Minimum Energy Fault Tolerant Sensor Networks

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***Abstract*— We devise a scheme which can provide reliable transport services in sensor networks and give an algorithm which minimizes the energy use of our scheme. We use a *distributed sink* where information arrives at the sink via multiple proxy nodes, called "prongs" in this paper. The sender node uses Forward Error Correction (FEC) erasure coding to encode each packet into multiple fragments and transmits the fragments to each of the prongs over a path which is disjoint from the paths to the other prongs. The erasure coding allows the sink to reconstruct the original packet even if some of the fragments are lost.**

We use a cross-layer design where higher network layers use information about packet loss and energy consumption to distribute the load in the network. We show that the source can distribute the load so that energy consumption is minimized. The optimization takes fault tolerance into account with a bound on the probability of packet loss. The extra fragments increase both the reliability and the energy in the network. However, we show with simulations that it is possible for the sensor to decrease energy use in the network by using the diversity available with multiple network paths.

***Index Terms*— Sensor networks, energy-aware communications, wireless communication, fault tolerance, network monitoring.**

I. Introduction

SENSOR networks are wireless ad hoc networks used for monitoring and information gathering. They consist of many small, self-organized nodes that form an ad hoc network that reports to a common sink at the edge of the network. Sensor networks are used to observe natural phenomena such as seismological and weather data, collect data in battlefields, and monitor traffic in urban areas.

We show a typical sensor network in Fig. 1. The sink sends a query which is disseminated throughout the sensor network with flooding. The query requests a subset of nodes to send their collected information. The nodes, which have the requested information, send it to the sink by forming a routing tree with the root at the sink receiver. However, collection of information with a single sink may not be appropriate for sensor networks. In a sensor network with a single sink, the top level of the tree contains relatively few nodes, compared to the total number of nodes. So, most of the energy is consumed by the few nodes close to the sink. Therefore, the nodes closer to the sink will run out of battery power earlier than other nodes. A node closer to the sink may also be a bottleneck. For example, a routing protocol which takes fault tolerance (reliability) into account [1] would not have too many options in choosing the last hop.

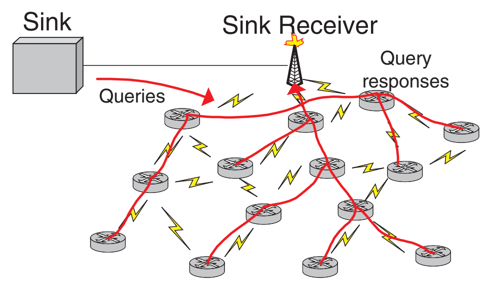

Fig. 1. The conventional sensor network design. The sink has only one receiver which connects to the network.

We propose a sensor network architecture which assumes that the sink is distributed throughout the sensor network. The sink uses a number of receivers, called "prongs", which connect to it with reliable and high bandwidth links. We assume that if a packet arrives at a prong, it will be delivered intact to the sink. This creates a hierarchical architecture as shown in Fig. 2. The figure shows a number of sensor nodes and a sink with four receivers. As illustrated in the figure, the nodes can be connected directly to a prong or with multiple hops through other sensor nodes. The advantage of this architecture is that it distributes the load on the last hop among a larger set of nodes than the architecture with a single receiver design.

The presence of multiple prongs also allow us to increase the number of distinct paths from every node in the network to the sink. This allows the sensor node to use *path diversification* to increase fault tolerance (reliability) in the network. In path diversification, each node in the network (a source) sends packets over multiple disjoint paths.[1] One way to increase reliability would be to send copies of the same packet over the multiple paths. However, this would be very inefficient. Instead, path diversification increases reliability efficiently with Forward Error Correction (FEC). The sensor encodes each packet of M fragments into $M + K$ fragments with an erasure code [2]. The fragments are then distributed over the paths and simultaneously sent to the sink. The sink can reconstruct the packet if it receives more than M fragments.

[1]We use multipaths in the network layer, as opposed to a scheme that may use multipaths in the physical layer.

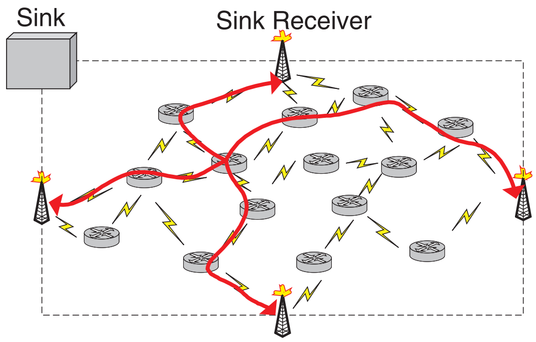

Fig. 2. Our sensor network design. The sink has many receivers scattered throughout the network. The source uses multiple disjoint paths to send information to the sink. This improves load-balancing and increases reliability.

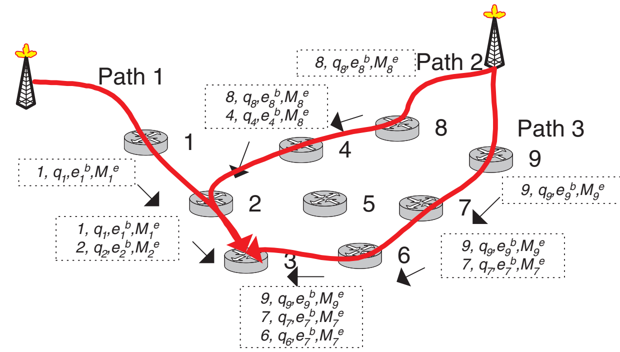

Fig. 3. Our sensor network design. The sink sends queries by flooding the network. The query carries information about the conditions on every hop in the network. This improves load-balancing and increases reliability.

The approach of using erasure codes with multiple paths to increase reliability was shown to be effective previously in [3], [4], [5]. We show in [3] that erasure codes are not sufficient to efficiently increase the reliability. This is because a single path may be more likely to fail on its own than to fail together with other disjoint paths. This means that the source needs to use more parity fragments with a single path than with multiple disjoint paths in order to achieve the same level of reliability. In effect, the simultaneous use of multiple paths takes advantage of the diversity available in dense sensor networks.

In this paper, we show that it is possible to distribute the fragments on the paths so that the energy consumption is minimized. We propose an algorithm that allows a sensor to minimize the energy use while maintaining the reliability in the network. Our algorithm uses standard linear programming techniques. Our simulations show that path diversification uses two types of diversity, which decrease energy use in the sensor network. The first type of diversity comes from the use of multiple paths, which increases reliability and decreases energy required to achieve reliable transmissions. In this case, the energy is decreased with more efficient FEC. The second type of diversity comes from the variation in energy required to transmit information on each path. As the variation increases, the sensor can increase network reliability with less efficient FEC and still decrease the energy use by using paths with lower energy consumption.

II. Sensor Network Model

We position our design within the context of existing sensor networks. We assume that our network operates with a system similar to TinyDB [6], which is a query based system where the queries are flooded through the network. The query operation is also a mechanism used to find routes to the sink.

The sink sends a query to the network which asks a subset of nodes to respond. The queries are usually of the type "All nodes with temperature higher than $15°$C respond". Each query is flooded through the network with some optimizations. For example, if the sink knows that a subset of nodes cannot answer a query it does not send the query to that subset of nodes. As the query propagates through the network, the nodes also build a routing table of routes to the sink. A query may also specify if the responses should be aggregated at one of the sensor nodes. This allows for removal of redundant information.

Normally, the query is sent from the single sink, but in our case we change the system to send the query from the different prongs of the sink at the same time. Fig. 3 illustrates an example for the query/route discovery operation. The sink floods the network by sending the query through each of the prongs. The query records the path it took on the way to the source node as well as the conditions on each hop of the path. The conditions are given as reliability (q_i) and energy parameters ($M_i^{(e)}, e_i^{(b)}$) on each hop. The information about the reliability and energy on each hop is used in the algorithm to minimize the total energy.

Each sensor node records its information in the query and forwards the query to the next set of nodes. The sensor nodes also keep track of each path that arrives at a receiver prong. If a node needs to send back a reply to the query, it selects a set of *arc-disjoint*[2] paths to the distributed receivers on which it can send the reply. For example, in Fig. 3 node 3 can use path 2 or 3 to the receiver on the right and path 1 to the node on the left. In this case node 3 should select paths 1 and 3. Since the selection scheme is suboptimal, it may happen that the source may not communicate with all of the prongs of the sink.

Each sensor node may select the paths with the shortest hop count, or with the highest reliability. The shortest path metric is not the best metric for sensor networks due to the poor quality of links. [1] shows that the shortest path metric results in the nodes using a few long hops, which decreases reliability. The selection methods, which take reliability into account, perform much better. Nevertheless, the contribution of this paper is to analyze the network once the paths have

[2]Two arc-disjoint paths have no vertices or edges in common.

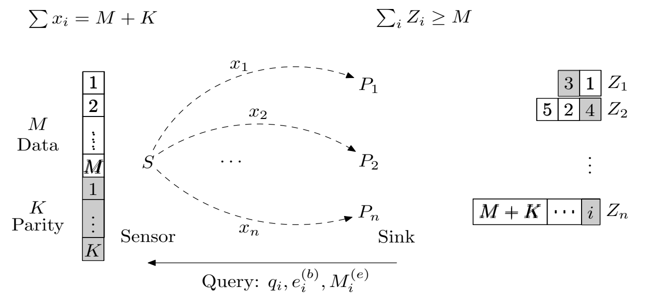

Fig. 4. Transmission to a distributed sink. $P_1, \ldots, P_n$ are the prongs of the sink. The sensor transmits $x_1, \ldots, x_n$ fragments of information to each of the n prongs of the distributed sink. The sink can reconstruct the original information if it receives more than M fragments in total.

been selected.

We note that the size of the packets sent by the nodes may be quite small, compared to packet sizes used in regular networks. However, our scheme can still work on the nodes which aggregate data from other sensor nodes. This serves a dual purpose. First, the aggregation nodes can eliminate the redundant data reported by the sensors with coding techniques [7]. Therefore, the total amount of data, which is transmitted to the sink, is reduced. Second, the aggregation nodes have more data to send than the sensors, which makes subdivision of packets on the aggregation nodes more practical.

III. Analytical Model and Assumptions

In this section, we give a mathematical model for path diversification. We assume that a sensor node generates packets of size bM bits. The packet is split into M fragments, each with size b, and an additional K parity fragments with size b are generated using a linear erasure code [2]. The source node then transmits the fragments over multiple parallel paths as shown in Fig. 4. The source transmits x_i fragments on path i where $i = 1, \ldots, n$ and n is the total number of available paths, and:

$$\sum_{i=1}^{n} x_i = \mathbf{x}^T \mathbf{1} = M + K, \tag{1}$$

where $\mathbf{x} = [x_1, x_2, .., x_n]^T$ is the allocation vector of fragments, and $\mathbf{1}$ is a vector of all 1s.

A. Fault tolerance in the network

The destination node needs to receive a total of M fragments in order to reconstruct the packet. We use random variables $\mathcal{Z}_i$ to indicate the number of fragments received on path i. So, the probability that the packet can be reconstructed is given as:

$$P_{\text{succ}} = \Pr\left[\sum_{i=1}^{n} \mathcal{Z}_i > M\right]. \tag{2}$$

P_{succ} is a function of $\mathbf{x}$ the allocation of fragments on each path, $\mathbf{q} = [q_1, \ldots, q_n]^T$ the vector indicating the probability that a fragment will be successfully transmitted on each path, and K the number of parity fragments. We will use P_{succ} and $P_{\text{succ}}(\mathbf{x}, \mathbf{q}, K)$ interchangeably in the rest of the paper.

We measure the effectiveness of the scheme, in terms of the overhead introduced by the erasure code, as:

$$\eta(\mathbf{x}, K) = \frac{\mathbf{x}^T\mathbf{1} - K}{\mathbf{x}^T\mathbf{1}} = \frac{M}{M+K} \tag{3}$$

where $\eta(\mathbf{x}, K)$ is the efficiency we can achieve for a given reliability level. The scheme is more effective as $\eta(\mathbf{x}, K)$ approaches 1. We use a lower bound on $\eta(\mathbf{x}, K)$ in the minimization of energy.

B. Energy Consumption in the Network

We assume that each sensor has the ability to measure the average amount of energy it uses to transmit a bit of information $e_i^{(k)}$ and the amount of available energy on the sensor $E_i^{(k)}$, where we have indexed the sensor as "sensor k on path i". In order to simplify the optimization, we assume that $e_i^{(k)}$ and $E_i^{(k)}$ do not change during the transmission of a single packet. The information about the energy consumption is disseminated during the query process. The query contains the information about the per-bit energy required to transfer a packet between the source and the destination on a path, $e_i^{(\text{b})}$. The per-bit energy consumption on a path can be determined by adding up the energy required to transfer a bit at every node on the path $e_i^{(\text{b})} = \sum_{k=0}^{n_i-1} e_i^{(k)}$, where n_i is the number of nodes on path i. The vector of per bit energy consumption in the network is given by $\mathbf{e}_\text{b} = [e_1^{(\text{b})}, e_2^{(\text{b})}, .., e_n^{(\text{b})}]^T$. So, the total amount of energy spent to transmit the $M+K$ fragments is given by:

$$E_{\text{Total}}(\mathbf{x}, \mathbf{e}_b) = b\mathbf{x}^T\mathbf{e}_\text{b}. \tag{4}$$

The sensor node obtains the vector $\mathbf{e}_b$ periodically when it receives the query from the sink. So, it should also perform the minimization periodically, when it receives the updates. The query also contains information about the maximum number of fragments, $M_i^{(\text{e})}$, that can be transmitted on each path before the energy on the path runs out:

$$M_i^{(\text{e})} = \min_{1 \leq k \leq n_i} \left\{ \frac{E_i^{(k)}}{b e_i^{(k)}} \right\}. \tag{5}$$

We denote with $\mathbf{M}_\text{e}$ the vector of maximum number of fragments that we can transmit on each path, i.e. $\mathbf{M}_\text{e} = [M_1^{(\text{e})}, M_2^{(\text{e})}, .., M_n^{(\text{e})}]^T$.

IV. Minimum Energy for Fault Tolerant Sensor Networks

We minimize the total consumed energy (i.e. the sum of energy use across each path in the network) with a given bound on reliability and efficiency in the network. The minimization finds the optimum number of parity fragments k needed to satisfy the reliability bounds, as well as the allocation of fragments on each path $\mathbf{x}$:

$$\underset{\mathbf{x},k}{\text{Minimize:}}\ E_{\text{Total}}(\mathbf{x}, \mathbf{e}_{\text{b}}) = b\mathbf{x}^T\mathbf{e}_{\text{b}} \tag{6a}$$

$$\text{Subject to: } P_{\text{succ}} \geq \epsilon \tag{6b}$$

$$\eta(\mathbf{x}, k) \geq \delta \tag{6c}$$

$$\mathbf{x}^T\mathbf{1} - k = M \tag{6d}$$

$$0 \preceq \mathbf{x} \preceq \mathbf{M}_{\text{e}} \tag{6e}$$

where $\preceq$ is pointwise comparison and k is in lower case since it is now a variable.

The reliability constraint (6b) is the guarantee that the packet can be reconstructed at the receiver node with some minimum probability ϵ. The efficiency constraint (6c) puts a bound on the maximum number of fragments K that can be used to achieve the reliability, i.e. $k \leq K_{\max} = M(1-\delta)/\delta$. The last two constraints take into account that the total number of fragments is $M + k$ and that the maximum number of fragments, which can be transmitted on each path (due to energy constraints), is given by $\mathbf{M}_{\text{e}}$.

We transform the optimization (6) into a linear program in two steps. First, we use the Poisson cumulative distribution function (c.d.f) to calculate the lower bound for the network reliability P_{succ} in (6b). We approximate the loss of consecutive fragments on each path to be independent and identical to each other and approximate $P_{\text{succ}}(\mathbf{x}, \mathbf{q}, k)$ with:

$$Q(\mathbf{x}, \mathbf{q}, k) \leq P_{\text{succ}}, \tag{7}$$

where

$$Q(\mathbf{x}, \mathbf{q}, k) = \sum_{j=0}^{k} \frac{e^{-\lambda(\mathbf{x})}[\lambda(\mathbf{x})]^j}{j!}, \tag{8}$$

$\ln(\mathbf{q}) = [\ln(q_1), \ldots, \ln(q_n)]^T$ and $\lambda(\mathbf{x}) = -\mathbf{x}^T \ln(\mathbf{q})$.

The approximation of P_{succ} with the Poisson c.d.f. follows from the results of [8]. The approximation (7) allows us to replace P_{succ} in (6b) with $Q(\mathbf{x}, \mathbf{q}, k)$. Since, in practice we would only be interested in fairly high values of $Q(\mathbf{x}, \mathbf{q}, k)$, we can conclude that the bound in (7) is tight. For example, if $Q(\mathbf{x}, \mathbf{q}, k) = 0.999$, the error can be at most 10^{-3}.

Second, we linearize the Poisson approximation and convert the minimization into a linear program:

$$\underset{\mathbf{x},k}{\text{Minimize:}}\ E_{\text{Total}}(\mathbf{x}, \mathbf{e}_{\text{b}}) = b\mathbf{x}^T\mathbf{e}_{\text{b}} \tag{9a}$$

$$\text{Subject to: } -\mathbf{x}^T\ln(\mathbf{q}) - s(\epsilon)k \leq c(\epsilon) \tag{9b}$$

$$0 \leq k \leq K_{\max} \tag{9c}$$

$$\mathbf{x}^T\mathbf{1} - k = M \tag{9d}$$

$$0 \preceq \mathbf{x} \preceq \mathbf{M}_{\text{e}}, \tag{9e}$$

where $s(\epsilon)$ and $c(\epsilon)$ are constants relating the Poisson c.d.f with ϵ. We show some values for $s(\epsilon)$ and $c(\epsilon)$ in Table I. The constraint (9b) is derived by linearizing (7). We give mathematical details of the linearization in Appendix I.

TABLE I

VALUES OF $s(\epsilon)$ AND $c(\epsilon)$

ϵ	$\log_{10}\left(\frac{\epsilon}{1-\epsilon}\right)$	$s(\epsilon)$	$c(\epsilon)$
0.9	1.0	0.940	-5.21
0.99	2.0	0.896	-8.86
0.999	3.0	0.862	-11.3
0.9999	4.0	0.835	-13.0

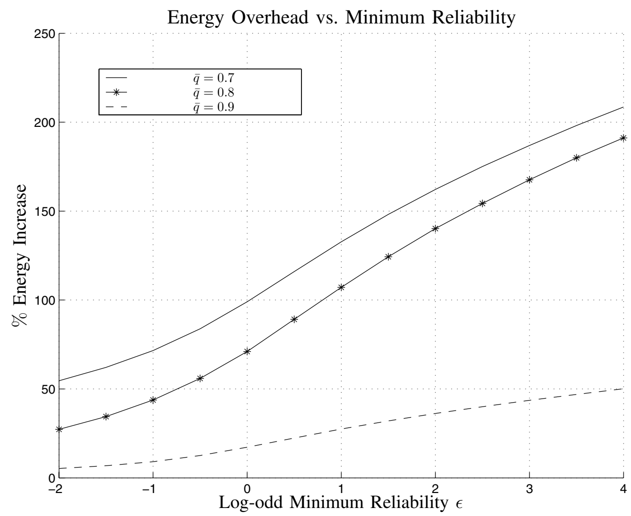

Fig. 5. Energy Overhead vs. Minimum Reliability ϵ.

V. RESULTS

We simulated a network in which there is a large number of sensors and few sinks. In each simulation, the source performs 1000 packet transmissions to the prongs of the sinks. The conditions in the network (q_i and $e_i^{(b)}$) change before every transmission and the sensor has perfect knowledge of these changes.

In our first simulation, we model the channel as a binary symmetric channel (BSC) with the average probability of fragment success $\bar{q}$. Fig. 5 shows the effect that the increase in reliability has on energy consumption. We plot the percent increase in energy consumption — from the minimum energy — when the minimum reliability ϵ changes. The minimum energy is achieved when the sensor does not use any parity fragments to send the information to the sink, i.e. the optimization is constrained with (6e) only. In this case, $P_{\text{succ}} = \bar{q}$. We show ϵ in the log-odd format where we plot ϵ as $\log(\epsilon/(1-\epsilon))$. The log-odd scale allows us to map the set $[0, 1]$ uniformly to the set $[-\infty, \infty]$, so that we can observe the asymptotic effect when $\epsilon \to 0$ or $\epsilon \to 1$. We can see from Fig. 5 that with less than three times the increase in total energy consumption, we can make the minimum reliability $\epsilon > 0.999$ even for channels where the probability of fragment loss is 20% on average.

In our second simulation, we model the reliability on the path with a Markov-Chain model, where the channel's fragment success rate is changing through the following states

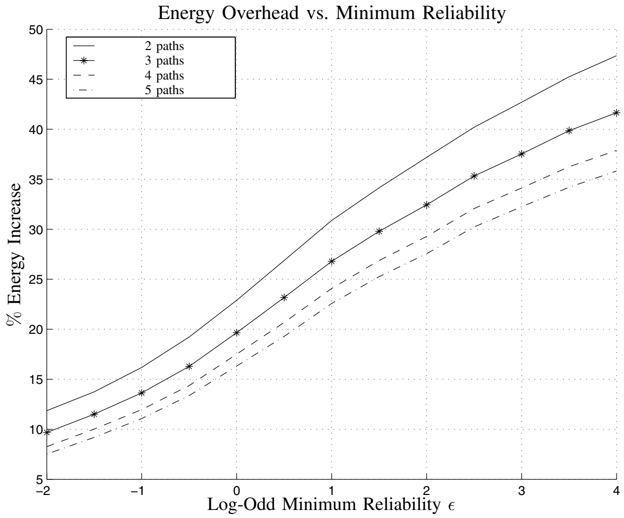

Fig. 6. Energy Overhead vs. Minimum Reliability ϵ.

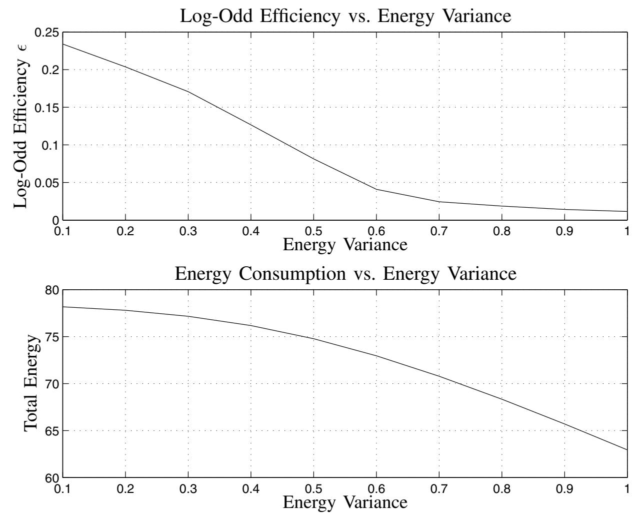

Fig. 7. Efficiency vs. Energy Variance.

$q = [0.7, 0.8, 0.9]$, to give an average fragment success rate of $\bar{q} = 0.7$. Fig. 6 shows the effect of the increased number of prongs on energy consumption. We observe from Fig. 6 that as the number of prongs increases the energy use decreases. Indeed, the distributed sink allows the sensor to take advantage of the diversity available in the network, which decreases the energy consumption. The diversity comes from the variation in the reliability of the paths and so the sensor can increase the reliability by using better paths without increasing the number of parity fragments K.

Fig. 7 illustrates the effect of variance of energy consumption on the efficiency η. Efficiency η and total energy E_{Total} are plotted for the fragment allocation that has the minimum energy consumption. We can see that the efficiency does not depend on the minimum reliability alone. The sensor can decrease the energy consumption with transmissions which use more parity fragments; these transmissions decrease efficiency. This is because the sensor can send most of the fragments on the path with the lowest energy and still achieve the desired reliability by increasing the number of parity fragments used in the transmission. The increased variance of energy consumption means that there is a higher likelihood that a path with low energy consumption also has a fragment success rate that allows the sensor to achieve the reliability bound.

APPENDIX I
LINEARIZATION OF NETWORK RELIABILITY

We transform $Q(\mathbf{x}, \mathbf{q}, k)$ into a linear function by noting that $Q(\mathbf{x}, \mathbf{q}, k)$ is a monotonically decreasing function of $\lambda(\mathbf{x})$. It can be easily shown that:

$$\frac{\partial Q(\mathbf{x}, \mathbf{q}, k)}{\partial x_i} = -\frac{\partial \lambda}{\partial x_i} e^{-\lambda(\mathbf{x})} \frac{[\lambda(\mathbf{x})]^k}{k!} < 0 \tag{10}$$

which means that $Q(\mathbf{x}, \mathbf{q}, k)$ is a decreasing function of $\lambda(\mathbf{x})$. So, for a given k there exists $\alpha_\epsilon(k)$ such that:

$$\lambda(\mathbf{x}) \leq \alpha_\epsilon(k) \leftrightarrow Q(\mathbf{x}, \mathbf{q}, k) \geq \epsilon \rightarrow P_{\text{succ}} \geq \epsilon. \tag{11}$$

When we plot $\alpha_\epsilon(k)$ for a range of ϵ, we notice that for $k > 5$, $\alpha_\epsilon(k)$ is almost a linear function. So, we approximate $\alpha_\epsilon(k)$ as:

$$\alpha_\epsilon(k) \approx s(\epsilon)k + c(\epsilon). \tag{12}$$

In our simulations, we have used the least-square method to obtain the slope $s(\epsilon)$ and constant $c(\epsilon)$. We show some of the values, for $s(\epsilon)$ and $c(\epsilon)$, in Table I.

We have examined this approximation more closely in [3] where we have shown that the increase in minimum energy in (9) over the non-linear optimization (6) is almost negligible. At the same time, the linear constraint decreases the complexity of the optimization in the order of $\Theta(M + K_{\max})$.

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