Joint Sparse Beamforming and Network Coding for Downlink Multi-Hop Cloud Radio Access Networks

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***Abstract*—This paper proposes a joint design of the routing strategy over the fronthaul network and the transmission strategy over the wireless network in a downlink cloud radio access network (C-RAN), in which the remote radio heads (RRHs) are connected to the central processor (CP) via multi-hop routers. The data-sharing strategy is adopted, where the CP multicasts each user's data to all the RRHs serving this user via the multi-hop fronthaul network, which then cooperatively serve the users through joint beamforming. Such a setting naturally provides an opportunity for applying the technique of network coding to efficiently reduce the multicast traffic in the fronthaul network. A novel cross-layer optimization framework is then investigated, where the RRH's beamforming vectors as well as the user-RRH association in the physical-layer, and the network coding design in the network-layer are jointly optimized to maximize the throughput of C-RAN subject to fronthaul link capacity constraints. This paper proposes a two-stage algorithm to solve this problem using the techniques of sparse optimization and successive convex approximation. Simulation results are provided to verify the effectiveness of the proposed cross-layer design in the downlink multi-hop C-RAN.**

I. Introduction

Cloud radio access network (C-RAN), in which multiple distributed access points known as remote radio heads (RRHs) serve mobile users cooperatively under the coordination of the central processor (CP), is envisioned as a promising candidate for the 5G cellular network on the roadmap. In the literature, a considerable amount of effort has been dedicated to reducing the fronthaul capacity required in the downlink communication in C-RAN (see e.g., [1] and the references therein). Among them, the data-sharing scheme has attracted a great deal of attention, where the CP multicasts each user's data to all the RRHs serving this user over the fronthaul network, which then encode the user messages into wireless signals and cooperatively transmit them to users [2]–[5].

Most previous works in this area, however, focus on the beamforming design across the RRHs alone and ignore the routing of user data in the fronthaul network. This paper points out that the joint optimization of the transmission strategy in the physical-layer together with the routing strategy in the network-layer can significantly improve the throughput of downlink C-RAN, especially when the fronthaul network consists of edge routers and network processors over multiple hops, as illustrated in Fig. 1. A key observation is that such a cross-layer design provides an opportunity to leverage the network coding technique [6] for multicasting user data to the corresponding RRHs over the multi-hop fronthaul network.

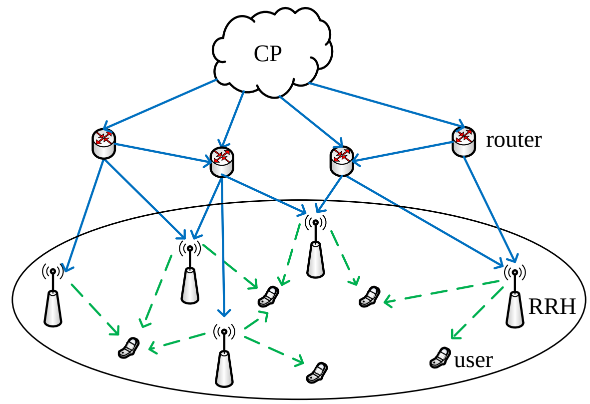

Fig. 1. System model of downlink multi-hop C-RAN.

This paper formulates a throughput maximization problem for C-RAN subject to multiple fronthaul link capacity constraints and proposes to jointly optimize the RRH's beamforming vectors, user-RRH association, and network coding based routing in an overall design. By applying the techniques of sparse optimization and successive convex approximation, we propose a two-stage algorithm to efficiently solve the studied problem: first, we approximate each user-RRH's discrete association indicator function by a continuous function and obtain a user-RRH association solution; then we fix this user-RRH association and find the corresponding beamforming and network coding strategy. Numerical examples show that the proposed scheme outperforms other data-sharing based benchmark schemes in terms of the network throughput.

It is worth noting that the joint beamforming and user-RRH association design in the downlink C-RAN has been previously investigated in [3], but without considering the optimization of the routing strategy. On the other hand, [7] proposes to jointly design the transmission and routing strategy in the downlink C-RAN, but in the model of [7] each user is solely served by one RRH, and the CP unicasts the data of each user to its associated RRH. Our paper differs from [3], [7] in allowing cooperative beamforming among RRHs and in the utilization of network coding technique over the fronthaul network for information multicast. Finally, the cross-layer design of the multi-hop C-RAN has been studied in the uplink in [8], where the RRHs utilize a compress-and-forward strategy. The downlink cross-layer design is different because of the need to optimize both the user-RRH association as well as the information multicast in the fronthaul network.

II. System Model

Consider the downlink communication in C-RAN where N RRHs, denoted by the set $\mathcal{N} = \{1, \cdots, N\}$, cooperatively serve K users, denoted by the set $\mathcal{K} = \{1, \cdots, K\}$, under the coordination of the CP. It is assumed that each RRH is equipped with $M \geq 1$ antennas, while each user is equipped with one single antenna. Moreover, we assume that the CP and RRHs communicate over a multi-hop fronthaul network consisting of J routers, denoted by the set $\mathcal{J} = \{1, \cdots, J\}$, and L digital fronthaul links, denoted by the set $\mathcal{L} = \{1, \cdots, L\}$, as shown in Fig. 1. The capacity of each link $l \in \mathcal{L}$ is denoted by C_l bits per second (bps). A novel network coding based data-sharing scheme is adopted in this paper, where the CP multicasts each user's message to all the RRHs serving it via the multi-hop fronthaul network using network coding technique [6], and each RRH then encodes the user messages into wireless signals and sends them to the users. In the following, we introduce the proposed cross-layer architecture for the downlink multi-hop C-RAN in detail.

A. Beamforming over wireless network

For the wireless network, it is assumed that the N RRHs communicate with the K users over quasi-static flat-fading channels over a given bandwidth of B Hz. The equivalent baseband transmit signal of RRH n is

$$\boldsymbol{x}_n = \sum_{k=1}^{K} \boldsymbol{w}_{k,n} s_k, \quad \forall n, \tag{1}$$

where $s_k \sim \mathcal{CN}(0, 1)$ denotes the message intended for user k, which is modeled as a circularly symmetric complex Gaussian (CSCG) random variable with zero-mean and unit-variance, and $\boldsymbol{w}_{k,n} \in \mathbb{C}^{M \times 1}$ denotes RRH n's beamforming vector for user k. Suppose that RRH n has a transmit sum-power constraint P_n; from (1), we have

$$\mathcal{E}[\boldsymbol{x}_n \boldsymbol{x}_n^H] = \sum_{k=1}^{K} \|\boldsymbol{w}_{k,n}\|^2 \leq P_n, \quad \forall n. \tag{2}$$

The received signal of user k can be expressed as

$$\begin{aligned} y_k &= \sum_{n=1}^{N} \boldsymbol{h}_{k,n}^H \boldsymbol{x}_n + z_k \\ &= \sum_{n=1}^{N} \boldsymbol{h}_{k,n}^H \boldsymbol{w}_{k,n} s_k + \sum_{n=1}^{N} \boldsymbol{h}_{k,n}^H \sum_{i \neq k} \boldsymbol{w}_{i,n} s_i + z_k, \ \forall k, \end{aligned} \tag{3}$$

where $\boldsymbol{h}_{k,n} \in \mathbb{C}^{M \times 1}$ denotes the channel from RRH n to user k, and $z_k \sim \mathcal{CN}(0, \sigma^2)$ denotes the additive white Gaussian noise (AWGN) at user k. In this paper, it is assumed that the channels to all the K users are perfectly known at the CP.

The signal-to-interference-plus-noise ratio (SINR) for user k is expressed as

$$\gamma_k = \frac{\left| \sum_{n=1}^{N} \boldsymbol{h}_{k,n}^H \boldsymbol{w}_{k,n} \right|^2}{\sum_{i \neq k} \left| \sum_{n=1}^{N} \boldsymbol{h}_{k,n}^H \boldsymbol{w}_{i,n} \right|^2 + \sigma^2} = \frac{|\boldsymbol{h}_k^H \boldsymbol{w}_k|^2}{\sum_{i \neq k} |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 + \sigma^2}, \ \forall k, \tag{4}$$

where $\boldsymbol{h}_k = [\boldsymbol{h}_{k,1}^T, \cdots, \boldsymbol{h}_{k,N}^T]^T$ denotes the effective channel from all RRHs to user k, and $\boldsymbol{w}_k = [\boldsymbol{w}_{k,1}^T, \cdots, \boldsymbol{w}_{k,N}^T]^T$ denotes the effective beamforming vector for user k across all RRHs. The achievable rate of user k in bps is given by

$$r_k \leq B \log_2(1 + \gamma_k), \quad \forall k. \tag{5}$$

B. Network coding over fronthaul network

Next, consider the data transmission from the CP to RRHs over the digital multi-hop fronthaul network. It is worth noting that if $\boldsymbol{w}_{k,n} \neq \boldsymbol{0}$, then user k is served by RRH n; otherwise, user k is not served by RRH n. As a result, we can define the user-RRH association indicator function $\alpha_{k,n}(\boldsymbol{w}_{k,n})$ as follows:

$$\alpha_{k,n}(\boldsymbol{w}_{k,n}) = \begin{cases} 1, & \text{if } \|\boldsymbol{w}_{k,n}\|^2 \neq \boldsymbol{0}, \\ 0, & \text{otherwise}, \end{cases} \quad \forall k, n. \tag{6}$$

If user k is served by RRH n, i.e., $\alpha_{k,n}(\boldsymbol{w}_{k,n}) = 1$, the CP needs to send the digital messages s_k to RRH n over the multi-hop fronthaul network at a rate of r_k bps; otherwise, the CP does not need to send s_k to RRH n. To summarize, there are K multicast sessions in the multi-hop fronthaul network, i.e., $s_1, \cdots, s_K$, and each session s_k has a set $\mathcal{D}_k = \{n : \alpha_{k,n}(\boldsymbol{w}_{k,n}) = 1, n = 1, \cdots, N\}$ of destinations.

The traditional approach for information multicast is to make each router replicate and forward its received information to the downstream routers. However, the optimization of such multicast routing is equivalent to the Steiner tree packing problem, which is NP-hard [9], [10]. Moreover, this replicate-and-forward based routing strategy is suboptimal since the coding operations at routers are necessary to achieve the multicast capacity [6]. In this paper, we propose to apply the network coding technique to multicast each session to its destinations independently, but do not code between different sessions for the following reasons. First, this strategy results in an easy characterization of the routing region, therefore makes the optimal multicast routing problem polynomial time computable. Second, intersession coding provides marginal throughput gains over this approach [9], [10].

Network coding enables flows for different destinations of a multicast session to share network capacity without competition: the actual physical flow rate on each link only needs to be the maximum rate of the individual destination's flows. According to [9], [10], the routing constraints for the multi-hop fronthaul network can be formulated as

$$\alpha_{k,n}(\boldsymbol{w}_{k,n}) r_k \leq \sum_{l \in \mathcal{I}(\mathcal{N}_n)} d_l^{k,n}, \quad \forall k, n, \tag{7}$$

$$\sum_{l \in \mathcal{O}(\mathcal{J}_j)} d_l^{k,n} = \sum_{l \in \mathcal{I}(\mathcal{J}_j)} d_l^{k,n}, \quad \forall k, n, j, \tag{8}$$

$$d_l^{k,n} \leq f_l^k, \quad \forall n, k, l, \tag{9}$$

$$\sum_{k=1}^{K} f_l^k \leq C_l, \quad \forall l, \tag{10}$$

$$f_l^k \geq 0, \quad d_l^{k,n} \geq 0, \quad \forall k, n, l, \tag{11}$$

where $d_l^{k,n}$ denotes the conceptual flow rate on link $l \in \mathcal{L}$ for the kth multicast session to its potential destination RRH

n, f_l^k denotes the actual flow rate on link l for multicast session k, $\mathcal{N}_n$ and $\mathcal{J}_j$ denote RRH n and router j, respectively, $\mathcal{I}(\mathcal{N}_n)$ denotes the set of links that are incoming to RRH n, and $\mathcal{I}(\mathcal{J}_j)$ and $\mathcal{O}(\mathcal{J}_j)$ denote the set of links that are incoming to and outgoing from router j, respectively. The first constraint guarantees that if $n \in \mathcal{D}_k$, then the kth session must flow at rate r_k to its destination RRH n. The second constraint represents the law of flow conservation for conceptual flows. Note that the flow conservation constraint for the CP is not considered because it is automatically guaranteed by constraints (7) and (8). The third constraint indicates that the actual flow rate of the kth multicast session at each link l is the maximum rate of the conceptual flows of that link to all the destinations, which is the benefit of network coding. The fourth constraint guarantees that the overall information flow rate at each link does not exceed the link capacity. The last constraint guarantees a positive flow rate for all the multicast sessions on all the links.[1]

III. Problem Formulation

In this paper, we aim to maximize the throughput of downlink multi-hop C-RAN via a joint optimization of the resources available in the physical-layer and network-layer. Specifically, we design the beamforming vectors at all RRHs, i.e., $\boldsymbol{w}_{k,n}$'s, and network coding strategy, i.e., $d_l^{k,n}$'s and f_l^k's, to maximize the sum-rate of all the users subject to each RRH's transmit power constraint over the wireless network as well as the network coding constraints in the multi-hop fronthaul network, i.e.,

$$\underset{\{\boldsymbol{w}_{k,n},r_k,d_l^{k,n},f_l^k\}}{\text{maximize}} \quad \sum_{k=1}^{K} r_k \tag{12a}$$

$$\text{subject to} \quad (2),\ (5),\ (7)-(11). \tag{12b}$$

It is worth noting that without the network coding constraints given in (7) – (11), each user should be served by all the RRHs, i.e., $\alpha_{k,n}(\boldsymbol{w}_{k,n}) = 1$. However, with the newly introduced network coding constraints given in (7) – (11), in general each RRH cannot support all the users in the downlink transmission, and as a result, from (6), for each RRH n, only a subset of users are associated with it, for which the corresponding user association function $\alpha_{k,n}(\boldsymbol{w}_{k,n})$ and beamforming vector $\boldsymbol{w}_{k,n}$ are non-zero. Moreover, the user association functions $\alpha_{k,n}(\boldsymbol{w}_{k,n})$'s also affect the network coding design since they determine the destinations of each multicast session. Therefore, the RRH's beamforming, user-RRH association, and network coding are coupled together and need to be jointly optimized in problem (12), which is a challenging problem in general.

It is also worth noting that constraint (7) induces a sparse beamforming solution to problem (12). In the literature, sparse optimization technique has been previously used for the downlink beamforming design problem [5], [12]. Problem (12) differs from prior work in two aspects. First, [5], [12] encourage a sparse beamforming solution by penalizing the objective function with a sparsity term. However, problem (12) considered in this paper imposes a set of sparsity constraints which need to be strictly satisfied. Second, in [5], [12] the sparsity penalty is independent of the beamforming solution, but in constraint (7) of our studied problem they are coupled. As a result, the existing sparse optimization techniques, e.g., least-absolute shrinkage and selection operator (LASSO), cannot be applied in this paper.

IV. Proposed Two-Stage Algorithm

In this section, we propose an efficient algorithm to solve problem (12) based on the techniques of sparse optimization as well as successive convex approximation. One main challenge for solving problem (12) is the discrete indicator function $\alpha_{k,n}(\boldsymbol{w}_{k,n})$ defined in (6). By applying standard sparse optimization technique, in this paper we use the following continuous function to approximate $\alpha_{k,n}(\boldsymbol{w}_{k,n})$:

$$g_\Phi(\boldsymbol{w}_{k,n}) = 1 - e^{-\Phi\|\boldsymbol{w}_{k,n}\|^2}, \quad \forall k, n, \tag{13}$$

where $\Phi \gg 1$. It can be observed that when $\|\boldsymbol{w}_{k,n}\|^2 = 0$, then $g_\Phi(\boldsymbol{w}_{k,n}) = \alpha_{k,n}(\boldsymbol{w}_{k,n}) = 0$. Otherwise, if $\|\boldsymbol{w}_{k,n}\|^2 > 0$, we have $g_\Phi(\boldsymbol{w}_{k,n}) \to \alpha_{k,n}(\boldsymbol{w}_{k,n}) = 1$ with $\Phi \gg 1$.

By using $g_\Phi(\boldsymbol{w}_{k,n})$ to approximate $\alpha_{k,n}(\boldsymbol{w}_{k,n})$, $\forall k, n$, problem (12) becomes the following continuous problem.

$$\underset{\{\boldsymbol{w}_{k,n},r_k,d_l^{k,n},f_l^k\}}{\text{maximize}} \quad \sum_{k=1}^{K} r_k \tag{14a}$$

$$\text{subject to} \quad g_\Phi(\boldsymbol{w}_{k,n})r_k \le \sum_{l\in\mathcal{I}(\mathcal{N}_n)} d_l^{k,n},\ \forall k, n, \tag{14b}$$

$$(2),\ (5),\ (8)-(11). \tag{14c}$$

However, since $g_\Phi(\boldsymbol{w}_{k,n})$ is strictly less than one when $\|\boldsymbol{w}_{k,n}\|^2 > 0$, the solution to problem (14), which satisfies constraint (14b), may not satisfy constraint (7) in problem (12). As a result, in this paper we propose to solve problem (12) in two steps as follows. First, we solve problem (14) and obtain the beamforming solution, denoted by $\hat{\boldsymbol{w}}_{k,n}$'s. The user-RRH association solution is then obtained as follows:

$$\alpha_{k,n}(\hat{\boldsymbol{w}}_{k,n}) = \begin{cases} 1, & \text{if } g_\Phi(\hat{\boldsymbol{w}}_{k,n}) \ge \psi, \\ 0, & \text{otherwise}, \end{cases} \quad \forall k, n, \tag{15}$$

where $0 \le \psi \le 1$ is a threshold to control the user association solution.[2] Second, we fix this user association solution in problem (12) and solve the following simplified problem to refine the beamforming and network coding strategy:

$$\underset{\{\boldsymbol{w}_{k,n},r_k,d_l^{k,n},f_l^k\}}{\text{maximize}} \quad \sum_{k=1}^{K} r_k \tag{16a}$$

$$\text{subject to} \quad \alpha_{k,n}(\hat{\boldsymbol{w}}_{k,n})r_k \le \sum_{l\in\mathcal{I}(\mathcal{N}_n)} d_l^{k,n},\ \forall k, n, \tag{16b}$$

$$\|\boldsymbol{w}_{k,n}\|^2 = 0,\ \forall\ \alpha_{k,n}(\hat{\boldsymbol{w}}_{k,n}) = 0, \tag{16c}$$

$$(2),\ (5),\ (8)-(11). \tag{16d}$$

In the following, we show how to solve problems (14) and (16), respectively.

[1] Given any flow rate solution satisfying constraints (7) – (11), the code design which determines the content of each flow being transmitted across the network can be found according to [11].

[2] In our simulation, we set $\Phi = 50$ and $\psi = 0.5$.

A. The First Stage: Solution to Problem (14)

Problem (14) is a non-convex problem due to constraints (5) and (14b). As a result, the conventional convex optimization technique cannot be directly applied to solve it. In this section, we propose an efficient algorithm to solve problem (14) suboptimally based on the technique of successive convex approximation.

First, we consider constraint (5), which is equivalent to

$$\frac{|\boldsymbol{h}_k^H \boldsymbol{w}_k|^2}{\sum\limits_{i\neq k} |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 + \sigma^2} \geq 2^{\frac{r_k}{B}} - 1, \quad \forall k. \tag{17}$$

By introducing a set of auxiliary variables $\eta_k \geq 0$'s, $k = 1, \cdots, K$, it can be shown that constraint (17) is equivalent to the following two constraints:

$$\boldsymbol{h}_k^H \boldsymbol{w}_k \geq \sqrt{(2^{\frac{r_k}{B}} - 1)\eta_k}, \quad \forall k, \tag{18}$$

$$\sqrt{\sum_{i\neq k} |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 + \sigma^2} \leq \sqrt{\eta_k}, \quad \forall k. \tag{19}$$

As a result, η_k can be interpreted as the interference constraint for user k. Constraint (19) can be further transformed into the following convex second-order cone (SOC) constraint:

$$\left\| [\boldsymbol{h}_k^H \boldsymbol{w}_1, \cdots, \boldsymbol{h}_k^H \boldsymbol{w}_{k-1}, \boldsymbol{h}_k^H \boldsymbol{w}_{k+1}, \cdots, \boldsymbol{h}_k^H \boldsymbol{w}_K]^T \right\| \leq \sqrt{\eta_k - \sigma^2}, \quad \forall k. \tag{20}$$

For constraint (18), $\sqrt{(2^{r_k/B} - 1)\eta_k}$ is not a convex function. However, given any $\tilde{\beta}_k$, the following convex function is an upper bound for $\sqrt{(2^{r_k/B} - 1)\eta_k}$:

$$f_{\tilde{\beta}_k}(r_k, \eta_k) = \frac{\tilde{\beta}_k \eta_k}{2} + \frac{2^{\frac{r_k}{B}} - 1}{2\tilde{\beta}_k} \geq \sqrt{(2^{\frac{r_k}{B}} - 1)\eta_k}, \ \forall k, \tag{21}$$

where the equality holds if and only if $\tilde{\beta}_k = \sqrt{(2^{r_k/B} - 1)/\eta_k}$. As a result, we use the following convex constraint to approximate constraint (18):

$$\boldsymbol{h}_k^H \boldsymbol{w}_k \geq \frac{\tilde{\beta}_k \eta_k}{2} + \frac{2^{\frac{r_k}{B}} - 1}{2\tilde{\beta}_k}, \ \forall k. \tag{22}$$

After approximating the non-convex constraint (5) by the convex ones (20) and (22), we come to constraint (14b). First, we take the natural logarithm of the left-hand side (LHS) and right-hand side (RHS) of inequality constraint (14b), which results in

$$\log(1 - e^{-\Phi\|\boldsymbol{w}_{k,n}\|^2}) + \log(r_k) \leq \log\left(\sum_{l \in \mathcal{I}(\mathcal{N}_n)} d_l^{k,n} \right), \ \forall k, n. \tag{23}$$

It can be shown that $\log(\sum_{l\in\mathcal{I}(\mathcal{N}_n)} d_l^{k,n})$ is a concave function over $d_l^{k,n}$'s. However, the LHS of constraint (23) is still non-convex. Since $\log(1 - e^{-\Phi x})$ is a concave function over x, its first-order approximation serves as its upper bound. Specifically, given any $\tilde{x}$, the first-order approximation of $\log(1 - e^{-\Phi x})$ can be expressed as

$$\log(1 - e^{-\Phi x}) \leq \frac{\Phi e^{-\Phi\tilde{x}}(x - \tilde{x})}{1 - e^{-\Phi\tilde{x}}} + \log(1 - e^{-\Phi\tilde{x}}), \tag{24}$$

where the equality holds if and only if $x = \tilde{x}$. By substituting x with $\|\boldsymbol{w}_{k,n}\|^2$, given any $\tilde{\boldsymbol{w}}_{k,n}$, a convex upper bound for $\log(1 - e^{-\Phi\|\boldsymbol{w}_{k,n}\|^2})$ is expresses as

$$\log(1 - e^{-\Phi\|\boldsymbol{w}_{k,n}\|^2}) \leq \frac{\Phi e^{-\Phi\|\tilde{\boldsymbol{w}}_{k,n}\|^2}\|\boldsymbol{w}_{k,n}\|^2}{1 - e^{-\Phi\|\tilde{\boldsymbol{w}}_{k,n}\|^2}} + \phi(\tilde{\boldsymbol{w}}_{k,n}), \ \forall k, n, \tag{25}$$

where

$$\phi(\tilde{\boldsymbol{w}}_{k,n}) = -\frac{\Phi e^{-\Phi\|\tilde{\boldsymbol{w}}_{k,n}\|^2}\|\tilde{\boldsymbol{w}}_{k,n}\|^2}{1 - e^{-\Phi\|\tilde{\boldsymbol{w}}_{k,n}\|^2}} + \log(1 - e^{-\Phi\|\tilde{\boldsymbol{w}}_{k,n}\|^2}).$$

The equality holds if and only if $\boldsymbol{w}_{k,n} = \tilde{\boldsymbol{w}}_{k,n}$.

Similarly, given any point $\tilde{r}_k$, the concave function $\log(r_k)$ can be approximated by its first-order approximation as follows:

$$\log(r_k) \leq \frac{r_k - \tilde{r}_k}{\tilde{r}_k} + \log(\tilde{r}_k), \quad \forall k, \tag{26}$$

where the equality holds if and only if $r_k = \tilde{r}_k$.

With (25) and (26), the non-convex constraint (23) can be approximated by the following convex constraint:

$$\frac{\Phi e^{-\Phi\|\tilde{\boldsymbol{w}}_{k,n}\|^2}\|\boldsymbol{w}_{k,n}\|^2}{1 - e^{-\Phi\|\tilde{\boldsymbol{w}}_{k,n}\|^2}} + \frac{r_k - \tilde{r}_k}{\tilde{r}_k} + \phi(\tilde{\boldsymbol{w}}_{k,n}) + \log(\tilde{r}_k) \leq \log\left(\sum_{l\in\mathcal{I}(\mathcal{N}_n)} d_l^{k,n} \right), \quad \forall k, n. \tag{27}$$

To summarize, given $\tilde{r}_k$'s, $\tilde{\boldsymbol{w}}_{k,n}$'s, and $\tilde{\beta}_k$'s, the non-convex constraints (5) and (14b) in problem (14) are approximated by the convex constraints given in (20), (22), and (27). As a result, with any given $\tilde{r}_k$'s, $\tilde{\boldsymbol{w}}_{k,n}$'s, and $\tilde{\beta}_k$'s, problem (14) is approximated by the following convex problem.

$$\underset{\{\boldsymbol{w}_{k,n}, r_k, \eta_k, d_l^{k,n}, f_l^k\}}{\text{maximize}} \quad \sum_{k=1}^{K} r_k \tag{28a}$$

$$\text{subject to} \quad (2), (20), (22), (27), (8) - (11). \tag{28b}$$

Since problem (28) is a convex problem, it can be globally solved by CVX. The successive convex approximation method based algorithm to problem (14) is summarized in Algorithm 1, which iteratively updates $\tilde{r}_k$'s, $\tilde{\boldsymbol{w}}_{k,n}$'s, and $\tilde{\beta}_k$'s based on the solution to problem (28) as shown in Step b.2). The convergence behaviour of Algorithm 1 is guaranteed in the following proposition.

Proposition 1: Monotonic convergence of Algorithm 1 is guaranteed, i.e., $\sum_{k=1}^K r_k^{(t)} \geq \sum_{k=1}^K r_k^{(t-1)}$. Moreover, the converged solution satisfies all the constraints as well as the Karush-Kuhn-Tucker (KKT) conditions of problem (14).

Proof: First, it can be shown that in the tth iteration of Algorithm 1, the solution obtained in the $(t-1)$th iteration is also feasible to problem (28) given $\tilde{\boldsymbol{w}}_{k,n} = \boldsymbol{w}_{k,n}^{(t-1)}$,

Algorithm 1 Proposed Algorithm for Solving Problem (14)

Initialization: Set the initial values for $\tilde{\boldsymbol{w}}_{k,n}$'s, $\tilde{r}_k$'s, and $\tilde{\beta}_k$'s and set $t = 1$;

Repeat:

1) Find the optimal solution to problem (28) using CVX as $\{\boldsymbol{w}_{k,n}^{(t)}, r_k^{(t)}, \eta_k^{(t)}, (d_l^{k,n})^{(t)}, (f_l^k)^{(t)}\}$;
2) Update $\tilde{\boldsymbol{w}}_{k,n} = \boldsymbol{w}_{k,n}^{(t)}$, $\tilde{r}_k = r_k^{(t)}$, and $\tilde{\beta}_k = \sqrt{(2^{r_k^{(t)}} - 1)/\eta_k^{(t)}}$, $\forall k, n$;
3) $t = t + 1$.

Until convergence

Algorithm 2 Proposed Algorithm for Solving Problem (16)

Initialization: Set the initial values for $\tilde{\beta}_k$'s and set $t = 1$;

Repeat:

1) Find the optimal solution to problem (29) using CVX as $\{\boldsymbol{w}_{k,n}^{(t)}, r_k^{(t)}, \eta_k^{(t)}, (d_l^{k,n})^{(t)}, (f_l^k)^{(t)}\}$;
2) Update $\tilde{\beta}_k = \sqrt{\eta_k^{(t)}/(2^{r_k^{(t)}} - 1)}$, $\forall k, n$;
3) $t = t + 1$.

Until convergence

$\tilde{r}_k = r_k^{(t-1)}$, and $\tilde{\beta}_k = \sqrt{(2^{r_k^{(t-1)}} - 1)/\eta_k^{(t-1)}}$, $\forall k, n$. In other words, $\sum_{k=1}^K r_k^{(t-1)}$ is achievable to problem (28) in the tth iteration. As a result, the optimal weighted sum-rate to problem (28) in the tth iteration, i.e., $\sum_{k=1}^K r_k^{(t)}$, is no smaller than the optimal weighted sum-rate achieved in the $(t-1)$th iteration, i.e., $\sum_{k=1}^K r_k^{(t-1)}$. Monotonic convergence of Algorithm 1 is thus proved.

Next, since in Algorithm 1 we use upper-bound to approximate the non-convex functions in problem (14), as shown in (21), (24), and (26), any feasible solution to problem (28) satisfies all the constraints of problem (14). As a result, the solution from Algorithm 1 must be feasible to problem (14).

Last, according to [13, Theorem1], the solution obtained by the successive convex approximation based Algorithm 1 must satisfy the KKT conditions of problem (14). ■

B. The Second Stage: Solution to Problem (16)

Given the user association in problem (16), constraint (16b) becomes convex. By using (20) and (22) to approximate the non-convex constraint (5), given any $\tilde{\beta}_k$'s, problem (16) can be approximated by the following convex problem.

$$\underset{\{\boldsymbol{w}_{k,n}, r_k, d_l^{k,n}, f_l^k\}}{\text{maximize}} \quad \sum_{k=1}^K r_k \tag{29a}$$

$$\text{subject to} \quad \|\boldsymbol{w}_{k,n}\|^2 \leq 0, \ \forall \alpha_{k,n}(\hat{\boldsymbol{w}}_{k,n}) = 0, \tag{29b}$$

$$(2), (20), (22), (16b), (8) - (11). \tag{29c}$$

Since problem (29) is a convex problem, it can be efficiently solved. The successive convex approximation based algorithm to problem (16) is summarized in Algorithm 2. Similar to Proposition 1, the convergence behaviour of Algorithm 2 is guaranteed in the following proposition.

Algorithm 3 Overall Algorithm for Solving Problem (12)

1) Solve problem (14) based on Algorithm 1 and obtain the user-RRH association according to (15);
2) Solve problem (16) based on Algorithm 2 and obtain the beamforming and network coding solution.

Proposition 2: Monotonic convergence of Algorithm 2 is guaranteed, i.e., $\sum_{k=1}^K r_k^{(t)} \geq \sum_{k=1}^K r_k^{(t-1)}$. Moreover, the converged solution satisfies all the constraints as well as the KKT conditions of problem (16).

The overall two-stage algorithm to problem (12) is summarized in Algorithm 3.

Remark 1: It is worth noting that [3] studies a similar problem of jointly optimizing the user-RRH association with the beamforming vectors. To deal with the discrete user-RRH association indicator functions (6), in [3] the reweighted ℓ_1-norm technique is employed to approximate the fronthaul constraint (7) by a set of weighted per-RRH power constraints. Then, an alternating optimization based iterative algorithm is proposed to find a beamforming and user-RRH association solution. Although the algorithm in [3] works well in practice, a rigorous convergence proof is not available. In contrast, the algorithm proposed in this paper always converge, but the performance depends on the tuning of the approximation parameters Φ and ψ.

V. NUMERICAL RESULTS

In this section, we provide one numerical example to verify the effectiveness of our proposed network coding based data-sharing strategy in the downlink multi-hop C-RAN. In this example, there are $N = 5$ RRHs, each equipped with $M = 2$ antennas, and $K = 10$ users randomly distributed in a circle area of radius 1000m. The bandwidth of the wireless link is $B = 10$MHz. The channel vectors are generated from independent Rayleigh fading, while the path loss model of the wireless channel is given as $128.1 + 37.6\log_{10}(D)$ in dB, where D (in kilometer) denotes the distance between the user and the RRH. The transmit power constraint for each RRH is $P_n = 43$dBm, $\forall n$. The power spectral density of the AWGN at each user receiver is assumed to be -169dBm/Hz, and the noise figure due to the receiver processing is 7dB. Moreover, the fronthaul network topology together with the capacities of the fronthaul links (denoted by $2C$, $C/2$, or $C/3$) are shown in Fig. 2.

Besides the proposed algorithm, we also consider the following two benchmark schemes for performance comparison. For the first benchmark scheme, we consider a strategy where each user is only served by one RRH, as proposed in [7]. Specifically, we first allocate each user to the RRH with the strongest channel power, i.e.,

$$\alpha_{k,n} = \begin{cases} 1, & \text{if } n = \arg\max\limits_{1 \leq n \leq N} \|\boldsymbol{h}_{k,n}\|^2, \\ 0, & \text{otherwise}, \end{cases} \quad \forall k, n. \tag{30}$$

Given the above user-RRH association solution, the CP unicasts each user's data to its associated RRH via routing over the fronthaul network. Note that in a unicast network, the network

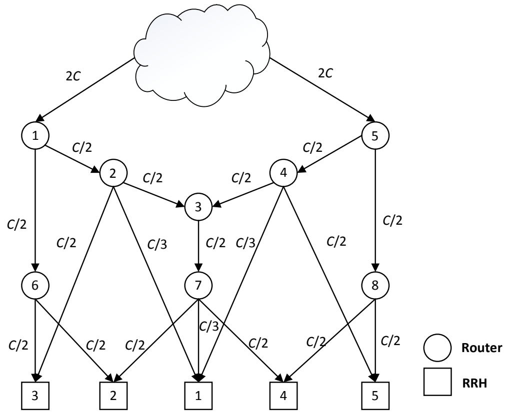

Fig. 2. The fronthaul network topology.

coding constraints given in (7) – (11) reduce to the unicasting constraints. As a result, the sum-rate of all the users achieved by this scheme can be obtained by solving problem (16) with the user-RRH association solution given in (30).

For the second benchmark scheme, we allow each user to be served by multiple RRHs, and consider a strategy where each user's data is treated as a commodity and unicast to each of the RRHs that serve this user. With this scheme, the actual flow rate on link l to transmit s_k is

$$f_l^k = \sum_{n=1}^{N} d_l^{k,n}, \quad \forall k, l. \tag{31}$$

By replacing (9) with (31), the routing constraints can be modeled by (7), (8), (31), (10), and (11). Algorithm 3 can thus be applied to jointly optimize the RRH's beamforming, user-RRH association, and unicast routing.

Fig. 3 shows the users' sum-rate achieved by different schemes versus different values of C, which determines the capacities of fronthaul links as shown in Fig. 2. It is observed that the proposed data-sharing strategy achieves much higher throughput than its counterpart without cooperation between RRHs, especially when the value of C is large. This is because our proposed scheme provides a joint beamforming design gain. It is also observed that the proposed network coding based scheme provides up to 25% throughput gain as compared to the unicast scheme. This is because network coding can significantly reduce the fronthaul traffic. Specifically, with network coding, the actual flow rate to transmit s_k at each link l, i.e., f_l^k, is the maximum rate of $d_l^{k,n}$'s, $\forall n$, as shown in (9), while with unicast, f_l^k is the summation of $d_l^{k,n}$'s over n, as shown in (31). Finally, we remark that the user-RRH association strategy in [3] could also been incorporated with network coding to achieve a performance very close to the two-stage method proposed in this paper.

VI. Conclusion

In this paper, we propose a novel network coding based data-sharing scheme in the downlink multi-hop C-RAN, where each user's data is multicast to the corresponding RRHs using network coding. To maximize the throughput of C-RAN subject to the fronthaul network capacity constraints, we design a cross-layer optimization framework where the RRH's beamforming as well as user-RRH association in the physical-layer, and network coding design in the network-layer are jointly optimized. Simulation results verify that the proposed network coding based data-sharing strategy is a promising solution to implement downlink communication in the future C-RAN with a multi-hop fronthaul network.

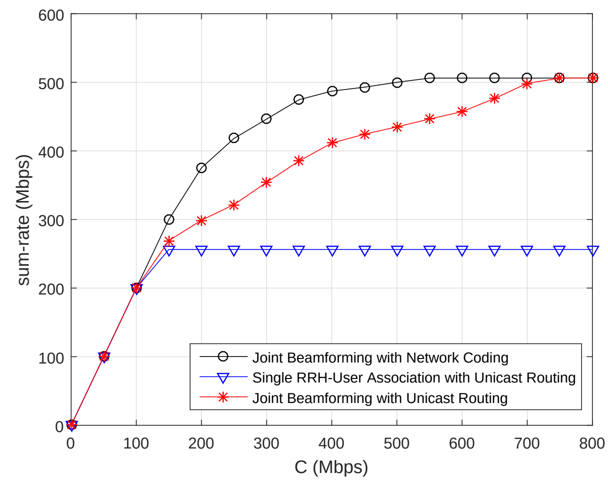

Fig. 3. Throughput versus fronthaul link capacity of network coding vs. unicast.

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