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Received Signal Strength based Indoor Positioning using Compressive Sensing

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Abstract—The recent growing interest for indoor Location-Based Services (LBSs) has created a need for more accurate and real-time indoor positioning solutions. The sparse nature of location finding makes the theory of Compressive Sensing (CS) desirable for accurate indoor positioning using Received Signal Strength (RSS) from Wireless Local Area Network (WLAN) Access Points (APs). We propose an accurate RSS-based indoor positioning system using the theory of compressive sensing, which is a method to recover sparse signals from a small number of noisy measurements by solving an ℓ_1-minimization problem. Our location estimator consists of a *coarse* localizer, where the RSS is compared to a number of clusters to detect in which cluster the node is located, followed by a *fine* localization step, using the theory of comperssive sensing, to further refine the location estimation. We have investiagted different coarse localization schemes and AP selection approaches to increase the accuracy. We also show that the CS theory can be used to reconstruct the RSS radio map from measurements at only a small number of fingerprints, reducing the number of measurements significantly. We have implemented the proposed system on a WiFi-integrated mobile device and have evaluated the performance. Experimental results indicate that the proposed system leads to substantial improvement on localization accuracy and complexity over the widely used traditional fingerprinting methods.

Index Terms—Indoor positioning, Fingerprinting, Compressive sensing, Clustering, Radio map, WLANs.

1 Introduction

RECENT advances in smartphones have made it feasible to provide indoor Location-Based Services (LBSs) such as indoor positioning, tracking, navigation, and location-based security [1][2]. However, due to the complexity of the indoor environment, it is usually difficult to provide a satisfactory level of accuracy in most applications. Thus, one of the key challenges is to design accurate and real-time indoor positioning systems that can be easily deployed on commercially available mobile devices without any hardware installation or modification.

Received Signal Strength (RSS)-based localization algorithms have been extensively studied as an inexpensive solution for indoor positioning in recent years [3][4][5][6]. Compared with other measurement-based algorithms (*e.g.*, time-of-arrival (TOA) or angle-of-arrival (AOA) measurements of ultra-wideband (UWB) signals [7]), RSS can be easily obtained by a WiFi-integrated mobile device, without any additional hardware. Several RSS-based indoor positioning and tracking algorithms have been proposed using the location information of access points (APs), which may not be available or hard to obtain in practice [8]. The positioning scheme proposed in this paper only measures RSS readings from available APs, without knowing their location in advance.

The major challenge for accurate RSS-based positioning comes from the variations of RSS due to the dynamic and unpredictable nature of radio channel, such as shadowing, multipath, the orientation of wireless device, etc [9]. Thus, instead of using a propagation model to describe the relationship between RSS and position [10], a pre-built radio map is used in *fingerprinting* methods to localize a Wi-Fi device [11][12]. The position of a mobile user is estimated by comparing online RSS readings with offline observations. One simple solution is the k-nearest neighbor algorithm (kNN), which estimates the mobile user's location by computing the centroid of the k closest neighbors that have the smallest Euclidean distance to the online RSS reading [13][14]. Such a system is easy to implement but the estimation is not very accurate.

Another solution to the fingerprinting approach is to solve the problem by a statistical method, in which the probability of each potential position is analyzed using the Bayesian theory and kernel functions [5][15], assuming that the RSS readings from different APs are independent at every time instant. However, an explicit formulation of RSS distribution is challenging and the independence may not hold in real environments. Meanwhile, these probabilistic-based systems often have high computational complexity, which makes it difficult to run on mobile devices with limited processing power and small memory.

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In this paper, we use the theory of Compressive Sensing (CS) to match the signal strength measured by the mobile phone to the fingerprint database. Compressive sensing provides a novel framework for recovering sparse or compressible signals with far fewer noisy measurements than that needed by the Nyquist sampling theorem [16][17][18]. The sparse signal can be reconstructed exactly with high probability by solving an ℓ_1-minimization problem [19][20].

The localization problem can be modeled as a sparse problem since at each time instant the user is located at a specific point in space. If an identifier function can be formed for the localization problem, it takes the value 1 at the position of the mobile user and 0 elsewhere. The sparse nature of location estimation in the spatial domain motivates us to exploit the CS theory for indoor positioning system [21][22][23], which offers exact deterministic recovery using linear programming that can be computed on mobile devices in real-time.

We also show that the theory of compressive sensing can be used to reconstruct the radio map based on RSS measurements at only a subset of fingerprints, reducing the number of measurements significantly for updating the database in real applications. This is an important property since collecting and maintaining an accurate radio map is a labour intensive operation, which needs to be repeated every time that the number or the power of WiFi access points in the environment changes significantly.

The proposed localizer consists of two phases: an *offline phase*, and an *online phase*. In the off-line phase, RSS readings are collected on a grid of reference points. The RSS readings are then decomposed into multiple clusters using the *affinity propagation* algorithm, and the outliers are identified and adjusted accordingly. The online phase consists of the mobile device measuring RRS, using a *coarse* localizer to find the clusters to which it belongs, and a *fine* location estimation using CS. In this paper, different coarse localization metrics are investigated to reduce the maximum error of the positioning system. In the fine localization stage, AP selection schemes are studied to further improve the accuracy of the estimation.

We have implemented the proposed positioning system on a Personal Digital Assistant (PDA) with Windows Mobile 2003 to evaluate the performance. The positioning accuracy, the computational complexity, and the use of memory on resource limited devices are considered when designing the system. The coarse localization stage reduces the computational time for solving the ℓ_1-minimization problem, which allows this procedure to be executed on mobile devices. In the actual implementation, the process latency for location estimation is in the order of 100msec on a PDA with 624 MHz processor and 64M RAM. We have shown that the proposed system is able to estimate the location in real-time with an average error of 1.5 m on the resource limited PDA.

The remainder of this paper is organized as follows. Section 2 sets up the problem and describes the proposed two-phase positioning scheme in detail. Section 3 reconstructs the radio map by the theory of CS to reduce the redundancy of collecting fingerprints. The performance is evaluated through implementations in Section 4. Finally, Section 5 concludes the paper.

2 Indoor Positioning System

We start with a typical WLAN positioning scenario, where a user carries a mobile device equipped with a WLAN adapter, taking RSS measurements from available APs in an indoor environment. The location of these APs is unknown. The main task of the positioning system is to estimate and illustrate the user's current location on a map (floor plan) on the device, by only using RSS readings.

The location of the mobile is estimated by comparing the current RSS reading to a prestored database called the *fingerprints*, which is the table of measured RSS for a similar device over a grid of points on the map. Several methods can be suggested to compare the RSS reading and the fingerprints. In this paper, we use the theory of compressive sensing to find the best match between the received signal strength and the fingerprint database. The proposed CS-based localization scheme reformulates the location finding problem into a sparse signal recovery problem and thus finds the location estimation accurately by solving a linear program.

As depicted in Fig. 1, the proposed compressive sensing-based positioning system consists of two phases: an *offline phase*, in which RSS samples at specific positions within the area of interest are collected; and an *online phase*, which performs the actual localization. The offline phase is composed of a clustering scheme using the *affinity propagation*, followed by outlier adjustment. The online phase comprises two stages: the coarse localization stage, which reduces the area of interest into a smaller region by using cluster matching, and the fine localization stage that uses the theory of compressive sensing to estimate the actual location. The individual blocks are described in details in the following subsections.

2.1 Offline Phase

2.1.1 Collecting Database

During an offline phase, the time samples of RSS readings are collected at known locations, referred to as the *Reference Points* (RPs), by pointing the mobile device to different orientations (*i.e.*, north, south, east, west). The raw set of RSS time samples collected from AP i at RP j with orientation o is denoted as $\{\psi_{i,j}^{(o)}(\tau), \tau = 1, ..., q, q > 1\}$, with the q representing

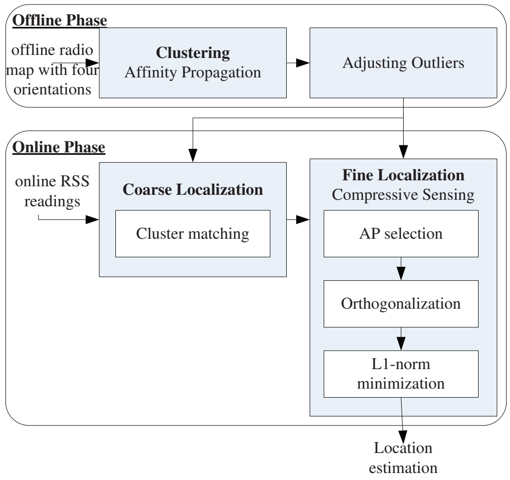

Fig. 1: Block diagram of the proposed indoor positioning system.

the total number of time samples collected. Then, the average of the RSS time samples is computed and stored in a database, known as the *radio map*. Such radio map can be represented by $\mathbf{\Psi}^{(o)}$:

$$\mathbf{\Psi}^{(o)} = \begin{pmatrix} \psi_{1,1}^{(o)} & \psi_{1,2}^{(o)} & \cdots & \psi_{1,N}^{(o)} \\ \psi_{2,1}^{(o)} & \psi_{2,2}^{(o)} & \cdots & \psi_{2,N}^{(o)} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{L,1}^{(o)} & \psi_{L,2}^{(o)} & \cdots & \psi_{L,N}^{(o)} \end{pmatrix} \tag{1}$$

where $\psi_{i,j}^{(o)} = \frac{1}{q}\sum_{\tau=1}^{q} \psi_{i,j}^{(o)}(\tau)$ is the average of RSS readings (in dBm scale) over time domain from AP i at RP j with orientation o, for $i = 1, 2, ..., L$, $j = 1, 2, ..., N$, and $o \in \mathcal{O} = \{0°, 90°, 180°, 270°\}$. L is the total number of APs that can be detected, and N is the number of RPs. The columns of $\mathbf{\Psi}^{(o)}$, radio map vectors, represent the RSS readings at each RP with a particular orientation o, which can be referred to as

$$\boldsymbol{\psi}_j^{(o)} = [\psi_{1,j}^{(o)}, \psi_{2,j}^{(o)}, \cdots, \psi_{L,j}^{(o)}]^T, j = 1, 2, ..., N \tag{2}$$

where the superscript T denotes transposition.

The variance vector for each RP is defined as

$$\mathbf{\Delta}_j^{(o)} = [\Delta_{1,j}^{(o)}, \Delta_{2,j}^{(o)}, \cdots, \Delta_{L,j}^{(o)}]^T, j = 1, 2, ..., N \tag{3}$$

where $\Delta_{i,j}^{(o)} = \frac{1}{q-1}\sum_{\tau=1}^{q}(\psi_{i,j}^{(o)}(\tau) - \psi_{i,j}^{(o)})^2$ is the unbiased estimated variance of RSS readings from AP i at RP j with orientation o. The variance can be used to select the APs that should be included in the localization scheme. The radio map is then the table $(x_j, y_j;\ \boldsymbol{\psi}_j^{(o)}, \mathbf{\Delta}_j^{(o)}), j = 1, ..., N, o \in \mathcal{O}$, where (x_j, y_j) is the coordinates of the jth RP. If no RSS reading is found for an AP at a RP, the corresponding RSS entity in the radio map is set to a small value (*e.g.*, -110 dBm in our implementation) to imply its invalidity.

2.1.2 Clustering by Affinity Propagation

The RPs collected in the offline phase are divided into a number of clusters. Since the database at different orientations has a different set of RSS readings, the clustering is performed independently for each orientation. The affinity propagation algorithm [24] is used to generate the clusters, as it does not require initialization of exemplars in the traditional K-means clustering algorithm [25]. Affinity propagation considers all RPs equally as potential exemplars by assigning the same real number, known as *preference*, for each RP as an input. Then, real-valued messages are recursively transmitted between pairs of RPs based on a measure of similarity, until exemplars and corresponding clusters are generated. The pairwise similarity $s(i,j)^{(o)}$ indicates how well the RP j is suited to be the exemplar for RP i, which in this paper is defined as

$$\begin{aligned} s(i,j)^{(o)} &= - \parallel \boldsymbol{\psi}_i^{(o)} - \boldsymbol{\psi}_j^{(o)} \parallel^2, \\ \forall i, j &\in \{1, 2, ..., N\}, \quad j \neq i, \quad o \in \mathcal{O} \end{aligned} \tag{4}$$

The self-similarity value $s(j,j)^{(o)}, j = 1, 2, ..., N$, indicates the possibility that RP j may become an exemplar. Since all the RPs are equally desirable to be exemplars, their preferences are set to a common value. In order to generate a moderate number of clusters, the common preference for each orientation is defined as

$$p^{(o)} = \gamma^{(o)} \cdot \text{median}\{s(i,j)^{(o)}, \forall i, j \in \{1, 2, ..., N\}, j \neq i\} \tag{5}$$

where $\gamma^{(o)}$ is a real number which is experimentally determined, such that a desired number of clusters is generated (see parameter settings for implementing the positioning system in Table 1, Section 4.1). Its effect on the complexity and the accuracy of the positioning system will be discussed in Section 4.2 .

The core operation of the algorithm is the transmission of two kinds of real-valued messages between pairs of RPs. The *responsibility message* $r(i,j)^{(o)}$, sent from RP i to candidate exemplar RP j, is given by

$$r(i,j)^{(o)} = s(i,j)^{(o)} - \max_{j' \neq j} \left\{ a(i,j')^{(o)} + s(i,j')^{(o)} \right\} \tag{6}$$

where $i \neq j$, and the *availability message* $a(i,j)^{(o)}$, sent from candidate exemplar RP j to RP i, is defined as

$$a(i,j)^{(o)} = \min\{0, r(j,j)^{(o)} + \sum_{i' \neq i,j} \max\{0, r(i',j)^{(o)}\}\}. \tag{7}$$

The messages are passed recursively between pairs of RPs within each radio map and the above updating rules are followed until a good set of exemplars and corresponding clusters emerges.

This process is conducted after the fingerprints are collected during the offline phase. For each radio map with a particular orientation o, let $\mathcal{H}^{(o)}$ be the set of exemplars; and for each RP $j \in \mathcal{H}^{(o)}$, let $\mathcal{C}_j^{(o)}$ denote the

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set of RPs for which RP j is an exemplar. We further adjust each outlier, referred to as a RP, which is in the set of $\mathcal{C}_j^{(o)}$ but physically far away from its exemplar j on the map, by assigning a new exemplar that is in its close proximity. Therefore, using the set of exemplars and their corresponding radio map vector, we will propose a coarse localization procedure to select the clusters that match the online RSS observations, and then the RPs of these candidate clusters will be used to localize the mobile device during the fine localization stage.

2.2 Online Phase

The actual localization of the mobile device takes place in the online phase. During the online phase, an RSS measurement vector, denoted as

$$\boldsymbol{\psi}_r = [\psi_{1,r}, \cdots, \psi_{L,r}]^T \tag{8}$$

where $\{\psi_{k,r}, k = 1, ..., L\}$, is collected by the mobile device in an arbitrary orientation. As shown in Fig. 1, there are two stages in the online phase, the coarse localization by cluster matching to reduce the area of interest; and the fine localization by using compressive sensing to recover the location estimation.

2.2.1 *Coarse Localization by Cluster Matching*

The goal of the coarse localization stage is to reduce the region of interest from the whole fingerprint database to a subset of it. Thus, it removes outliers, and reduces the computational complexity of the fine localization stage, as fewer RPs are considered. Furthermore, it confines the maximum localization error to the size of this subset, whereas this error can be much larger when no coarse localization is implemented.

The coarse localization is operated by comparing the similarity between the online RSS measurement vector and each exemplar to identify the cluster to which the online readings belong. Instead of selecting one cluster, we keep a few best-matched exemplars $\mathcal{S}$ with their corresponding cluster member set $\mathcal{C}$ to avoid the edge problem, which can lead to inaccurate estimation when the location of the mobile device is at the cluster boundaries. Meanwhile, due to the time variation of the RSS, the online measurement can deviate from the values stored in the database. Four coarse localization schemes are investigated in this paper to define the appropriate similarity function. The clusters with the largest similarity values are selected as the candidate clusters.

- *Criterion I - Similarity to the RSS of exemplar*
  Similar to (4), the similarity function is defined as the negative of Euclidean distance of the online measurement vector $\boldsymbol{\psi}_r$ to the individual exemplar's RSS radio map vector.

$$s(r,j)^{(o)} = - \parallel \boldsymbol{\psi}_r - \boldsymbol{\psi}_j^{(o)} \parallel^2, \forall j \in \mathcal{H}^{(o)}, \forall o \in \mathcal{O}. \tag{9}$$

- *Criterion II - Similarity to the averaged RSS of cluster members*
  Instead of using the RSS radio map vector of each exemplar for cluster matching, the average of the RSS radio map vectors of all the cluster members is used to average out the possible RSS variations that come from a specific exemplar. It gives a more comprehensive and representative readings of the cluster. In this case, the Euclidean distance of the online measurement vector $\boldsymbol{\psi}_r$ to the jth cluster can be computed by:

$$s(r,j)^{(o)} = - \parallel \boldsymbol{\psi}_r - \boldsymbol{\psi}_c \parallel^2, \tag{10}$$

  with

$$\boldsymbol{\psi}_c = \frac{1}{|\mathcal{C}_j^{(o)}|} \sum_{k \in \mathcal{C}_j^{(o)}} \boldsymbol{\psi}_k^{(o)}, \forall j \in \mathcal{H}^{(o)}, \forall o \in \mathcal{O} \tag{11}$$

  where $|\mathcal{C}_j^{(o)}|$ denotes the number of members in the jth cluster
- *Criterion III - Similarity to the weighted average RSS of cluster members*
  Since the variance of the RSS readings from each AP at each RP is calculated during the offline phase, the stability of the RSS readings from a specific AP within a certain cluster can be considered as a weight for cluster matching. In this scheme, different weights are added to the above similarity function for each AP, so that stable RSS readings have larger weights. The corresponding similarity function is defined as

$$s(r,j)^{(o)} = - \parallel \boldsymbol{\omega}_j^{(o)} \odot (\boldsymbol{\psi}_r - \boldsymbol{\psi}_c) \parallel^2 \tag{12}$$

  where $\boldsymbol{\psi}_c$ is the same as the one defined in (10) and $\odot$ is the element-wise multiplication between two vectors. $\boldsymbol{\omega}_j^{(o)} = [\omega_{1,j}^{(o)}, \cdots, \omega_{l,j}^{(o)}, \cdots, \omega_{L,j}^{(o)}]^T$, $l \in \{1, \cdots, L\}$, where $\omega_{l,j}^{(o)}$ represents the weight for the RSS reading from AP l in the cluster j with orientation o, which is proportional to the inverse of the corresponding RSS variance, namely,

$$\omega_{l,j}^{(o)} \propto \frac{1}{\bar{\Delta}_{l,j}^{(o)}}, \text{ and } \bar{\Delta}_{l,j}^{(o)} = \frac{1}{|\mathcal{C}_j^{(o)}|} \sum_{k \in \mathcal{C}_j^{(o)}} \Delta_{l,k}^{(o)}. \tag{13}$$

  The weights are normalized, so that $\sum_{l=1}^{L} \omega_{l,j}^{(o)} = 1$.
- *Criterion IV - Similarity using the strongest APs*
  Since the strongest APs provide the highest probability of the coverage over time, only the set of the APs with the highest online RSS readings is selected. Thus, in this scheme, the similarity is calculated using any of the above schemes by only considering these selected APs.

Different cluster matching schemes do not affect the average localization error of the positioning system significantly. However, choosing the wrong cluster at this stage is the main source of the maximum localization error — defined as the largest localization error that can be observed during the experiment. Therefore, all the above cluster matching schemes attempt to reduce the possibility of choosing the wrong cluster and thus, reduce the maximum localization error. Experimental results for different cluster matching schemes will be shown in Section 4.2 .

The best-matched clusters can be found by using one or more of the above schemes. By evaluating the similarity function described above, the set of best-matched exemplars $\mathcal{S}$ with their corresponding cluster member set $\mathcal{C}$ can be found as,

$$\mathcal{S} = \{(j,o) : s(r,j)^{(o)} > \alpha, j \in \mathcal{H}^{(o)}, \forall o \in \mathcal{O}\} \quad (14)$$

$$\mathcal{C} = \bigcup_{(j,o)\in\mathcal{S}} \mathcal{C}_j^{(o)} \quad (15)$$

where α is a pre-defined threshold to obtain a moderate number of clusters in $\mathcal{S}$ (see parameter settings for implementing the positioning system in Table 1, Section 4.1). Instead of fixing the number of selected clusters, we use a percentage-based approach such that the number of matched clusters might vary at different runs and at different orientations. This is in particular important since the correct orientation of the device is unknown and the RSS readings might match to clusters from different orientations. Since only a small number of clusters is desired to be included in $\mathcal{S}$, in this paper, α is set to be a large percentage of the maximum similarity, namely,

$$\alpha = \alpha_1 \cdot \max_{j\in\mathcal{H}^{(o)},\forall o\in\mathcal{O}} \{s(r,j)^{(o)}\} + \alpha_2 \cdot \min_{j\in\mathcal{H}^{(o)},\forall o\in\mathcal{O}} \{s(r,j)^{(o)}\} \quad (16)$$

where $\alpha_1 + \alpha_2 = 1$. ($\alpha_1 = 0.95$ in our implementation.)

After the coarse localization, the set of interest can be reduced to the set $\mathcal{C}$. The partial radio map matrix $\widetilde{\boldsymbol{\Psi}}_{L\times\widetilde{N}}$, with $\widetilde{N} = |\mathcal{C}|$ can be obtained by

$$\widetilde{\boldsymbol{\Psi}} = [\boldsymbol{\psi}_j^{(o)} : \forall (j,o) \in \mathcal{C}]. \quad (17)$$

The matrix $\widetilde{\boldsymbol{\Psi}}$ will be used by the following fine localization stage. Note that it is possible that more than two columns in $\widetilde{\boldsymbol{\Psi}}$ represent the same RP, but with different orientations, as all clusters from different orientations are considered for the cluster matching.

2.2.2 Fine Localization by Compressive Sensing

The localization problem setup in Section 2 has a sparse nature, as the position of the mobile user is unique in the discrete spatial domain at a certain time. Ideally, assuming that the mobile user is located exactly at one of the RPs pointing at one of the orientations, the user's location can be formulated as a 1-sparse vector, denoted as $\boldsymbol{\theta}$. Thus, $\boldsymbol{\theta}$ is a $\widetilde{N}\times 1$ vector with all elements equal to zero except $\theta(n) = 1$, where n is the index of the RP at which the mobile user is located, namely:

$$\boldsymbol{\theta} = [0, ..., 0, 1, 0, ..., 0]^T. \quad (18)$$

Then, the online RSS reading measured by the mobile device can be expressed as:

$$\boldsymbol{y} = \boldsymbol{\Phi}\widetilde{\boldsymbol{\Psi}}\boldsymbol{\theta} + \boldsymbol{\varepsilon} \quad (19)$$

where $\widetilde{\boldsymbol{\Psi}}$ is the partial radio map matrix as defined in (17), and $\boldsymbol{\varepsilon}$ is an unknown measurement noise that comes from RSS deviations. The $M \times L$ matrix $\boldsymbol{\Phi}$ is an AP selection operator applied on the online RSS measurement vector $\boldsymbol{\psi}_r$, such that

$$\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{\psi}_r. \quad (20)$$

Next, we discuss how $\boldsymbol{\Phi}$ can be determined.

Due to the wide deployment of APs, the total number of detectable APs is generally much greater than that required for positioning, which leads to redundant computations. Furthermore, unreliable APs with large RSS variances may also lead to biased estimation and affect the stability of the positioning system. This motivates the use of AP selection techniques to select a subset of available APs for positioning. In this section, we will introduce different AP selection schemes to increase the accuracy of the fine localization.

According to (1), the set of APs covering the RPs can be denoted as $\mathcal{L}$, with $|\mathcal{L}| = L$. The objective of AP selection is to determine a set $\mathcal{M} \subseteq \mathcal{L}$ such that $|\mathcal{M}| = M \leq L$. This process is carried out by using the AP selection matrix $\boldsymbol{\Phi}$. Each row of $\boldsymbol{\Phi}$ is a $1\times L$ vector with all elements equal to zero except $\phi(\ell) = 1$, where ℓ is the index of the AP that is selected for positioning:

$$\boldsymbol{\phi}_m = [0, ..., 0, 1, 0, ..., 0], \forall m \in \{1, 2, ..., M\}. \quad (21)$$

We introduce three different approaches to determine the matrix $\boldsymbol{\Phi}$.

- *Strongest APs* [26]
  Same as what we used in the coarse localization, the set of APs with the highest RSS readings is selected, arguing that the strongest APs provide the highest probability of the coverage over time. The measurement vector (8) is sorted in the decreasing order of RSS readings, and the APs corresponding to the least indices are used. Since $\boldsymbol{\Phi}$ is created based on the current online measurement vector, this criterion may create different $\boldsymbol{\Phi}$ for each location update.
- *Fisher criterion* [27][5]
  The Fisher criterion is used to quantify the discrimination ability for each AP across RPs over four orientations, by comparing the metric ξ_i, $\forall i \in \{1, ..., L\}$, defined as

$$\xi_i = \frac{\sum_{(j,o)\in\mathcal{C}} (\psi_{i,j}^{(o)} - \bar{\psi}_i)^2}{\sum_{(j,o)\in\mathcal{C}} (\Delta_{i,j}^{(o)})} \quad (22)$$

  where $\bar{\psi}_i = \frac{1}{\widetilde{N}} \sum_{(j,o)\in\mathcal{C}} \psi_{i,j}^{(o)}$. The denominator of ξ_i ensures that RSS values do not vary much over time so that the offline and online values are similar; while the numerator represents the discrimination ability of each AP by evaluating the strength of variations of mean RSS across RPs. The APs with the highest ξ_i are selected to construct the matrix $\mathbf{\Phi}$ for the fine localization.

- *Random combination*
  Unlike the above two schemes, which select the APs based on a certain criteria and create the matrix $\mathbf{\Phi}$ dynamically for each location update, in this scheme, the AP selection operator $\mathbf{\Phi}$ is defined as a random $M \times L$ matrix with *i.i.d* Gaussian entries. The random combination scheme has less computational complexity, as the matrix is fixed at different runs, and it does not require as much RSS time samples to calculate the variance as required by the Fisher criterion.

Besides sparsity in (19), incoherence between $\mathbf{\Phi}$ and $\widetilde{\mathbf{\Psi}}$ is another important property that should be satisfied to enable the use of the CS theory for a sparse signal recovery from a small number of measurements [28]. As indicated in [16], the smaller the coherence, the fewer the number of samples needed by the CS. In CS, the Restricted Isometry Property (RIP) provides a sufficient condition for robust recovery of a sparse signal from a small number of noisy measurements. An equivalent description of the RIP is to say that the columns of matrix $\mathbf{\Phi}\widetilde{\mathbf{\Psi}}$ should be nearly orthogonal [16]. This incoherence holds with high probability between $\mathbf{\Phi}$ generated by random combination and the fixed basis $\widetilde{\mathbf{\Psi}}$. However, $\mathbf{\Phi}$ (generated by either the strongest APs or the Fisher criterion) and $\widetilde{\mathbf{\Psi}}$ are in general coherent in the spatial domain, which violates the incoherence requirement for the CS theory. We propose the following orthogonalization preprocessing procedure that restores such property.

Define an orthogonalization operator $\mathbf{T}$ as

$$\mathbf{T} = \mathbf{Q}\mathbf{R}^{\dagger} \tag{23}$$

where $\mathbf{R} = \mathbf{\Phi}\widetilde{\mathbf{\Psi}}$, and $\mathbf{Q} = \mathrm{orth}(\mathbf{R}^T)^T$, where $\mathrm{orth}(\mathbf{R})$ is an orthogonal basis for the range of $\mathbf{R}$, and $\mathbf{R}^{\dagger}$ is a pseudo-inverse of matrix $\mathbf{R}$.

The orthogonalization process is done by applying the operator $\mathbf{T}$ on the measurement vector $\boldsymbol{y}$, such that

$$\boldsymbol{z} = \mathbf{T}\boldsymbol{y} = \mathbf{Q}\mathbf{R}^{\dagger}\mathbf{R}\boldsymbol{\theta} + \boldsymbol{\varepsilon}' \tag{24}$$

where $\boldsymbol{\varepsilon}' = \mathbf{T}\boldsymbol{\varepsilon}$. It is straightforward to show that $\mathbf{Q}\mathbf{R}^{\dagger}\mathbf{R} = \mathbf{Q}$. Therefore, the localization problem formulated in (19) can be reformulated as

$$\boldsymbol{z} = \mathbf{Q}\boldsymbol{\theta} + \boldsymbol{\varepsilon}'. \tag{25}$$

Here $\mathbf{Q}$ is a nearly orthogonal matrix with unit norm (we have more columns than rows). It is shown in [29] that $\mathbf{Q}$ obeys the Restricted Isomatry Property (RIP) that is needed by the CS [30]. Since $\boldsymbol{\theta}$ has a sparse nature, according to the theory of compressive sensing [29][31][32], if the number of APs M is in the order of $\log(\widetilde{N})$, the location indicator $\boldsymbol{\theta}$ can be well recovered from $\boldsymbol{z}$ with very high probability, by solving the following ℓ_1-minimization problem.

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta} \in R^{\widetilde{N}}} \parallel \boldsymbol{\theta} \parallel_1, \quad s.t.\ \boldsymbol{z} = \mathbf{Q}\boldsymbol{\theta} + \boldsymbol{\varepsilon}'. \tag{26}$$

where $\parallel . \parallel_1$ is the ℓ_1-norm of a vector.

On a special note, the complexity of the ℓ_1-minimization algorithm grows proportional to the dimension of vector $\boldsymbol{\theta}$, which represents the number of potential RPs. Therefore, the coarse localization stage, which reduces the area of interest from all the N RPs into a subset of $\widetilde{N}$ RPs ($\widetilde{N} \ll N$), reduces the computational time for solving the ℓ_1-minimization problem and thus, allows this procedure to be executed on resource-limited mobile devices.

If the mobile user is located at one of the RPs pointing at one of the measured orientations, the recovered position is almost exact. However, in real scenario, the mobile user may not be exactly located at a certain RP facing a certain orientation. In such cases, the recovered location $\hat{\boldsymbol{\theta}}$ is not an exact 1-sparse vector, but with a few non-zero coefficients. Therefore, a post-processing procedure is conducted for real applications. We choose the dominant coefficients in $\hat{\boldsymbol{\theta}}$ whose values are above a certain threshold λ (see Table 1 in Section 4.1), and take the normalized value in $\hat{\boldsymbol{\theta}}$ as the corresponding weight for each potential RP to calculate the location estimation. Let $\mathcal{R}$ be the set of all indices of the elements of $\hat{\boldsymbol{\theta}}$ such that

$$\mathcal{R} = \{n | \hat{\theta}(n) > \lambda\}. \tag{27}$$

The location of the mobile user can be estimated by a weighted linear combination of these candidate points, which is

$$(\hat{x}, \hat{y}) = \frac{1}{\sum_{n \in \mathcal{R}} \hat{\theta}(n)} \sum_{n \in \mathcal{R}} \hat{\theta}(n) \cdot (x_n, y_n). \tag{28}$$

3 Reducing The Number of Fingerprints by Compressive Sensing

Due to the dynamic and unpredictable nature of indoor radio propagation, a pre-built radio map is always needed in fingerprinting methods to localize a Wi-Fi device. Moreover, the database may need to be updated if the number or the power of WiFi access points in the environment changes significantly. This may increase the labor cost during the offline calibration. In this section, we show that the CS scheme can also be used in the offline phase to reconstruct the radio map based on a small number of RSS measurements. The intuition behind this technique is that the RSS readings vary smoothly over the area of interest, and the corresponding Fourier coefficients of the radio map have a sparse nature.

Specifically, let vector $\boldsymbol{r}_i$ represent the RSS readings from AP i over the RPs that cover the experimental area, which is also the transpose of the ith row of the RSS radio map database defined in (1), namely, $\boldsymbol{r}_i = \boldsymbol{\Psi}(i,:)^T$. Let $\boldsymbol{F}$ denote the linear operator that transforms the $\boldsymbol{r}_i$ from the pixel representation in the spatial domain into the sparse representation in the frequency domain (*e.g.*, F can be generated by taking DFT of an identity matrix in our case, which is non-singular), namely,

$$\boldsymbol{x} = \boldsymbol{F}\boldsymbol{r_i}, \tag{29}$$

Further define a selection matrix $\mathbf{G}_{M\times N}$. Each row of $\mathbf{G}$, represented by $\boldsymbol{g}$, is a $1 \times N$ vector with all elements equal to zero except $\boldsymbol{g}(n) = 1$, where n is the index of the RP that is measured on the radio map by the mobile device during the offline phase. For simplicity, the measured RPs are randomly selected such that they are not placed densely at one region. The matrix $\mathbf{G}$ is filled accordingly.

Therefore, the offline RSS vector from a certain AP measured by the mobile device can be expressed as:

$$\boldsymbol{m} = \mathbf{G}\boldsymbol{r_i} = \mathbf{G}\mathbf{F}^{-1}\boldsymbol{x} = \boldsymbol{R}\boldsymbol{x}. \tag{30}$$

Since $\boldsymbol{x}$ is sparse, with the similar orthogonalization procedure in (24), namely, $\boldsymbol{z} = \mathbf{T}\boldsymbol{m}$, the radio map can be reconstructed by solving the following ℓ_1-minimization problem. Here, we use total variation (TV) minimization, a special case of the ℓ_1-minimization, to solve the sparse signal recovery problem, as it is widely used for $2D$ images recovery. The use of TV regularization makes the recovered radio map quality sharper by preserving the edges or boundaries more accurately [20][33]. (Other ℓ_1-minimization methods can also be used for the signal recovery.)

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \parallel \nabla(\boldsymbol{x}) \parallel_1 \quad s.t.\ \boldsymbol{z} = \mathbf{Q}\boldsymbol{x} + \varepsilon \tag{31}$$

where $\parallel \nabla(\boldsymbol{x}) \parallel_1$, defined as the *total variation* of $\boldsymbol{x}$, is the sum of the magnitudes of the gradient at every point [34], and ε is the measurement noise.

Finally, the reconstructed RSS radio map from AP i over the area of interest can be obtained by

$$\boldsymbol{r_i} = \mathbf{F}^{-1}\boldsymbol{x}. \tag{32}$$

The radio map from the rest of the APs can be recovered using the same approach. Therefore, RSS is measured on a small number of grid points and (31) is used to reconstruct the radio map on the whole grid. Since the number of measurements (M) needed for the radio map recovery obeys $O(k\log(N))$, where N is the total number of RPs, and k indicates the sparsity level of signal $\boldsymbol{x}$, significant reduction in the number of measured RPs can be expected.

4 Simulation and Implementation Results

This section provides details on the experimental evaluation of the proposed positioning system. The positioning software was developed in C# using Microsoft .Net Compact Framework version 3.5, and installed on a PDA (HP iPAQ hx4700 with Windows Mobile 2003 pocket PC) to provide the localization service. In addition, two open source libraries: OpenNetCF [35] and DotNetMatrix [36] were used to provide the RSS scanning function and matrix operations. The MAC address and RSS values of available WLAN APs were collected on the device, with a sampling interval of 1 second.

Real data were obtained from an office building. Specifically, the experiments were carried out on a 30 m $\times$ 46 m area of the fourth floor of an eight-story building (Bahen Centre at the University of Toronto), which is comparable to those reported in [5] and [10]. A total of 26 APs were detected throughout the area of interest. During the offline phase, the RSS observations from 26 APs were recorded for a period of 50 seconds (one sample per second) over 72 RPs with an average grid spacing of 1.5 m. At each RP, RSS values from four orientations were recorded. The online observations were collected on a different day by the device at 60 independent unknown locations with 2 repetitions for each as the testing points to evaluate the actual performance of the system in time-varying environment.

The localization error, which is measured by averaging the Euclidean distance between the estimated locations of the mobile user and the actual location over the testing points, has been reported as the performance measure. The performance of the positioning system is affected by the RSS variation, the number of available APs and the reliability of the APs. Reducing the RSS variation by taking more RSS samples averaging out can improve the localization accuracy. However, that will result in a longer RSS scanning interval hence slowing the process. Therefore, in our experiments, each online observation is an average of 2 RSS time samples, which takes 2 seconds for Wi-Fi scanning on the device. In the following subsections, the number of APs needed by the CS for accurate recovery, different coarse localization schemes, and different AP selection schemes will be analyzed through experiments.

4.1 Offline Stage: Clustering by Affinity Propagation

In order to mitigate the RSS variations and to remove potential outliers for coarse localization, affinity propagation is applied on each radio map to generate clusters and their corresponding exemplars during the offline phase. Fig. 2 shows an example of the clustering result on the PDA for the radio map at the north orientation. Each point represents one RP at which RSS readings are collected, and each color represents one cluster. It shows that the 72 RPs are divided into 13 clusters, and most of RPs belonging to the same cluster are geographically close to each other.

Table 1 lists the parameter settings for implementing the proposed positioning system. We note that the number of clusters might be different at different orientations.

TABLE 1: PARAMETERS SETTINGS FOR PROPOSED POSITIONING SYSTEM

γ for clustering	0.40(N)	0.35(S)	0.45(W)	0.25(E)
# of clusters	13	15	15	15
α_1	0.95			
λ	$0.8 \times \max_{n \in \tilde{N}} \hat{\theta}(n)$			

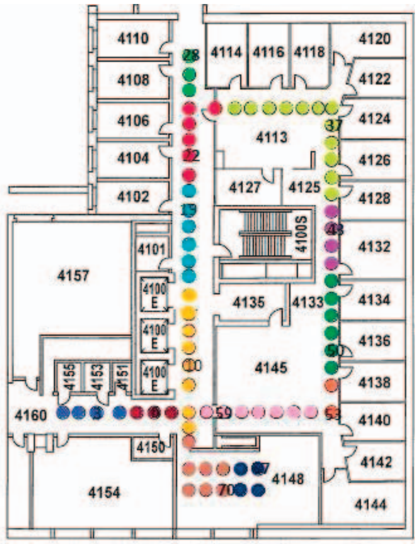

Fig. 2: An example of the clustering result on the PDA, using affinity propagation on the radio map at the north orientation (13 clusters are generated). Each point represents one RP, and different clusters are indicated by different colors for the RPs. The point indicated by a number inside represents the exemplar for cluster members that share the same color.

4.2 Online Stage: Coarse Localization

According to the theory of compressive sensing, the location indicator can be well recovered when the number of APs is in the order of $\log(\widetilde{N})$, where $\widetilde{N}$ is the number of selected RPs for the fine localization. Fig. 3 shows the average localization error as a function of the number of APs used in the algorithm under different number of clusters generated by the affinity propagation. As illustrated in Fig. 3, when no clustering scheme is used, the number of APs needed for reasonable recovery is 8, which is approximately equal to $\log(\widetilde{N}) = \log(72 \times 4) \approx 8$, considering four orientation database. Therefore, when the number of APs conforms with the CS theory, the system can achieve a high performance in terms of localization accuracy.

In addition, as mentioned in Section 2.2.1 , since the coarse localization is used to reduce the area of interest for location estimate into a subset $\mathcal{C}$, the dimension of the sparse signal in the CS algorithm is reduced. This allows the system to reduce the number of APs required for accurate location recovery. As shown in Fig. 3, only 8 APs are needed to achieve about 1.1 m error when a total of 58 clusters are generated in all four directions, and 1.9 m error under 29 clusters. However, 18 APs are needed to achieve 1.8 m error if no clustering scheme is applied.

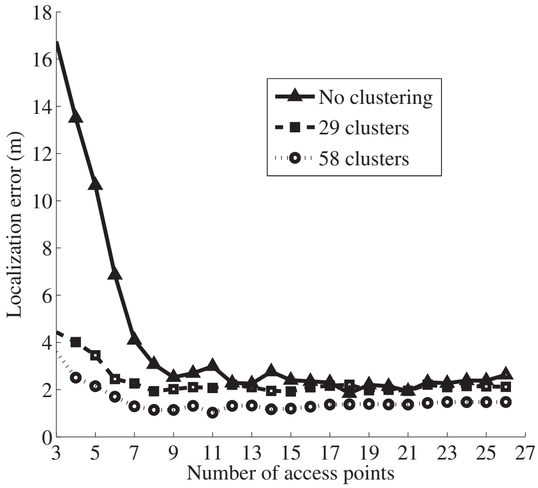

Fig. 3: The implementation result of the average localization error of the system with respect to the number of access points used, under different number of clusters generated in all four orientations.

Furthermore, the number of clusters generated by the affinity propagation is determined by the input of preference value, which is experimentally set. On one hand, increasing the number of clusters helps to reduce the area of interest into a smaller region after the coarse localization and thus, improves the average localization accuracy and also reduces the complexity for the fine localization. In the other hand, this increases the chance of choosing the wrong cluster, which induces a large localization error of the positioning system. Therefore, we studied different coarse localization schemes to reduce the possibility of choosing the wrong cluster. Fig. 4 illustrates the Cumulative Distribution Function (CDF) of the localization error of the positioning system under different coarse localization schemes that are proposed in Section 2.2.1 , when 10 APs are used. The strongest APs selection scheme is used for the fine localization at this stage. Different cluster matching schemes do not affect the system performance in terms of the average localization error significantly. However, it affects the maximum error of the positioning system. It is shown that the weighted cluster matching scheme reduces the maximum localization error from 9.1 m to 6.2

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m over the experiments, as it takes into account the stabilities of the RSS readings from different APs.

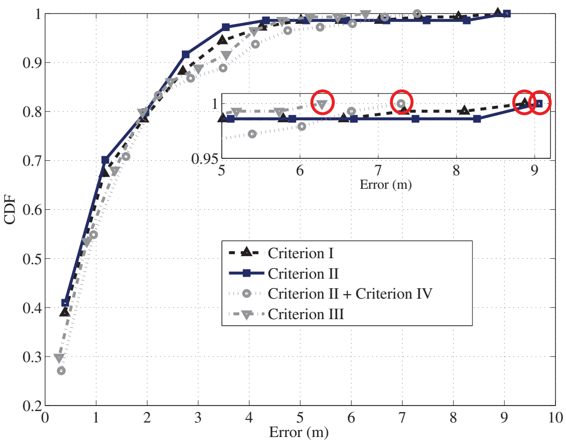


Fig. 4: The CDF of the localization error of the positioning system under different coarse localization schemes. (The Strongest APs selection is used for fine localization).

4.3 Online Stage: Fine Localization

Fig. 5 shows the average localization error under different AP selection schemes for the fine localization. In the random combination scheme, according to (20), the value of x-axis implies the number of linear random combinations of online RSS values from the L APs. Among the three schemes proposed in Section 2.2.2 , AP selection using the Fisher criterion achieves the best performance especially when the number of APs is less than 5, while the strongest APs selection achieves the worst performance. The proposed random selection scheme achieves a localization error comparable to that of the Fisher criterion, but does not require a large number of offline RSS time samples to calculate the variance. In addition, the matrix $\mathbf{\Phi}$ can be reused for each location update, saving the computational time for the fine localization.

Meanwhile, since each location is only covered by a certain number of APs ($6-12$ in our experiments), using more APs for the fine localization may not necessarily increase the accuracy, as a biased estimation generated by unreliable APs is introduced. As shown in Fig. 5, when the number of APs is above 11, the performance of the positioning system decreases. It is not affected by the way we choose the APs, as redundant APs are introduced for all of the three cases.

4.4 Comparison to Prior Work

We compare the proposed positioning system with the traditional fingerprinting approaches, known as the kNN [10] and the kernel-based methods [5], in terms of the cumulative error distribution ($M = 6$). The proposed positioning system used affinity propagation to generate overall 58 clusters at four orientations during the offline phase, and then performed coarse localization by weighted cluster matching, followed by a fine localization stage consisting of a random AP combination, an ℓ_1-minimization algorithm and a post-processing procedure. For fairness, the three positioning systems use the same coarse localization scheme, but are different in the fine localization stage. Fig. 6 shows the cumulative error distribution for these three schemes, and Table 2 shows the corresponding statistical result. As noticed, the proposed CS-based method provides a 90^{th} percentile error of 2.7 m, which outperforms the kNN and the kernel-based method by 25% and 27%, respectively.

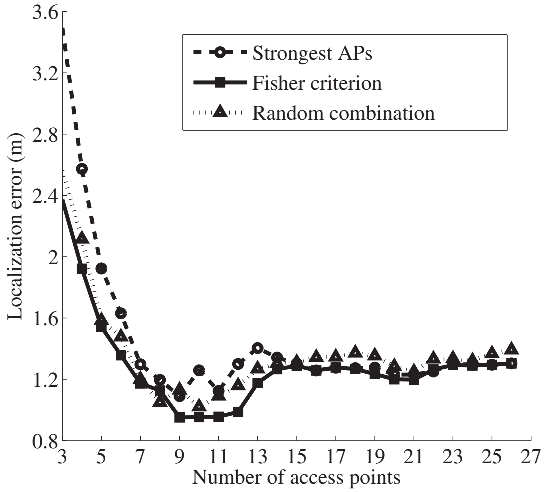


Fig. 5: The Implementation result of the average localization error under different AP selection schemes.

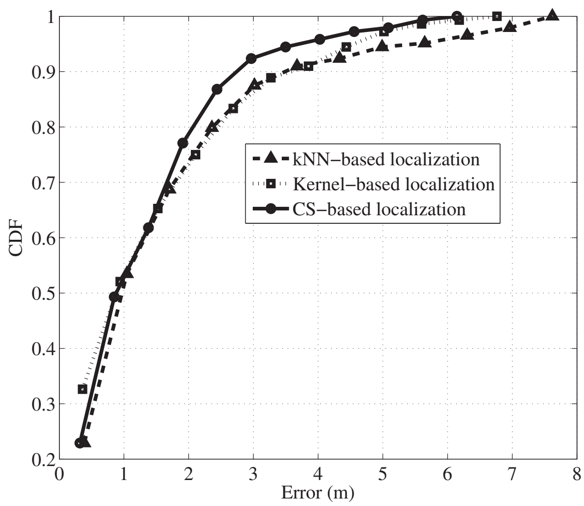


Fig. 6: The cumulative error distribution for kernel-based method, kNN, and the proposed CS-based method.

TABLE 2: POSITION ERROR STATISTICS

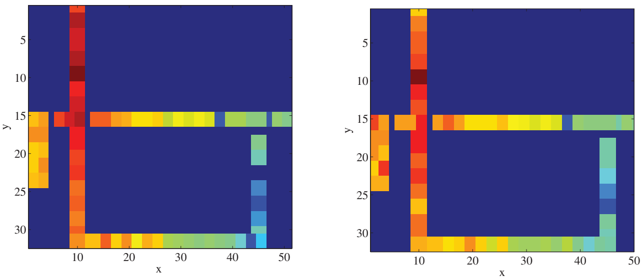
Method	Mean[m]	90^{th}[m]	Max[m]	Var[m^2]
kNN($k=3$)	1.8	3.7	7.9	2.81
Kernel-based	1.6	3.6	7.1	2.28
CS-based	1.5	2.7	6.2	1.47

In addition, since the dimension of the sparse signal is reduced through coarse localization, and the location finding using compressive sensing is implemented through a linear program, it is fairly quick for the mobile device to perform the actual sparse signal recovery. During our experiments, the process latency for location estimation is in the order of 100msec on a PDA with 624 MHz processor and 64M RAM. However, since the kernel-based localization scheme incorporates all RSS time samples from the fingerprint database for computation [5], it requires much more time to obtain the estimated position than the proposed scheme. Meanwhile, its computational time also increases as the number of used APs increases. Due to its high-volume computational cost, it is not desirable to implement on the resource-limited PDA as a real-time positioning system.

4.5 Reduce the size of FPs

In order to reduce the number of RPs for the fingerprinting approach, the same CS scheme is also used in the offline phase to reconstruct the radio map based on RSS measurements at only a small number of RPs. In general, small-scale variations happen when the user moves over a small distance (in the order of wavelength) [37]. For example, the variation in the average RSS could reach up to 10 dBm in a distance as small as 10 cm in our experiments. It is noticed that since the wavelength for the 802.11b/g networks working at the 2.4 GHz range is 12.5 cm, the RPs are placed with an average of 1.5 m apart in our experiments. This means that the radio map does not capture the small-scale variations and thus, it is smooth. The smoothness of the radio map across the whole experimental site allows the CS theory to be used for sparse signal recovery. In the simulation, only 36 of the RPs are randomly picked, and we show that the radio map at the overall 72 RPs can be well recovered compared with the values we actually measured at these 72 RPs. Note that increasing the density of RPs may increase the sparsity level of the radio map signal, which results more measurements for accurate recovery in practice.

Fig. 7a shows an example of the actual measured RSS radio map (in average over 50 time samples) from AP 1 at 72 RPs over the experimental area, while Fig. 7b is the corresponding reconstructed radio map, based on samples from 36 randomly picked FPs using the same PDA. The same technique is used for recovering RSS readings for the rest of APs.



(a) Measured radio map (b) Recovered radio map

Fig. 7: Recover the radio map during the offline phase using the theory of compressive sensing.

In addition, based on our experiments, the averaged recovery error of the CS-based scheme, defined as the averaged absolute RSS difference between the actual measured RSS and the recovered RSS at those non-measured locations, is 1.7dBm.

$$\epsilon = \frac{1}{N \times L \times |\mathcal{O}|} \sum_{o \in \mathcal{O}} \sum_{i=1}^{L} \sum_{j=1}^{N} |\psi_{i,j}^{(o)} - \hat{\psi}_{i,j}^{(o)}| \approx 1.7dBm \quad (33)$$

where o represents the orientation ($|\mathcal{O}| = 4$); L is the number of APs ($L = 26$); N is the number of non-measured locations ($N = 36$); $\psi_{i,j}^{o}$ is the actual measured RSS from AP i at RP j pointing at direction o, and $\hat{\psi}_{i,j}^{o}$ is the corresponding recovered RSS by the CS scheme.

We further use all the reconstructed RSS radio map for localization, compared with the localization using the actual measured radio map. Fig. 8 shows that the proposed scheme is able to achieve an average error of 1.6 m by using the reconstructed radio map, when 10 APs are used. However, using the radio map recovered by the traditional interpolation approach [38] reduces the average localization error to 2.6 m, when 10 APs are used.

5 Conclusion

In this paper, we have proposed an accurate RSS-based indoor positioning system using compressive sensing. The intuition behind this technique is that location estimation is a sparse problem and thus according to the CS theory, the location can be well recovered from only a small number of noisy measurements through an ℓ_1-minimization program. For accurate location recovery, the number of APs needs to be enough to conform with the CS theory. We have used different coarse localization schemes to compensate for the complex radio channel effects, and a pre-processing to induce incoherence needed by the CS theory. Meanwhile, the CS scheme is also used to recover the overall radio map from measurements at only a small number of random RPs. The positioning system is implemented on a PDA. The experimental

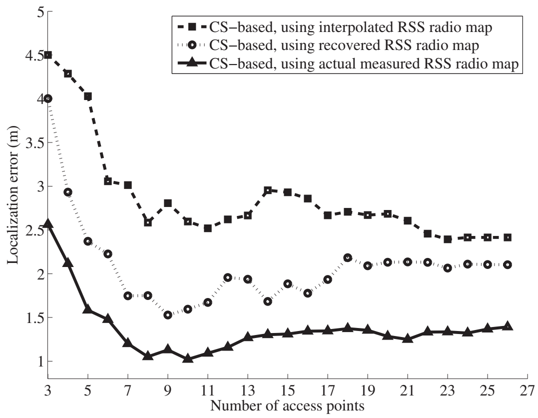


Fig. 8: Comparison of the localization accuracy, using actual RSS measurements at 72 RPs, the recovered RSS radio map from samples on 36 RPs, and that using interpolated RSS radio map. The 36 RPs are randomly picked but fixed during the radio map recovery for each AP at each orientation. Y axis indicates the average result over 60 independent locations with 2 repetitions for each.

results demonstrate that the proposed two-stage localization method leads to substantial improvements on the localization accuracy and the complexity over the widely used traditional fingerprinting methods.

The feasibility of using the CS theory to reconstruct the RSS radio map from measurements at a small number of fingerprints is studied, which reduces the labor cost when updating the database. Future research direction includes an analysis on the the density of FPs needed for accurate positioning systems. Meanwhile, new algorithms to build a more accurate and robust system that generates and maintains the fingerprint database automatically at a server without manual offline calibration are needed .

Acknowledgments

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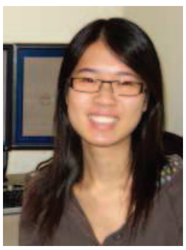

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