{P_s L(D) + P_b} = \gamma \quad (10)$$

where P_p and P_s are the primary transmitter and the cognitive radio's transmit powers, P_b is the power of background interference at the primary receiver and $L(d)$ is the function of total path loss at distance d [6]. Consequently, the cognitive radio should be able to sense any signal coming from a distance of maximum $R + D$ or equivalently any signal with power equal to or greater than $P_{min} = P_p L(D + R)$. So in each channel, if $P_{min} > BN_o$, where N_o is the noise spectral density, the threshold should be set above the noise level and below P_{min} . Otherwise the cognitive radio is not in the interference range of the primary receiver and can always transmit in the underlying channel. The parameters γ , R and P_b should be provided by the regulator or the corresponding primary system [6].

2.4. Cooperative Spectrum Sensing in Ad-hoc Networks

The compressive detection method proposed could be adopted by ad-hoc networks for efficient spectrum utilization. Assume a number of nodes communicating within an ad-hoc network and spectrum of W Hz is assigned to the whole network. This spectrum is divided into N service channels and a low bandwidth control channel. The control channel is used to convey control commands such as connection initialization commands and channel occupancy estimation. One-to-one communication is assumed so whenever one node in the network has information to share with another, an empty channel has to be selected and used for the transmission.

A major challenge in such scenario is the hidden terminal problem. Suppose that node A wants to transmit data to node B. Node A senses the spectrum and chooses a channel for the transmission. However, if node C which is out of the detection range of A but in the interference range of B, uses the same channel for transmission, then the signals of A and C interfere in B and the transmission fails.

In order to prevent the hidden terminal problem, the nodes communicating in an ad-hoc network should cooperate in finding the empty channels. Whenever data is available at one node intended to be sent to another, the destination is also notified through the control channel and then both nodes sense the spectrum and exchange their estimates of the available channels through the control channel. The estimates made at the two nodes might be different since each may pickup signals from close by nodes that are communicating through one of the N channels that are not detected by peer node. Next, based on the two estimates of the occupancy pattern, they agree on one or a number of channels that are empty on the location of both nodes.

Exchange of spectrum estimates can be performed by sending N bits over the control channel in which ones show the locations of occupied channels. Upon receiving this decision bit stream, each node can use bitwise OR operation to obtain the channels available at both ends.

To find the thresholds that nodes should adopt in the channel occupancy detection, assume that a minimum frequency reuse distance D is determined for the network. In other words, the same channel can be re-used to connect two other nodes if both are in a distance of at least D from the nodes initially using the channel. Using the same path loss function $L(\cdot)$, the threshold at each channel can be set to $\gamma = PL(D)$ where P denotes the maximum transmit power of the nodes. As both nodes detect the channel occupancy using the proposed compressive detection approach, it is guaranteed that no interference or hidden terminal problem occurs.

2.5. Advantages over Current Algorithms

The proposed algorithm has several advantages over the algorithms already suggested in the literature. First, unlike methods suggested in [3,4], in our compressive detection, the frequency domain representation of the signal is not being reconstructed. Instead, just the vector of channel energies is obtained by solving the ℓ_1 norm minimization. This benefits the complexity of the problem in two aspects. First, the energy vector in our algorithm has exactly N elements and hence the optimization problem has dimension N , while in spectrum reconstruction, the dimension is nN where n is the number of samples per channel and depends on the resolution of the spectrum reconstruction.

Second, the optimization variable in spectrum reconstruction is a complex vector. It is shown that the complexity of the ℓ_1 norm minimization in this case is $\mathcal{O}(n^3)$ [ref]. In the proposed energy detection, the optimization variable is a vector of real and nonnegative numbers. Authors of [7] show that in this case, the optimization problem can be solved with linear programming. To compare the complexities of the two methods, as an example, suppose that a spectrum consisting of 100 channels is being sensed. If just two samples per channel are used in the spectrum reconstruction scheme, the proposed method has a lower complexity factor of 80000 ($\frac{(nN)^3}{N}$).

It is also worth mentioning that comparing with non wide-band spectrum sensing methods, i.e. channel-by-channel scanning, the proposed method outperforms in complexity in spite of the added compressive sensing algorithm. First, in channel-by-channel scanning, a bank of N narrow-band filters are needed to scan each channel while the proposed method exploits just $K \ll N$ filters. Second, the filters used in the proposed method are wide-band filters having a much shorter impulse response and hence lower filtering complexity. Considering that our K filters have bandwidth N times larger than a narrow band single channel filter, and the fact that we have only K filters, leads to the conclusion that the filtering complexity of the proposed method over the conventional channel-by-channel scanning is $\frac{1}{N} \times \frac{K}{N} = \frac{K}{N^2}$.

Another advantage of the proposed method, is the effect of the compressive sensing algorithm on the performance of the detection in presence of noise. If there is no noise, the energy vector \mathbf{e} has lots of zeros and a few nonzero elements representing the occupied channels. This sparse vector hence can be reconstructed based on the compressive sensing theory. In real situations, the energy of the unoccupied channels is BN_o (the noise energy) and therefore \mathbf{e} has no zeros. In this case, the reconstructed signal is not an exact copy of the energy vector as the sparsity has changed. Nevertheless, numerical results suggest that as the compressive sensing algorithm searches for a vector with least number of nonzero elements, in the reconstructed energy vector, the noise effect has been suppressed compared to the original energy vector. In other words, as far as a reasonable signal power is present in the receiver, the output of the

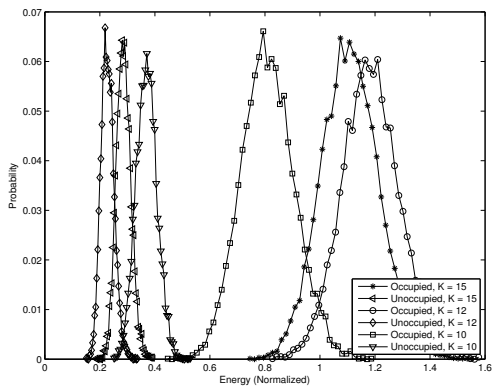


Fig. 1. PDF of energy in an occupied and an unoccupied channel for different number of filters K

compressive sensing algorithm has a higher SNR. We hope to be able to report on rigorous analysis of this result in a follow-up paper.

3. SIMULATIONS

In this section, we evaluate the performance of the compressive detection algorithm through simulations. The input signal to each node, is the wide-band noisy signal which is fed into K different filters and the energy of the output signals are then used in the ℓ_1 norm optimization problem to obtain the channel energy vector \mathbf{e} (10).

In the simulations, a spectrum bandwidth of 20 channels is considered and it is also assumed that at each node and at each instance of time, not more than 6 channels are occupied. Measurements show that a minimum of 12 filters ($K = 12$) is needed for successful reconstruction of the energy vector. Additive white Gaussian noise is added to the received time signal.

Fig. 1 illustrates the probability density function (PDF) of the estimated channel energy \mathbf{e} for an occupied and an unoccupied channel and the signal to noise ratio of 5 dB. As seen in this figure, the PDF of the detected energies in an occupied channel are very close for $K = 15$ and $K = 12$. On the other hand, for $K = 10$ the mean of the detected energy degrades significantly. This shows the threshold effect of K based on compressive sensing theory. Interestingly, the mean of the unoccupied channel energy, which is the mean of the reconstructed noise energy, decreases with K as far as K remains above the threshold. In other words, the compressive sensing algorithm is suppressing the input noise at the output while keeping the signal almost constant and hence increasing the SNR. This is of course true just if K is above the threshold. As seen in Fig. 1, for $K = 10$ the noise has been actually amplified.

Fig. 2 depicts the probability of error in channel occupancy detection versus SNR for different number of filters K . As seen in this figure, for number of filters equal and above $K = 12$, the performance of the detector is almost the same. The degradation is on the other hand apparent when less than 12 filters are used.

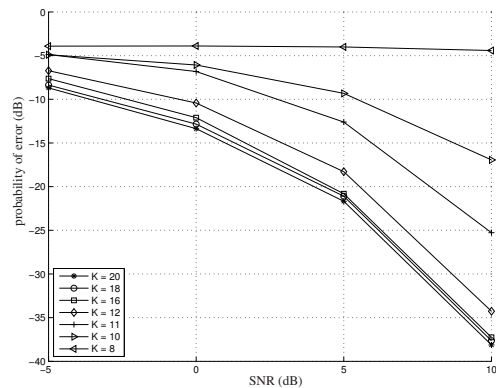


Fig. 2. Probability of detection error versus SNR for different number of filters

4. CONCLUSION

5. REFERENCES

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