Minimum Energy Reliable Multipath Ad Hoc Networks

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***Abstract*— We give an edge-based mechanism to provide reliable transport services and minimize the total energy in ad hoc networks. The implementation is based on *path diversification*, which uses spatial diversity in the network through multipath routing, and erasure codes to provide guaranteed reliability. We use a cross-layer design where higher network layers use information about packet loss and energy consumption to distribute the load in the network. We find a lower bound on the reliability in the network and use it to allocate network resources to minimize energy while the reliability of the network is guaranteed.**

I. INTRODUCTION

Ad hoc networks have unreliable end-to-end connections and limited battery power. The unreliable end-to-end connections make it difficult to provide QoS guarantees, such as maximum packet loss. The limited battery power makes it even harder to provide guaranteed QoS since any scheme that provides QoS guarantees must take into account its impact on the energy consumption in the network. In this paper, we give an edge-based mechanism that can guarantee maximum packet loss (reliability). In order to limit the impact of the mechanism on energy consumption, we minimize the total energy each connection must use to achieve the level of reliability requested by that connection.

The difficulty in implementing end-to-end QoS in ad hoc networks comes from unreliable connections due to channel characteristics and node mobility. For example, TCP algorithm is ineffective in cellular wireless networks [1] and in ad hoc networks due to high packet loss rates [2]; reliability is difficult to maintain with routing alone due to high node mobility [3]; and QoS reservation schemes are also ineffective due to node mobility [4]. We use *path diversification* as a more robust way to deal with the reliability problems in ad hoc networks.

II. PATH DIVERSIFICATION

Path diversification uses multiple parallel paths and erasure codes to provide reliability in the network. The mechanism encodes the data packets into fragments with an erasure code and distributes the fragments on multiple parallel paths in the network. The erasure code allows the destination node to reassemble the packets even if some of the fragments are lost. Path diversification allows us to use a cross-layer approach to provide end-to-end QoS in ad hoc networks. First, it uses spatial diversity available in relatively dense ad hoc networks to increase robustness of packet transmissions through multipath routing protocols [5]. Second, it uses cross-layer information collection to optimize the operation of the scheme in terms of reliability and energy consumption.

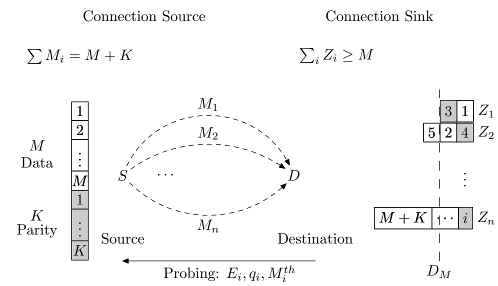

Fig. 1. Path Diversification

In path diversification, the source node encodes each packet of M *fragments* into $M+K$ fragments of the original data and parity, using a linear erasure code [6]. The destination can recover the packet if it receives more than M fragments. In addition to the erasure code, the source node also transmits the fragments over multiple parallel paths. In order to maximize spatial diversity in the network, we assume that the paths are chosen so that they have no nodes or edges in common (i.e. the paths are *arc-disjoint*).

We illustrate how path diversification works in Fig. 1. The connection supplies a packet of size bM bits every D_M seconds. The source node splits the packet into M fragments each with size b and generates K parity fragments, also with size b. The source node distributes the fragments on the n paths available for transmission, and transmits M_i fragments on path i. In this paper, we will show that path diversification provides an effective way to increase reliability.

III. NETWORK MODEL AND ASSUMPTIONS

We envision path diversification as a sub-layer of the network layer in which routing is implemented with a multipath routing protocol [5]. The network layer provides an interface which allows the path diversification module to access multiple paths in the network. The connections interact with the path diversification module to transmit their packets. The module allows specification of the level of reliability, ϵ, and the maximum cost the connection is willing to pay for the reliability, the overhead K. The path diversification module node uses this information when it allocates the fragments on the paths to minimize energy consumption. In the rest of this section, we introduce network parameters and quantities we will use for optimizations in Section IV.

A. Network Reliability (Packet Loss)

We define the level of network reliability. P_{succ}, as the probability that a packet can be reconstructed at the receiver. We use the Poisson cumulative distribution function (c.d.f) [7] to derive the lower bound on P_{succ}[1]:

$$Q(\mathbf{m}, K) \leq P_{\text{succ}} \leq Q(\mathbf{m}, K) + \frac{M_i}{2} \sum_{i=1}^{n} \ln^2(q_i) \tag{1}$$

where,

$$Q(\mathbf{m}, K) = \sum_{j=0}^{K} \frac{e^{-\lambda(\mathbf{m})} [\lambda(\mathbf{m})]^j}{j!} \text{ and } \lambda(\mathbf{m}) = -\sum_{i=1}^{n} M_i \ln[q_i], \tag{2}$$

and p_i is the probability that a fragment will be lost in a transmission on path i and $q_i = 1 - p_i$.

The upper bound in (1) is quite loose, especially if the total number of fragments $M + K$ is large. However, in practice we would only be interested in fairly high values of $Q(\mathbf{m}, K)$ which would make the actual error quite small. For example, if $Q(\mathbf{m}, K) = 0.999$, the error can be at most 10^{-3} regardless of the upper bound in (1).

B. Energy Consumption in the Network

We assume that every node in the network has the ability to measure the average amount of energy it uses to transmit a bit of information $e_i^{(k)}$ and the amount of available energy on the node $E_i^{(k)}$, where we have indexed the node as "node k on path i". In order to simplify the optimization, we assume that $e_i^{(k)}$ and $E_i^{(k)}$ do not change during the packet transmission. For example, each node in the network can assign a constant power level and the amount of energy available to each connection.

The destination can send a probe to the source node to inform it about the per-bit energy required to transfer a packet between the source and the destination on a path, $e_i^{(\text{b})}$. The per-bit energy consumption on a path can be determined by adding up the energy required to transfer a bit at every node on the path $e_i^{(\text{b})} = \sum_{k=0}^{n_i-1} e_i^{(k)}$, where n_i is the number of nodes on path i. The vector of per bit energy consumption in the network is given by $\mathbf{E}_{\text{b}} = [e_1^{(\text{b})}, e_2^{(\text{b})}, .., e_n^{(\text{b})}]^T$. So, the total amount of energy spent to transmit the $M + K$ fragments is given by:

$$E_{\text{Total}}(\mathbf{m}, \mathbf{E}_b) = b\mathbf{m}^T \mathbf{E}_{\text{b}}. \tag{3}$$

The source node obtains the vector $\mathbf{E}_b$ periodically from the network measurements. So, the source node should also perform the minimization periodically, when it receives the updates.

With network probing, we also have access to the maximum number of fragments, $M_i^{(\text{e})}$, that can be transmitted on each path before the energy on the path runs out:

$$M_i^{(\text{e})} = \min_{1 \leq k \leq n_i} \left\{ \frac{E_i^{(k)}}{b e_i^{(k)}} \right\}. \tag{4}$$

We denote with $\mathbf{M}_{\text{e}}$ the vector of maximum number of fragments that we can transmit on each path, i.e. $\mathbf{M}_{\text{e}} = [M_1^{(\text{e})}, M_2^{(\text{e})}, .., M_n^{(\text{e})}]^T$.

IV. Minimum Energy Reliable Ad Hoc Networks

We minimize the the total consumed energy (i.e. the sum of energy use across each path in the network) with a given bound on reliability and efficiency in the network. The minimization finds the optimum number of parity fragments K needed to satisfy the reliability bounds, as well as the allocation of fragments on each path $\mathbf{m} = [M_1, M_2, \ldots, M_n]$:

$$\underset{\mathbf{m},K}{\text{Minimize:}}\ E_{\text{Total}}(\mathbf{m}, \mathbf{E}_{\text{b}}) = b\mathbf{m}^T \mathbf{E}_{\text{b}} \tag{5a}$$
$$\text{Subject to: } P_{\text{succ}} \geq \epsilon \tag{5b}$$
$$\eta(\mathbf{m}, K) \geq \delta \tag{5c}$$
$$\mathbf{m}^T \mathbf{1} - K = M \tag{5d}$$
$$0 \preceq \mathbf{m} \preceq \mathbf{M}_{\text{e}} \tag{5e}$$

The reliability constraint (5b) is the guarantee that the packet can be reconstructed at the receiver node with some minimum probability ϵ. The efficiency constraint (5c) puts a bound on the maximum number of fragments K that can be used to achieve the reliability, i.e. $\eta(\mathbf{m}, K) = M/(M + K)$.

We solve the minimization in two ways. First, we assume that the source has full knowledge of fragment loss on each path. In this case, we use the Poisson cumulative distribution function (c.d.f) to give the lower bound for the network reliability P_{succ}. This allows us to convert the minimization into a linear program:

$$\underset{\mathbf{m},K}{\text{Minimize:}}\ E_{\text{Total}}(\mathbf{m}, \mathbf{E}_{\text{b}}) = b\mathbf{m}^T \mathbf{E}_{\text{b}} \tag{6a}$$
$$\text{Subject to: } -\mathbf{m}^T \ln(\mathbf{q}) - s(\epsilon)K \leq c(\epsilon) \tag{6b}$$
$$0 \leq K \leq K_{\max} \tag{6c}$$
$$\mathbf{m}^T \mathbf{1} - K = M \tag{6d}$$
$$0 \preceq \mathbf{m} \preceq \mathbf{M}_{\text{th}}. \tag{6e}$$

where $\ln(\mathbf{q}) = [\ln(q_1), \ldots, \ln(q_n)]$ and $s(\epsilon)$ and $c(\epsilon)$ are constants relating the Poisson c.d.f and the parameter ϵ[2].

Second, we enhance the optimization by adding a robust bound for the reliability in the network. We modify (6) to include the uncertainty of fragment loss information. The resulting optimization becomes a robust linear program [9] which can be solved with convex programming. We add robustness for the reliability constraint by noting that the probability that reliability is lower than what is given by (6b) is equal to the probability that $\lambda(\mathbf{m}) > s(\epsilon)K + c(\epsilon) = \alpha(K)$,

[1] We give mathematical details of this discussion in [8].

[2] We derive bound in (6b) from (1). We give mathematical details of the linearization in [8]. We also show there that even though the solution with the approximation is sub-optimal, the difference is almost negligible.

i.e.:

$$\Pr[P_{\text{succ}} < \epsilon] = \Pr[\lambda(\mathbf{m}) > \alpha(K)]. \tag{7}$$

So, we can replace the reliability bound (6b) with a confidence bound on the reliability. We note by (2) that $\lambda(\mathbf{m})$ is a sum of continuous random variables, so by the Central Limit Theorem (C.L.T.) we can approximate $\lambda(\mathbf{m})$ as a Normal random variable with mean μ_λ and variance σ_λ^2. With the confidence bound, the optimization is given as the following mathematical program:

$$\underset{\mathbf{m},K}{\text{Minimize:}}\ E_{\text{Total}}(\mathbf{m}, \mathbf{E}_{\text{b}}) = b\mathbf{m}^T\mathbf{E}_{\text{b}} \tag{8a}$$

$$\text{Subject to: } \mathbf{m}^T\mu + \Phi^{-1}(\gamma)\|\boldsymbol{\sigma} \circ \mathbf{m}\| \leq -\alpha(K) \tag{8b}$$

$$K_{\min} \leq K \leq K_{\max} \tag{8c}$$

$$\mathbf{m}^T\mathbf{1} - K = M \tag{8d}$$

$$0 \preceq \mathbf{m} \preceq \mathbf{M}_{\text{e}} \tag{8e}$$

where $\|x\| = \sqrt{x^T x}$ is the vector norm, and $\circ$ is the pointwise (Hadamard) matrix multiplication.

This is a Second-Order Conic Program (SOCP) that can be solved with convex programming [9]. In fact, we could also relax the bound on the reliability of the energy information required for the program and use the C.L.T. approximation to add a confidence bound on the minimum of the energy consumption. The robust bound allows us to validate our claim on the guaranteed reliability in the network, by taking into account that the information about the paths may be inaccurate. The robust bound transforms the minimization into a Second-Order Conic Program that can be solved with Convex programming.

V. Results

We use numerical simulations to give results for minimization of energy consumption. We show the effect of improving network reliability on the energy consumption in Fig. 2. We use packet loss ratio on the paths ϵ as a measure of reliability. We plot the percent increase from the minimum energy when ϵ changes. The minimum energy is achieved when the problem is optimized with no reliability constraints. We give the plot in log-odd format where we plot α as $\log(\alpha/(1-\alpha))$. The log-odd scale allows us map the set $[0, 1]$ uniformly to the set $[-\infty, \infty]$, so that we can observe the asymptotic effect when $\epsilon \to 0$ or $\epsilon \to 1$. We can see from Fig. 2 that with less than twice the increase in total energy consumption, we can make the minimum reliability $\epsilon > 0.999$ even for channels where the probability of fragment loss is 20% on average.

We show the effect of variance of energy consumption on the efficiency in Fig. 3. Efficiency is plotted for the packet allocation that has the minimum energy consumption. We can see that the efficiency does not depend on the minimum reliability alone. In fact, the variance of energy consumption has a great impact on the traffic efficiency we can expect in the network. Fig. 3 also shows that when the variance in energy consumption is large, we can decrease energy by adding more parity fragments. This is because there is a higher likelihood that a path with low energy consumption also has higher reliability.

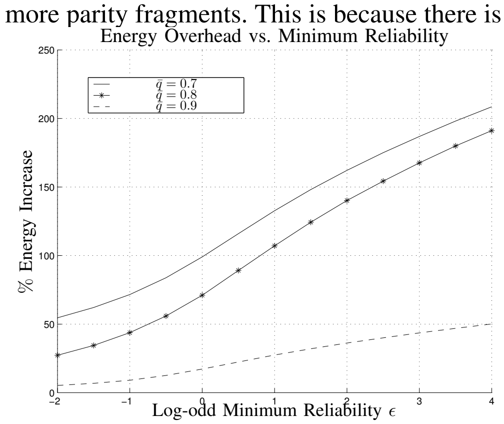

Fig. 2. Energy Overhead vs. Minimum Reliability ϵ

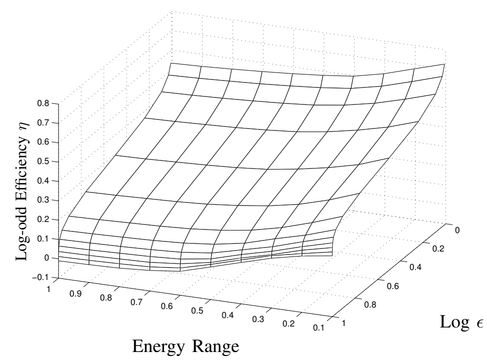

Fig. 3. Efficiency at Minimum Energy

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