# Optimizing User Association and Frequency Reuse for Heterogeneous Network under Stochastic Model

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***Abstract*—This paper considers the joint optimization of frequency reuse and base-station (BS) bias for user association in downlink heterogeneous networks for load balancing and inter-cell interference management. To make the analysis tractable, we assume that BSs are randomly deployed as point processes in multiple tiers, where BSs in each tier have different transmission powers and spatial densities. A utility maximization framework is formulated based on the user coverage rate, which is a function of the different BS biases for user association and different frequency reuse factors across BS tiers. Compared to previous works where the bias levels are heuristically determined and full reuse is adopted, we quantitatively compute the optimal user association bias and obtain the closed-form solution of the optimal frequency reuse. Interestingly, we find that the optimal bias and the optimal reuse factor of each BS tier have an inversely proportional relationship. Further, we also propose an iterative method for optimizing these two factors. In contrast to system-level optimization solutions based on specific channel realization and network topology, our approach is off-line and is useful for deriving deployment insights. Numerical results show that optimizing user association and frequency reuse for multi-tier heterogeneous networks can effectively improve cell-edge user rate performance and utility.**

## I. Introduction

The heterogeneity is a key feature of future wireless systems [1]. By deploying low-power nodes such as pico and femto base-stations (BSs) in addition to the tower-based macro BSs, the conventional cellular system is split into multi-tier topology, and users can be off-loaded to the small cells (see Fig. 1). The heterogeneous networks are expected to provide better coverage and higher throughput.

The deployment of heterogeneous networks, however, also faces two main challenges. First, system parameters such as transmission power and deployment density are distinct across BS tiers; this highlights the importance of load balancing. Second, the increased density of small cell transmitters also causes more interference, hence efficient and practical methods to reduce interference are critical to system performance. How to tackle these two problems jointly is thus an interesting question, which this paper seeks to address.

The coverage area of a BS is determined by the set of users it serves. An appropriate user association scheme should jointly consider the signal quality from the users' perspective and the load balancing from the BSs' perspective, since user rates are related to both the spectrum efficiency and the fraction of resources it gets, and the latter of which are limited and shared with other users. One approach to the user association

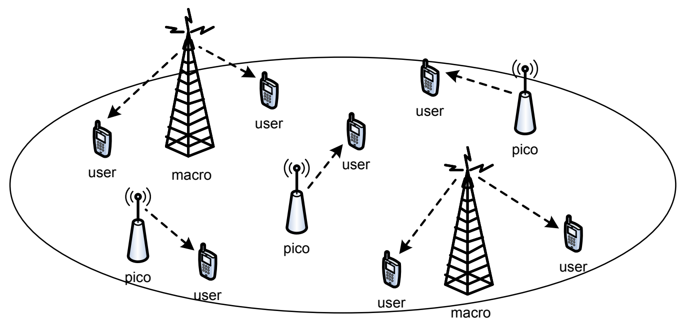


Fig. 1. An example of a 2-tier heterogeneous network. The macro BSs have greater transmission power and lower deployment density, while pico BSs have lower power and higher density.

problem is the greedy association, i.e., add users that improve a certain metric to the BS, as in [2] and [3] for single-tier and heterogeneous networks, respectively. Another approach constructs utility maximization framework and develop pricing based association method, see [4] for single-tier and [5], [6] for heterogeneous networks. In [7], [8], [9], the association problem is jointly considered with resource allocation using the game theoretical approach. These solutions are dynamic and rely on channel and topology realization, and may require iterations and real-time computation. In this paper, we adopt a simpler *cell range expansion* approach, also known as the *biased* user association [1], in which traffic can be effectively off-loaded to lower-power nodes by setting a power bias term towards them.[1] The effect of the biased off-loading has been investigated for heterogeneous networks in [10], [11], [12] in terms of coverage and rate.

Frequency reuse is simple and effective in reducing interference. The most representative static and semi-static strategies include the fractional frequency reuse (FFR) [13] and soft frequency reuse (SFR) [14], which aim to increase the frequency reuse for cell-edge users or to reduce the transmission power for cell-center users. In an irregular heterogeneous network, it is difficult to identify and partition resources for cell-center and cell-edge users. This paper therefore assumes a simple frequency reuse factor for each of the different BS tiers.

This paper aims to jointly optimize the user association bias and the frequency reuse of each tier in a heterogeneous

[1] As the load of BSs within a tier is statistically the same, only inter-tier biasing is considered where each tier has one bias factor for user association.

network. To make the analysis tractable and to account for the irregular deployment of low-power nodes for hot-spot or indoor coverage, we assume that BSs of each tier form a spatial random process and use tools from stochastic geometry to obtain system deployment insights. Our system is related to [10]. Specifically, we define the user coverage rate, based on which the network utility maximization problem is formulated. The utility is averaged over BS locations as well as the channel and is thus not dependent on specific network realization. Further, we adopt a stochastic model for the frequency reuse [15] in order to account for the randomness in the network topology and for ease of analysis. Under this model and by utilizing stochastic geometry, we can numerically compute the optimal bias and frequency reuse factor of each BS tier.

Unlike [15] which shows that the full reuse is optimal in single-tier random networks in terms of mean spectrum efficiency, we show that under our utility maximization model for heterogeneous networks the optimal frequency reuse is not necessarily universal across tiers. This is because BSs in different tiers have quite different powers, and macro BS can cause significant interference to users in lower tiers. Thus less aggressive frequency reuse in macro BS may be beneficial. Moreover, this paper analytically derives a closed-form expression of the optimal frequency reuse factor. Interestingly, the optimal frequency reuse factor is shown to be inversely proportional to the optimal user association bias. Intuitively, this can be explained by the fact that larger user association bias corresponds to more users at each BS, which would then require a smaller frequency reuse factor in order to gain more available resources for each BS. Finally, we propose an iterative method for optimizing the bias and reuse factors, and demonstrate the effectiveness of the optimization via numerical experiment.

## II. System Model

We assume $K$ tiers of BSs in the network. In order to model the spatial randomness, we model BSs in each tier $k$ as a homogeneous Poisson point process (PPP) $\Phi_k$ with density $\lambda_k$. The BS PPPs are independent across tiers. The users are also modeled as a homogeneous PPP with density $\lambda^{(u)}$.

### A. User Association

Using the biased maximum-signal-strength association rule, a typical user is associated with a BS in tier $k$ if

$$P_k \left( \min_{\mathcal{L}_i \in \Phi_k} \ell_{k,i} \right)^{-\alpha} B_k \geq P_j \left( \min_{\mathcal{L}_i \in \Phi_j} \ell_{j,i} \right)^{-\alpha} B_j, \forall j, \quad (1)$$

where $P_k$ is the transmission power of the BSs in tier $k$ which is fixed *a priori*, $\ell_{k,i}$ is the distance from BS $i$ in tier $k$ to the typical user, $\mathcal{L}_i$ is the location of BS $i$, $B_k$ is the corresponding user association bias factor, and $\alpha$ is the pathloss exponent (normally $\alpha > 2$). Note that the maximum-signal-strength based association with $B_k = 1, \forall k$ is equivalent to the maximum-average-SIR (signal-to-interference ratio) based association where only large-scale fading is considered; while setting $B_k = \frac{1}{P_k}, \forall k$ is equivalent to the distance based association. Setting larger bias toward low-power BSs can effectively off-load users from BSs in "hot-spots" with more-than-average users to their lighted-loaded neighbors.

Given BS density $\lambda_k$, power $P_k$ and bias factor $B_k$, the probability $A_k$ of a randomly chosen user being associated with BSs in tier $k$ is [10]

$$A_k = \frac{\lambda_k \left(P_k B_k\right)^{2/\alpha}}{\sum_{j=1}^{K} \lambda_j \left(P_j B_j\right)^{2/\alpha}} = \left[ \sum_{j=1}^{K} \hat{\lambda}_j \left( \hat{P}_j \hat{B}_j \right)^{2/\alpha} \right]^{-1}, \quad (2)$$

where $\hat{\lambda}_j \triangleq \frac{\lambda_j}{\lambda_k}$, $\hat{P}_j \triangleq \frac{P_j}{P_k}$, and $\hat{B}_j \triangleq \frac{B_j}{B_k}$.

### B. Frequency Reuse

All BS tiers in the network share the same frequency band $W$. Let $\delta_k \geq 1$ be the frequency reuse factor of the $k$th tier. Instead of using a fixed reuse pattern, we model $\delta_k$ statistically to account for the randomness of the network topology as follows: BS in tier $k$ randomly and independently picks one of a total of $\delta_k$ orthogonal frequency bands for transmission.[2]

## III. Utility Optimization

### A. User Coverage Rate

For the sake of analytical amenability while accounting for practical application, we define the user coverage spectrum efficiency for users associated with the $k$th tier as (in nats/s/Hz)

$$\begin{aligned} r_k &= \log\left(1+\tau_k\right) C_k \\ &= \log\left(1+\tau_k\right) \mathbb{P}\left(\text{SIR}_k > \tau_k\right), \end{aligned} \quad (3)$$

where $\tau_k$ is the target SIR of the $k$th BS tier, which is determined by the physical-layer requirement such as target bit error rate (BER), and $C_k$ is the corresponding coverage probability. We ignore the noise since typical heterogeneous networks are interference limited. Conditioned on a distance $x$ from the user to its associated BS, we have

$$\text{SIR}_k = \frac{x^{-\alpha} g_{k,0}}{\sum_{j=1}^{K} \hat{P}_j \sum_{\mathcal{L}_i \in \Phi_j(\delta_j) \setminus \text{BS}_{k,0}} \ell_{j,i}^{-\alpha} g_{j,i}}, \quad (4)$$

where $g_{j,i}$ is the exponentially distributed channel power gain from BS $i$ in tier $j$, $\Phi_j(\delta_j)$ is the PPP $\Phi_j$ thinned by the random reuse $\delta_j$, and $\text{BS}_{k,0}$ denotes the serving BS in tier $k$. Note that the coverage spectrum efficiency is binary and corresponds to the practical case where only one modulation and coding scheme is used. Since the user rate is summed across subcarriers on which that user is scheduled, the coverage rate is proportional to the number of subcarriers with non-zero rate.

### B. Average User Utility

We aim at optimizing both the user association bias and the frequency reuse of each BS tier. Towards this end, we formally formulate these questions as a proportional fairness utility maximizing problem for the typical user under consideration

$$\max_{B_k, \delta_k} U, \quad \text{where } U = \sum_{k=1}^{K} A_k U_k, \quad (5)$$

[2]Equivalently speaking, BS transmits on a subcarrier with a probability $\frac{1}{\delta_k}$.

where $U_k$ is the average utility of a typical user given it is associated to the $k$th BS tier, which is computed as a logarithm function of the coverage rate of that user

$$\begin{aligned} U_k &= \mathbb{E}\left\{\log\left[\beta_k \log\left(1+\tau_k\right) C_k\right]\right\} \\ &= \log\left[\log\left(1+\tau_k\right)\right] + \mathbb{E}\left[\log\left(\beta_k\right)\right] + \mathbb{E}\left[\log\left(C_k\right)\right], \end{aligned} \quad (6)$$

where $\beta_k$ is the per-user resources in tier $k$. We assume all the associated users of a particular BS are allocated the same resources. This can be achieved by round-robin scheduling. It is also shown in [5] that equal allocation of resources among users of a BS can maximize the log-utility. We approximate $\beta_k$ by a ratio of expectations as[3]

$$\begin{aligned} \beta_k &\approx \frac{\mathbb{E}\left(\text{Number of resources per BS in the } k\text{th tier}\right)}{\mathbb{E}\left(\text{Number of users per BS in the } k\text{th tier}\right)} \\ &= \frac{W/\delta_k}{A_k \lambda^{(u)}/\lambda_k}. \end{aligned} \quad (7)$$

The expected logarithm of the coverage probability is averaged over the user and BS locations as well as the channel. First the probability density function (PDF) of the distance between a user and its serving BS in tier $k$ is given as [10]

$$f_{X_k}(x) = \frac{2\pi\lambda_k}{A_k} x \exp\left[-\pi x^2 \sum_{j=1}^{K} \lambda_j \left(\hat{P}_j \hat{B}_j\right)^{2/\alpha}\right], \quad (8)$$

and we have

$$\mathbb{E}\left[\log\left(C_k\right)\right] = \int_0^\infty \mathbb{E}_{\Phi,\mathbf{g}}\left[\log\left(C_k\right)|X_k = x\right] f_{X_k}(x)\mathrm{d}x. \quad (9)$$

Conditioned on the distance $x$ between the typical user and its associated BS in tier $k$, we have

$$\begin{aligned} &\mathbb{E}_{\Phi,\mathbf{g}}\left[\log\left(C_k\right)|X_k = x\right] \\ =&\mathbb{E}_{\Phi,\mathbf{g}}\left\{\log\left[\mathbb{P}\left(\mathrm{SIR}_k > \tau_k\right)\right]|X_k = x\right\} \\ =&\mathbb{E}_{\Phi,\mathbf{g}}\left\{\log\left[\mathbb{P}\left(g_{k,0} > x^\alpha \tau_k \sum_{j=1}^{K} \hat{P}_j \sum_{\mathcal{L}_i \in \Phi_j(\delta_j)\backslash \mathrm{BS}_{k,0}} \ell_{j,i}^{-\alpha} g_{j,i}\right)\right]\right\} \\ \overset{(a)}{=}&\mathbb{E}_{\Phi,\mathbf{g}}\left\{\log\left[\exp\left(-x^\alpha \tau_k \sum_{j=1}^{K} \hat{P}_j \sum_{\mathcal{L}_i \in \Phi_j(\delta_j)\backslash \mathrm{BS}_{k,0}} \ell_{j,i}^{-\alpha} g_{j,i}\right)\right]\right\} \\ =&-x^\alpha \tau_k \sum_{j=1}^{K} \hat{P}_j \mathbb{E}_{\Phi}\left(\sum_{\mathcal{L}_i \in \Phi_j(\delta_j)\backslash \mathrm{BS}_{k,0}} \ell_{j,i}^{-\alpha}\right) \\ \overset{(b)}{=}&-x^\alpha \tau_k \sum_{j=1}^{K} \hat{P}_j \frac{2\pi\lambda_j}{\delta_j} \int_{x\left(\hat{P}_j \hat{B}_j\right)^{1/\alpha}}^{\infty} y^{-\alpha} y \mathrm{d}y \\ =&-\frac{2\pi}{\alpha-2} x^2 \tau_k \sum_{j=1}^{K} \frac{\lambda_j}{\delta_j} \hat{P}_j^{2/\alpha} \hat{B}_j^{2/\alpha-1}, \end{aligned} \quad (10)$$

where in (a) we assume $g_{k,0} \sim \exp(1)$, i.e., Rayleigh distributed, and (b) follows from the Campbell's Formula [16]. Due to frequency reuse, the equivalent density of the PPP $\Phi_j\left(\delta_j\right)$ of the $j$th tier BS is thinned to $\lambda_j/\delta_j$. The integration limit is obtained by noticing from (1) that the closest interfering BS in the $j$th tier is at least $x\left(\hat{P}_j \hat{B}_j\right)^{1/\alpha}$ away.

Substituting (8) and (10) into (9) and after some manipulations, we obtain

$$\begin{aligned} \mathbb{E}\left[\log\left(C_k\right)\right] &= \frac{-2\tau_k A_k}{(\alpha-2)\lambda_k} \sum_{j=1}^{K} \frac{\lambda_j}{\delta_j} \hat{P}_j^{2/\alpha} \hat{B}_j^{2/\alpha-1} \\ &= \frac{-2\tau_k}{(\alpha-2)} \sum_{j=1}^{K} \frac{A_j}{\delta_j} \hat{B}_j^{-1}. \end{aligned} \quad (11)$$

Combining (5), (6), (7), and (11), the average per-user utility can be written as

$$\begin{aligned} &U\left(\delta_1,...,\delta_K,B_1,...,B_K\right) \\ =&\sum_{k=1}^{K} A_k \left\{\log\left[\frac{W\lambda_k \log(1+\tau_k)}{\delta_k A_k \lambda^{(u)}}\right] - \frac{2\tau_k}{(\alpha-2)} \sum_{j=1}^{K} \frac{A_j}{\delta_j} \hat{B}_j^{-1}\right\}. \end{aligned} \quad (12)$$

### C. Optimization of User Association Bias

First, we consider the optimization of the average utiliy (12) over $B_k$ for fixed $\delta_k$. Instead of a direct optimization over $B_k$, we take the following approach. Observe in (2) that

$$\hat{B}_j = \hat{P}_j^{-1} \hat{\lambda}_j^{-\alpha/2} \left(\frac{A_j}{A_k}\right)^{\alpha/2}. \quad (13)$$

Plugging (13) into (12), we can eliminate the term $B_j$ and formulate the utility maximization problem over $A_k$:

$$\max_{A_1,...,A_k} \quad U\left(A_1,...,A_k\right), \quad (14a)$$

$$\text{s.t.} \quad \sum_{k=1}^{K} A_k = 1, \quad (14b)$$

$$A_k > 0, \ \forall k. \quad (14c)$$

Although the above optimization problem is nonconvex and does not have a closed form solution, numerical solutions can be obtained efficiently to achieve a local optimum.

Finally, to recover $B_k^*$ from $A_k^*$, we first re-formulate (2) as follows

$$\mathbf{Z} \times \left[B_1^{2/\alpha},\ldots,B_K^{2/\alpha}\right]^T = \mathbf{0}, \quad (15)$$

where $\mathbf{Z}$ is a matrix with its elements as

$$Z_{i,j} = \begin{cases} A_i \lambda_j P_j^{2/\alpha} & i \neq j \quad (16a) \\ \left(A_i - 1\right) \lambda_j P_j^{2/\alpha} & i = j. \quad (16b) \end{cases}$$

Since the rank of $\mathbf{Z}$ is $K-1$, there is one set of orthogonal base for equation (15). We have

$$B_k^* = \hat{\lambda}_1^{\alpha/2} \hat{P}_1 \left(\frac{A_k}{A_1}\right)^{\alpha/2} B_1^*. \quad (17)$$

Setting $B_1^* = 1$ we can get all the $B_k$. Finally we normalize the maximum of the bias factor to unit

$$\widetilde{B_k} = \frac{B_k^*}{\max_j\left(B_j^*\right)}. \quad (18)$$

[3] The approximation of the average user number per $k$th-tier BS in the denominator of (7) is adopted from [10]. More accurate value is given in [11] as $1 + 1.28 A_k \lambda^{(u)}/\lambda_k$ by considering the implicit area biasing. These additional contents do not affect the optimization procedures that follow.

### D. Optimization of Frequency Reuse

Next, we consider the optimization of the frequency reuse factor $\delta_k$ for fixed $B_k$. Reformulating (12) we have

$$U(\delta_1, ..., \delta_K) = \sum_{k=1}^{K} A_k \left[ T_k - \log(\delta_k) - \frac{T}{B_k \delta_k} \right], \tag{19}$$

where $T_k \triangleq \log\left[\frac{W\lambda_k \log(1+\tau_k)}{A_k \lambda^{(u)}}\right]$ and $T \triangleq \frac{2}{(\alpha-2)}\left(\sum_{j=1}^{K} A_j \tau_j B_j\right)$.

Setting $\partial U / \partial \delta_k = 0$ in (19) and after some simplification, we have the following stationary point

$$\delta_k^* = \frac{T}{B_k} = \frac{2}{\alpha - 2} \sum_{j=1}^{K} A_j \tau_j \hat{B}_j. \tag{20}$$

It can be easily verified by checking the second-order derivative that $U$ is concave in $\delta_k$ when $\delta_k < 2\delta_k^*$ and convex in $\delta_k$ when $\delta_k > 2\delta_k^*$. Since $\delta_k^*$ is the only stationary point, and there is no stationary point in the convex regime $(2\delta_k^*, +\infty)$, we conclude that $\delta_k^*$ maximizes the utility. For single tier networks where $K = 1$, $\delta^* = \frac{2\tau}{\alpha-2}$ is only determined by the pathloss exponent and the target SIR.

It is interesting to note from (20) that the optimal frequency reuse and user association bias of each tier are inversely proportional to each other, i.e.,

$$\delta_k^* B_k = \delta_j^* B_j = T, \forall k \neq j, \tag{21}$$

where $T$ is a function of $\alpha$ and $\{\tau_i, P_i, B_i, \lambda_i\}_{i=1,...,K}$, as defined in (19). This indicates that the BS tier with small bias would prefer large frequency reuse factor, and vice versa. Qualitatively, BSs with smaller bias associate with fewer users, and thus need fewer resources to serve these users. This explains the increased optimal frequency reuse factor.

In practice the frequency reuse factors need to be lower bounded by unit

$$\widetilde{\delta_k} = \max(\delta_k^*, 1). \tag{22}$$

Note that the reuse factor can be a fractional number, i.e., under random frequency reuse, $1/\widetilde{\delta_k}$ represents the probability of a subcarrier being used for transmission.

### E. Iterative Optimization of Frequency Reuse and Bias

From (21) we know that the optimal frequency reuse and user association bias depend closely on each other. Hence, we propose to iteratively optimize the two factors, the process of which can be summarized as

Initialize $B_k = 1, \delta_k = 1, \forall k$;
**while** $||\mathbf{B}[t] - \mathbf{B}[t-1]|| > \epsilon ||\mathbf{B}[t]||$ **do**
    1) Given fixed $\delta_k$, compute $A_k$ as in (14);
    2) Compute $B_k$ as in (17);
        $B_k \leftarrow B_k / \max_j (B_j)$;
    3) Given fixed $B_k$, compute $\delta_k$ as in (20);
        $\delta_k \leftarrow \max(\delta_k, 1)$.
**end while**

We use the BS bias factors to determine the termination condition of the iteration, since they are always bounded within unit while the reuse factors can be unbounded. The algorithm is guaranteed to converge as in each step the utility increases, although $B_k^*$ might not be a global optimum. At convergence, the optimized pair of frequency reuse and bias factors also have the relation

$$\delta_k^* B_k^* = \delta_j^* B_j^*, \forall k \neq j, \tag{23}$$

since in each step the optimal $\delta_k^*$ is computed as in (20) (assuming that any intermediate value of $\delta_k$ does not fall below unit, i.e., (22) is not used).

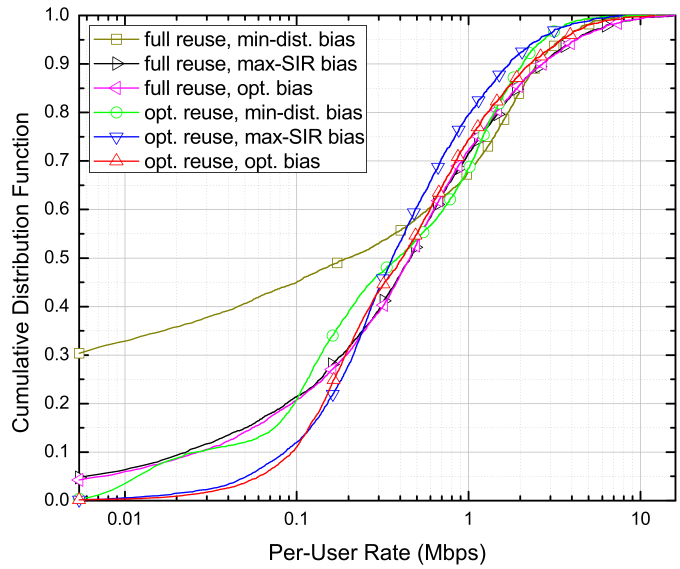


Fig. 2. CDF of per-user rate. $K = 3$, $\{P_1, P_2, P_3\} = \{46, 35, 24\}$dBm, $\{\lambda_1, \lambda_2, \lambda_3\} = \{0.01, 0.03, 0.06\}\, \lambda^{(u)}$, $\tau_k = 2, \forall k$.

## IV. Simulation

In this section, we present numerical results to demonstrate the effectiveness of the proposed results on optimal frequency reuse and user association bias. We assume $\alpha = 4$ and that there are $K = 3$ tiers of hierarchical BSs. The transmission power of the three tiers are $\{P_1, P_2, P_3\} = \{46, 35, 14 \sim 30\}$ dBm, representing typical macro, micro, and pico/femto power, respectively. Since the Poisson based model is used for spatial distribution of BSs, the cell topology should be derived as Voronoi tessellation. Let the user density be $\lambda^{(u)} = \frac{100}{\pi R^2}$, where $R = 1000$m is the average cell radius, and the BS density as $\lambda_k = a_k \lambda^{(u)}$, where $\{a_1, a_2, a_3\} = \{0.01, 0.03, 0.03 \sim 0.13\}$. We use the Monte-Carlo methods to generate multiple snapshots. The typical user is assumed to be located at the origin, while the locations of other users and BSs are drawn from PPPs with their given density. The system bandwidth is $W = 20$MHz and is divided to 2048 subcarriers, and we repeat for 20 time-slots in each snapshot. Round robin user scheduling is adopted at each BS. The typical user can be scheduled on multiple subcarriers, thus the coverage rate of the user is proportional to the number of subcarriers that offer non-zero rates; user can get zero rate if all of its subcarriers are below the SIR threshold. Note that fixing the frequency reuse

TABLE I

AVERAGE PER-USER PERFORMANCE. $K=3$, $\{P_1,P_2,P_3\}=\{46,35,24\}$ DBM, $\{\lambda_1,\lambda_2,\lambda_3\}=\{0.01,0.03,0.06\}\,\lambda^{(u)}$, $\tau_k=2,\forall k$.

| | reuse $\delta_1$, $\delta_2$, $\delta_3$ | bias $B_1$, $B_2$, $B_3$ | average rate (Mbps) | utility[a] | zero-rate probability |
|---|---|---|---|---|---|
| full reuse, min-dist. bias | 1, 1, 1 | 0.0063, 0.0794, 1 | 0.8766 | 5.6546 | 23.4% |
| full reuse, max-SIR bias | 1, 1, 1 | 1, 1, 1 | 1.0376 | 5.8511 | 1.4% |
| full reuse, opt. bias | 1, 1, 1 | 0.7470, 0.8688, 1 | 1.0360 | 5.8735 | 1.3% |
| opt. reuse, min-dist. bias | 197.94, 15.72, 1.25 | 0.0063, 0.0794, 1 | 0.8085 | 5.7671 | 0% |
| opt. reuse, max-SIR bias | 2, 2, 2 | 1, 1, 1 | 0.7091 | 5.9138 | 0% |
| opt. reuse, opt. bias | 3.99, 1.91, 1.31 | 0.3273, 0.6829, 1 | 0.8954 | 6.0355 | 0% |

[a] The utility is computed as the logarithm of the average user rate in Kbps. Zero-rate users are not counted for utility computation.

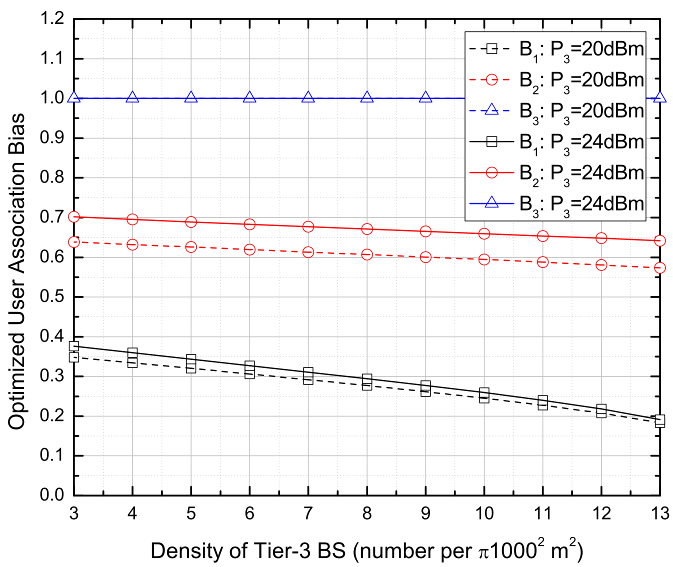

Fig. 3. User association bias as a function of tier-3 BS density. $K=3$, $\{P_1,P_2\}=\{46,35\}$dBm, $\{\lambda_1,\lambda_2\}=\{0.01,0.03\}\,\lambda^{(u)}$, $\tau_k=2,\forall k$.

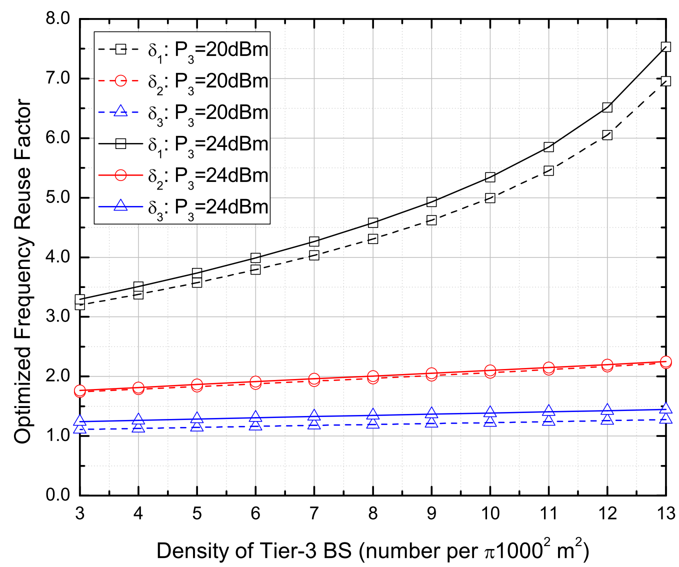

Fig. 4. Frequency reuse factor as a function of tier-3 BS density. $K=3$, $\{P_1,P_2\}=\{46,35\}$dBm, $\{\lambda_1,\lambda_2\}=\{0.01,0.03\}\,\lambda^{(u)}$, $\tau_k=2,\forall k$.

and bias factors, the optimal SIR threshold can be obtained from (12) as follows

$$\tau_k^*=\left[D_k\mathcal{L}\left(0,D_k^{-1}\right)\right]^{-1}-1, \quad (24)$$

where $\mathcal{L}(0,\cdot)$ is the Lambert W function of 0-th branch and $D_k\triangleq\frac{2}{(\alpha-2)}\sum_{j=1}^{K}\frac{A_j}{\delta_j}\hat{B}_j^{-1}$. Consequently the optimization of $\tau_k$ should be nested with the optimization of $B_k$ and $\delta_k$. However, the target SIR is usually determined by the physical layer modulation and coding schemes in practice, and is not considered in our evaluation. We set $\tau_k=3$dB, $\forall k$.

Fig. 2 shows the cumulative density function (CDF) of the user rate distribution. The $x$-axis for the user rate is presented in logarithm scale. We present results for cases where 1) only the user association bias is optimized, 2) only the frequency reuse is optimized, and 3) both bias and reuse factors are iteratively optimized. We consider the min-distance based association with $B_k=\frac{1}{P_k},\forall k$ as well as the max-SIR based association with $B_k=1,\forall k$. The CDF is computed only for the rate of the typical user at the origin over multiple trials, but can represent all the users in the network since the relative location of the typical user is random. The min-distance user association has the worst performance. The proposed *opt. reuse opt. bias* scheme significantly outperforms the benchmark *full reuse max-SIR bias* scheme for cell-edge users. It is noted that most of the cell-edge gain comes from the optimized frequency reuse while the gain from the optimized bias is small. However, with the optimized frequency reuse, the optimized bias factors can effectively improve the cell-center rate (red vs. blue in the upper-right of the figure), and consequently the overall average rate.

Table I summarizes for different schemes the association bias and frequency reuse factors that are analytically optimized, as well as the average per-user rate and utility computed from Monte Carlo simulation. The proposed scheme *opt. reuse opt. bias* has the highest utility and minimizes the probability of getting zero rate for the users as compared to all other schemes. We can also verify the inverse proportional relation for schemes with frequency reuse optimization, e.g., $\delta_kB_k\approx1.25,\forall k$ for the *opt. reuse min-dist. bias* scheme, and $\delta_kB_k\approx1.31,\forall k$ for the *opt. reuse opt. bias* scheme.

Fig. 3 and Fig. 4 plot the optimized bias and reuse factors of all BSs versus the density of the tier-3 BSs. As the number of the low-power tier-3 BSs increases, more users need to be offloaded to them and hence the bias factors of the other two BS tiers drop. This allows the tier-1 and tier-2 BSs to use fewer resources to accommodate fewer users, so their reuse factors

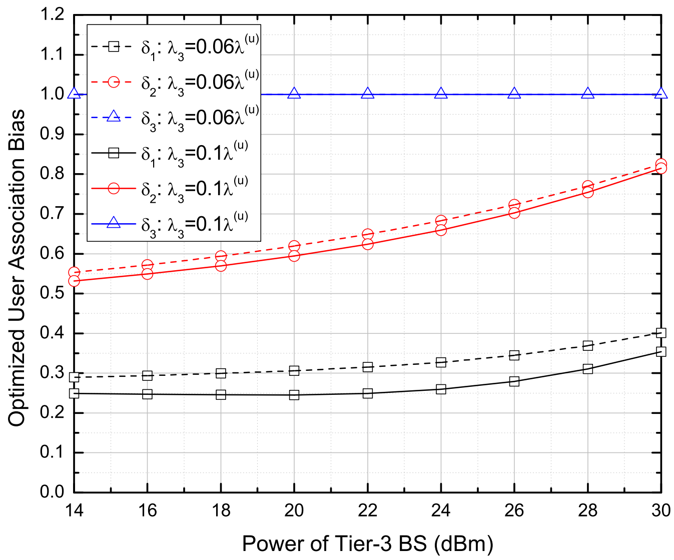


Fig. 5. User association bias as a function of tier-3 BS power. $K = 3$, $\{P_1, P_2\} = \{46, 35\}$dBm, $\{\lambda_1, \lambda_2\} = \{0.01, 0.03\}\,\lambda^{(u)}$, $\tau_k = 2, \forall k$.

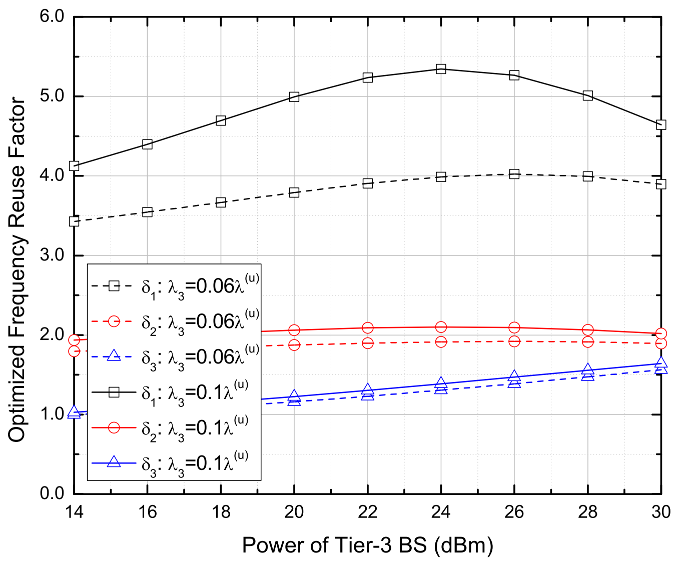


Fig. 6. Frequency reuse factor as a function of tier-3 BS power. $K = 3$, $\{P_1, P_2\} = \{46, 35\}$dBm, $\{\lambda_1, \lambda_2\} = \{0.01, 0.03\}\,\lambda^{(u)}$, $\tau_k = 2, \forall k$.

grow, which also creates less interferences to the densified tier-3. What is not shown in Fig. 3 and Fig. 4 is that, at the regime where $\delta_3 > \frac{13}{\pi 1000^2}$, the bias factor of tier-1 BSs drops to a number very close to zero and its reuse factor grows to a very large number ($> 10^{11}$), which indicates that under such scenario it is not beneficial to deploy tier-1 BSs in the network.

Fig. 5 and Fig. 6 show the optimized bias and reuse factors as a function of the power of the tier-3 BSs. As the tier-3 power rises, the bias factors of BSs in other tiers also becomes higher for better load balancing. The frequency reuse of the tier-3 BSs increases with their power so as to reduce their interference to other tiers. The reuse factors of other tiers, however, are not monotonic as the tier-3 BS power varies.

## V. Conclusion

In this paper, we consider the optimal frequency reuse and user association factors in a downlink heterogeneous networks. We assume random network topology for analytic tractability and construct a utility maximization framework based on a proportionally fair measure of users' coverage rate. The optimal frequency reuse and association bias of each tier turn out to be inversely proportional to each other. We further propose an iterative scheme for optimizing bias and frequency reuse. As verified by simulation, the system performance can be significantly improved using the optimized parameters. Compared to the approach of dynamically optimizing user association and frequency reuse for each channel realization, our solution is based on a stochastic model of the BS and user locations, and can therefore provide useful insight to deployment optimization of heterogeneous networks.

## References


[1] A. Damnjanovic, J. Montojo, Y. Wei, T. Ji, T. Luo, M. Vajapeyam, T. Yoo, O. Song, and D. Malladi, "A survey on 3GPP heterogeneous networks," *IEEE Wireless Commun.*, vol. 18, no. 3, pp. 10–21, June 2011.

[2] K. Son, S. Chong, and G. de Veciana, "Dynamic association for load balancing and interference avoidance in multi-cell networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3566–3576, July 2009.

[3] R. Madan, J. Borran, A. Sampath, N. Bhushan, A. Khandekar, and T. Ji, "Cell association and interference coordination in heterogeneous LTE-A cellular networks," *IEEE J. Sel. Areas Commun.*, vol. 28, no. 9, pp. 1479–1489, Dec. 2010.

[4] J.-W. Lee, R. R. Mazumdar, and N. B. Shroff, "Joint resource allocation and base-station assignment for the downlink in CDMA networks," *IEEE/ACM Trans. Netw.*, vol. 14, no. 1, pp. 1–14, Feb 2006.

[5] Q. Ye, B. Rong, Y. Chen, M. Al-Shalash, C. Caramanis, and J. G. Andrews, "User association for load balancing in heterogeneous cellular networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 6, pp. 2706–2716, June 2013.

[6] K. Shen and W. Yu, "Downlink cell association optimization for heterogeneous networks via dual coordinate descent," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Process. (ICASSP)*, 2013.

[7] M. Hong and A. Garcia, "Mechanism design for base station association and resource allocation in downlink OFDMA network," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 11, pp. 2238–2250, Dec. 2012.

[8] M. Hong, A. Garcia, J. Barrera, and S. G. Wilson, "Joint access point selection and power allocation for uplink wireless networks," 2012. [Online]. Available: arXiv:1207.4393

[9] M. Hong and Z.-Q. Luo, "Distributed linear precoder optimization and base station selection for an uplink heterogeneous network," *IEEE Trans. Signal Process.*, vol. 61, no. 12, pp. 3214–3228, June 2012.

[10] H. Jo, Y. Sang, P. Xia, and J. Andrews, "Heterogeneous cellular networks with flexible cell association: A comprehensive downlink SINR analysis," *IEEE Trans. Wireless Commun.*, vol. 11, no. 10, pp. 3484–3495, Oct. 2012.

[11] S. Singh, H. Dhillon, and J. Andrews, "Offloading in heterogeneous networks: Modeling, analysis, and design insights," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 2484–2497, May 2013.

[12] S. Singh and J. G. Andrews, "Joint resource partitioning and offloading in heterogeneous cellular networks," 2013. [Online]. Available: arXiv:1303.7039

[13] M. Sternad, T. Ottosson, A. Ahlén, and A. Svensson, "Attaining both coverage and high spectral efficiency with adaptive OFDM downlinks," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, 2003.

[14] Huawei, "R1-050507: Soft frequency reuse scheme for UTRAN LTE," in *3GPP TSG RAN WG1 Meeting #41*, May 2005.

[15] J. Andrews, F. Baccelli, and R. K. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Trans. Commun.*, vol. 59, no. 11, pp. 3122–3134, Nov. 2011.

[16] M. Haenggi, *Stochastic Geometry for Wireless Networks*. Cambridge University Press, 2012.