Enhance Cell-Edge Rates by Amplify-Forward Shared Relays in Dense Cellular Networks

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Abstract—This paper explores the benefits of deploying multi-antenna half-duplex amplify-and-forward shared relays at the cell-edge to assist the downlink transmission in a multiple-input multiple-output wireless cellular network. We design the relay node to provide extra spatial dimensions to multiple receivers at the same time for interference mitigation and signal enhancement. This paper proposes an efficient algorithm to solve the non-convex problem of jointly optimizing the transmit beamforming and relay combining matrices to a stationary point by extending the celebrated weighted minimum mean squared error (WMMSE) algorithm. We show that the optimized relaying strategy can significantly improve the long-term average rates of cell-edge users in a cellular network, even after accounting for the extra bandwidth required for half-duplex relaying.

I. INTRODUCTION

Increasing the density of base stations (BSs) and user terminals in the next generations of wireless cellular networks aggravates the adverse effect of uncoordinated interference on data rates, especially at the cell edges. This paper explores the idea of deploying multiple-antenna relays without dedicated backhaul to provide cell-edge receivers with extra spatial dimensions for interference mitigation and signal enhancement.

We consider the downlink of a cellular network consisting of multi-antenna BSs transmitting to their associated users, modeled as a multiple-input-multiple-output (MIMO) interfering broadcasts channel (IBC). We deploy multi-antenna relay nodes (without backhaul infrastructure) to help multiple nearby receivers at the same time over an out-of-band broadcast channel (BC). The receivers and the relay observe uncoordinated interference from surrounding BSs, so the noise processes across the receivers' and the relay's antennas are correlated. The main idea of this paper is that by amplifying-and-forwarding the relay observation, the receivers can take advantage of this noise correlation for interference cancellation. Since the relay node is infrastructureless, its deployment can be a viable candidate for enhancing performance of, e.g., vehicular or ad hoc networks.

In this paper, we choose amplify-and-forward (AF) as the relaying scheme to simultaneously help multiple receivers. Unlike decode-and-forward (DF) and compress-and-forward schemes, AF does not digitize the relaying links for serving multiple users. Moreover, the AF relaying operation is performed on a symbol-wise basis, which introduces less delay as compared to block-wise decoding or quantization operations.

Effective use of the relay requires joint optimization of beamforming (BF) at BSs and AF combining matrix at the relay node. In this work, we address optimization of weighted sum rates (WSR) of users over BF matrices and the relay combining matrix. This is a challenging optimization problem, because neither the objective nor the relay power constraint is convex. In fact, optimization of BF matrices in MIMO IBC is already a challenging problem even in the absence of the relay. In this paper, we propose using the weighted minimum mean squared error (WMMSE) algorithm to solve this problem to a stationary point. WMMSE is a well-known alternating optimization approach for BF optimization in a MIMO IBC [1]. The technical contribution of this paper is the extension of WMMSE algorithm for the MIMO IBC augmented by an AF relay node.

In [2], [3], MMSE-based coordinate ascent algorithms for joint optimization of transmit BF and AF relaying in MIMO relay-interference scenarios are developed. However, due to the coupled constraint, convergence to stationary point is not guaranteed [4]. This paper resolves this issue using a Lagrangian approach to achieve a Karush-Kuhn-Tucker (KKT) point of the problem.

Deploying a shared DF relay for mitigating downlink intercell interference is considered in [5], where focus is on bandwidth allocation and scheduling. An important consideration in deploying half-duplex relays is the extra bandwidth needed for the relay-destination links. Such extra bandwidth could have alternatively been used in the direct transmission from BSs to improve the user rates. To avoid such half-duplex loss, deploying two-way relays that simultaneously assist both uplink and downlink transmissions is proposed in [6].

Intuitively, if the signal-to-interference-and-noise ratio (SINR) of direct transmissions in the network is lower than the SINR of the relaying links, investing a part of bandwidth in relaying more than compensates for the half-duplex loss. This paper shows that this is indeed true

for cell-edge users. When the transmit BF vectors at the BSs and the AF relaying operation are jointly optimized, we illustrate that in a cellular network under proportional fairness scheduling, investing the extra bandwidth for relaying improves long-term average rates of cell-edge users more than what could have been achieved if the extra bandwidth is used for direct transmission from BSs. In particular, relaying brings in a more uniform throughput profile over the entire network.

II. SYSTEM MODEL

We consider downlink transmission in a cellular network, where a cluster of B BSs coordinate their transmit BF and are further assisted by a shared relay node, located at the cells intersection, to serve all the remote terminals in the B cells. The transmission takes place over two separate frequency bands; one for the BSs and the other for the relay. Each remote terminal receives the intended signal from its serving BS and treats the intera-cell and the inter-cell interference from all other BSs as noise; it also receives the relaying signal from the shared relay and likewise treat all interference from other relays in the network as noise. The shared relay node provides the users with extra spatial dimensions for both interference mitigation and signal enhancement.

Mathematically, the cluster of B BSs can be modeled as a MIMO Gaussian IBC augmented with a relay node. Each transmitter has an independent message for each of its associated receivers. The relay node receives signal with multiple antennas and transmits over an orthogonal BC to (a group of) receivers. Due to common interference, noise processes are correlated across relay and destinations' antennas.

The set of B BSs in the cluster is denoted by $\mathcal{B} = \{1, 2, \ldots B\}$. Each BS serves K users. The kth user served by the bth BS is referred to as user (b, k). The set of all users is $\mathcal{K} = \{(b, k) : 1 \leq b \leq B, 1 \leq k \leq K\}$. The bth BS transmits

$$\mathbf{X}_b = \sum_{k=1}^{K} V_{b,k} \mathbf{S}_{b,k}, \quad b \in \mathcal{B} \tag{1}$$

from s antennas. Vector $\mathbf{S}_{b,k} \in \mathbb{C}^{D \times 1}$ contains information symbols intended for user (b, k) that is selected from a Gaussian codebook $\mathbf{S}_{b,k} \sim \mathcal{CN}(\mathbf{0}_{D \times 1}, \mathbf{I}_D)$, where D denoted the number of independent data streams. Matrix $V_{b,k} \in \mathbb{C}^{s \times D}$ is the corresponding BF matrix. The bth BS has transmit power constraint

$$\mathbb{E}[\mathbf{X}_b^{\dagger} \mathbf{X}_b] = \sum_{k=1}^{K} \text{trace}(V_{b,k} V_{b,k}^{\dagger}) \leq P_b, \quad b \in \mathcal{B}. \tag{2}$$

User (b, k) receives

$$\mathbf{Y}_{b,k} = \sum_{i=1}^{B} H_{b,k,i} \mathbf{X}_i + H_{b,k,t} \mathbf{X}_t + \mathbf{N}_{b,k}, \tag{3}$$

using d antennas from BSs. Vector $\mathbf{X}_t \sim \mathcal{CN}(\mathbf{0}_{t \times 1}, S_{\mathbf{X}_t})$ models the total uncoordinated interference from BSs of surrounding clusters that is independent of everything else and is treated as noise, and $\mathbf{N}_{b,k} \sim \mathcal{CN}(\mathbf{0}_{d \times 1}, \sigma^2 \mathbf{I}_d)$ is the background AWGN. Here, $H_{b,k,i} \in \mathbb{C}^{d \times s}$ and $H_{b,k,t} \in \mathbb{C}^{d \times t}$ are the channel matrices from the ith transmitter and uncoordinated interferers to user (b, k) respectively. Relay receives

$$\mathbf{Y}_r = \sum_{i=1}^{B} H_{r,i} \mathbf{X}_i + H_{r,t} \mathbf{X}_t + \mathbf{N}_r \tag{4}$$

by r_i antennas. Similar to (3), here, $H_{r,t} \mathbf{X}_t$ is the total uncoordinated intercell interference from surrounding BSs and $\mathbf{N}_r \sim \mathcal{CN}(\mathbf{0}_{r \times 1}, \sigma^2 \mathbf{I}_{r_i})$ is the background AWGN. Here, $H_{r,i} \in \mathbb{C}^{r_i \times s}$ and $H_{r,t} \in \mathbb{C}^{r_i \times t}$ are channel matrices from the ith transmitter and uncoordinated interferers to the relay respectively.

Relay transmits over its separate frequency band. It performs AF, i.e., transmits

$$\mathbf{X}_r = A^{\dagger} \mathbf{Y}_r, \tag{5}$$

from r_o antennas, where $A \in \mathbb{C}^{r_i \times r_o}$ is the combining matrix, with power constraint

$$\mathbb{E}[\mathbf{X}_r^{\dagger} \mathbf{X}_r] = \text{trace}(A^{\dagger} S_{\mathbf{Y}_r} A) \leq P_r. \tag{6}$$

In the relay frequency band, user (b, k) receives

$$\mathbf{Y}'_{b,k} = H_{b,k,r} \mathbf{X}_r + H_{b,k,t'} \mathbf{X}_{t'} + \mathbf{N}'_{b,k} \tag{7}$$

using d' antennas. Vector $\mathbf{X}_{t'} \sim \mathcal{CN}(\mathbf{0}_{t' \times 1}, S_{\mathbf{X}_{t'}})$ models the total uncoordinated interference from surrounding relays in nearby clusters that is independent and treated as noise, and $\mathbf{N}'_{b,k} \sim \mathcal{CN}(\mathbf{0}_{d' \times 1}, \sigma^2 \mathbf{I}_{d'})$ is the background AWGN. $H_{b,k,r} \in \mathbb{C}^{d' \times r_o}$ and $H_{b,k,t'} \in \mathbb{C}^{d' \times t'}$ are channel matrices from relay and interferers to user (b, k) respectively.

We summarize the received signals of user (b, k) as

$$\bar{\mathbf{Y}}_{b,k} = \begin{bmatrix} \mathbf{Y}'_{b,k} \\ \mathbf{Y}_{b,k} \end{bmatrix} = \sum_{i=1}^{B} \bar{H}_{b,k,i} \mathbf{X}_i + \bar{H}_{b,k,t} \mathbf{X}_t + \bar{\mathbf{N}}_{b,k}, \tag{8}$$

where, $\bar{H}_{b,k,i} = \begin{bmatrix} H_{r,i}^{\dagger} A H_{b,k,r}^{\dagger} & H_{b,k,i}^{\dagger} \end{bmatrix}^{\dagger}$ and $\bar{H}_{b,k,t} = \begin{bmatrix} H_{r,t}^{\dagger} A H_{b,k,r}^{\dagger} & H_{b,k,t}^{\dagger} \end{bmatrix}^{\dagger}$ are effective channel matrices and $\bar{N}_{b,k} = \begin{bmatrix} N_r^{\dagger} A H_{b,k,r}^{\dagger} + \mathbf{X}_{t'}^{\dagger} H_{b,k,t'}^{\dagger} + N'^{\dagger}_{b,k} & N_{b,k}^{\dagger} \end{bmatrix}^{\dagger}$ is the effective noise vector.

III. PROBLEM FORMULATION

This paper aims to solve the problem of jointly optimizing transmit BF across the B coordinated BSs together with the AF relay combining matrix for the IBC channel model augmented with a relay in (8). For maximizing the proportional fairness network utility, see e.g., [7, Ch. 9], we formulate a WSR maximization

problem for all users under power constraints (2) and (6). For given values of user weights, we solve this maximization problem, and then update the weights according to the proportional fairness criterion, until sum of log-utilities converges. The WSR maximization is

$$\underset{A,V_{1,1},\ldots,V_{K,B}}{\text{maximize}} \quad \sum_{b,k} \alpha_{b,k} R_{b,k}(A, V_{1,1}, \ldots, V_{K,B})$$
$$\text{subject to} \quad \text{trace}(A^\dagger S_{\mathbf{Y}_r} A) \leq P_r, \tag{9}$$
$$\sum_{k=1}^{K} \text{trace}(V_{b,k} V_{b,k}^\dagger) \leq P_b, \quad b \in \mathcal{B},$$

where $\alpha_{b,k} \in \mathbb{R}^+$ is the given weight of user (b,k)'s rate in the objective function and

$$S_{\mathbf{Y}_r} = \sum_{m,j} H_{r,m} V_{m,j} V_{m,j}^\dagger H_{r,m}^\dagger + H_{r,t} S_{\mathbf{X}_t} H_{r,t}^\dagger + \sigma^2 \mathbf{I}_r$$

is the covariance matrix of the relay's observed vector. The achievable rate of user (b,k) is

$$R_{b,k} = \log \frac{\left| S_{\bar{\mathbf{Y}}_{b,k}} \right|}{\left| S_{\bar{\mathbf{Y}}_{b,k}|\mathbf{X}_{b,k}} \right|}, \tag{10}$$

where the covariance matrices are

$$S_{\bar{\mathbf{Y}}_{b,k}} = \sum_{m,j} \bar{H}_{b,k,m} V_{m,j} V_{m,j}^\dagger \bar{H}_{b,k,m}^\dagger$$
$$+ \bar{H}_{b,k,t} S_{\mathbf{X}_t} \bar{H}_{b,k,t}^\dagger + S_{\bar{\mathbf{N}}_{b,k}} \tag{11}$$

and

$$S_{\bar{\mathbf{Y}}_{b,k}|\mathbf{X}_{b,k}} = S_{\bar{\mathbf{Y}}_{b,k}} - \bar{H}_{b,k,b} V_{b,k} V_{b,k}^\dagger \bar{H}_{b,k,b}^\dagger. \tag{12}$$

In problem (9), BSs transmit power constraints are convex. But, relay's transmit power constraint and the objective function are neither convex nor concave.

IV. Optimization Algorithm

In this section we propose an efficient algorithm for solving problem (9) to a stationary point. We extend the weighted MMSE optimization technique of [1] for BF design in a MIMO IBC to the case where the network is augmented by an AF relay node. The WMMSE optimization algorithm converges to high quality BF solutions and is computationally efficient. The AF relaying scheme imposes a non-convex constraint on relay transmit power that depends on all of the optimization variables. To tackle the difficulty of this constraint, we write the Lagrangian as

$$\mathcal{L} = \sum_{b,k} \alpha_{b,k} R_{b,k} - \mu \text{trace}(A^\dagger S_{\mathbf{Y}_r} A) \tag{13}$$

and use the WMMSE optimization approach to solve

$$\underset{A,V_{1,1},\ldots,V_{B,K}}{\text{maximize}} \quad \mathcal{L}(A, V_{1,1}, \ldots, V_{B,K})$$
$$\text{subject to} \quad \sum_{k=1}^{K} \text{trace}(V_{b,k} V_{b,k}^\dagger) \leq P_b, \quad b \in \mathcal{B}, \tag{14}$$

to a stationary point for a given dual variable μ. Then, in an outer loop we use the bisection method to obtain the μ^* for which the relay power constraint is active

$$\text{trace}(A^\dagger S_{\mathbf{Y}_r} A)\big|_{(A^*(\mu^*), V_{1,1}^*(\mu^*), \ldots, V_{B,K}^*(\mu^*))} = P_r. \tag{15}$$

The objective function of (14) is not concave. We use the connection between user (b,k)'s rate expression (10) and the MMSE in estimating $\mathbf{S}_{b,k}$ from $\bar{\mathbf{Y}}_{b,k}$ through

$$\widehat{\mathbf{S}}_{b,k} = U_{b,k}^\dagger \bar{\mathbf{Y}}_{b,k}, \tag{16}$$

with $U_{b,k} \in \mathbb{C}^{(d'+d) \times D}$ to write an equivalent objective function that is concave in each variable when others are kept fixed. We solve the equivalent problem using coordinate ascent, which converges to a stationary point provided that in each subproblem the maximizer is uniquely attained. For each subproblem of the coordinate ascent, we derive the optimal solution in closed-form. The concavity of the objective function in each variable when others are fixed ensures the global optimality of solution in the corresponding subproblem.

The MMSE receiver in estimation (16) is

$$U_{b,k}^* = S_{\bar{\mathbf{Y}}_{b,k}}^{-1} \bar{H}_{b,k,b} V_{b,k}, \quad (b,k) \in \mathcal{K}, \tag{17}$$

and the resulting MMSE matrix is

$$E_{b,k}^* = \mathbf{I}_D - V_{b,k}^\dagger \bar{H}_{b,k,b}^\dagger S_{\bar{\mathbf{Y}}_{b,k}}^{-1} \bar{H}_{b,k,b} V_{b,k}. \tag{18}$$

It is straightforward to verify that

$$R_{b,k} = -\log \left| E_{b,k}^* \right|. \tag{19}$$

By exploiting (19), the following lemma states that maximizing

$$\mathcal{L}_{eq}(A, V, U, W) = \sum_{b,k} \alpha_{b,k} (\log |W_{b,k}| -$$
$$\text{trace}(W_{b,k} E_{b,k})) - \mu \text{trace}(A^\dagger S_{\mathbf{Y}_r} A) \tag{20}$$

under BSs power constraints is equivalent to solving (14). The positive definite matrices $W_{b,k} \in \mathbb{C}^{D \times D}$ are intermediate variables. Here, for brevity of notation, we define $V = (V_{1,1}, \ldots, V_{B,K})$, $U = (U_{1,1}, \ldots, U_{B,K})$, and $W = (W_{1,1}, \ldots, W_{B,K})$.

Lemma 1 *Coordinate ascent for solving*

$$\underset{A,V,U,W}{\text{maximize}} \quad \mathcal{L}_{eq}(A, V, U, W)$$
$$\text{subject to} \quad \sum_{k=1}^{K} \text{trace}(V_{b,k} V_{b,k}^\dagger) \leq P_b, \quad b \in \mathcal{B}, \tag{21}$$

over (A, V, U, W) converges to a stationary point of the Lagrangian maximization problem (14).

Proof: For $\mu \geq 0$, function $\mathcal{L}_{eq}$ in (20) is concave in each variable. Moreover, the power constraints in (21) are separable across BSs. In iterations of coordinate ascent for solving (21), the unique globally optimal

solution of each subproblem can be attained. Therefore, coordinate ascent converges to a stationary point of (21) [4]. Denote this stationary point by (A^*, V^*, U^*, W^*). With the same argument as in [1], we show that the stationary point of (21) is a stationary point of (14) as well. Since (21) has a convex constraint set, we have

$$\text{trace}(\nabla_V \mathcal{L}_{eq}(A^*, V^*, U^*, W^*) (V^* - V)) \geq 0,$$

for all feasible V's, and

$$\nabla_A \mathcal{L}_{eq}(A^*, V^*, U^*, W^*) = \mathbf{0}.$$

Now, note that in (21) the optimal value of $W_{b,k}$ is

$$W_{b,k}^* = E_{b,k}^{-1}, \quad (b,k) \in \mathcal{K}. \tag{22}$$

By (22), (17), (18) we have

$$\mathcal{L}_{eq}(A, V, U^*, W^*)$$
$$= -\sum_{b,k} \alpha_{b,k} \log \left| E_{b,k}^* \right| - \mu \text{trace}(A^\dagger S_{\bar{\mathbf{Y}}_r} A) + \text{const.} \tag{23}$$

Therefore, by substituting (19) in (13) we have

$$\mathcal{L}_{eq}(A, V, U^*, W^*) = \mathcal{L}(A, V) + \text{const.} \tag{24}$$

Using (24),

$$\text{trace}(\nabla_V \mathcal{L}(A^*, V^*) (V^* - V)) \geq 0,$$

for all feasible V's, and

$$\nabla_A \mathcal{L}(A^*, V^*) = \mathbf{0}.$$

Therefore, if (A^*, V^*, U^*, W^*) is a stationary point of (21), then (A^*, V^*) is a stationary point for (14). $\square$

We provide the update formulas for BF matrices and relay combining matrix bellow. The overall procedure for solving (9) is summarized in Algorithm 1.

For optimization of the relay combining matrix, the objective function of (21) is concave in A when other variables are fixed. Therefore, solving

$$\nabla_A \mathcal{L}_{eq} = \mathbf{0} \tag{25}$$

yields the optimum solution of the subproblem, which is

$$A^* = S_{\mathbf{Y}_r}^{-1}$$
$$\left(\sum_{b,k} \alpha_{b,k} \left[H_{r,b} V_{b,k} - S_{\mathbf{Y}_r \mathbf{Y}_{b,k}}^{(1,2)} U_{b,k}^d \right] W_{b,k} U_{b,k}^{r \, \dagger} H_{b,k,r} \right)$$
$$\left(\sum_{b,k} \alpha_{b,k} H_{b,k,r}^\dagger U_{b,k}^r W_{b,k} U_{b,k}^{r \, \dagger} H_{b,k,r} + \mu \mathbf{I}_{r_o} \right)^{-1} \tag{26}$$

where $U_{b,k}^r$ and $U_{b,k}^d$ are the first d' and last d rows of $U_{b,k}$ respectively and $S_{\mathbf{Y}_r \mathbf{Y}_{b,k}}^{(1,2)} = \mathbb{E}[\mathbf{Y}_r \mathbf{Y}_{b,k}^\dagger]$.

For optimization of BF matrices, problem (21) is convex in V when other variables are fixed. Therefore,

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Algorithm 1 BF and AF Relaying Optimization (9)

Initialize $(V_{1,1}, \ldots, V_{B,K})$ and A,
repeat
    Fix Lagrangian multiplier μ:
    repeat
        Update U by (17)
        Update W by (18) and (22)
        Update A by (26)
        Update V by (27)
    until $\sum_{b,k} \alpha_{b,k} \log |W_{b,k}| + \mu \, \text{trace}(A^\dagger S_{\mathbf{Y}_r} A)$ converges;
    Update μ using bisection;
until $\text{trace}(A^\dagger S_{\mathbf{Y}_r} A) = P_r$.

KKT conditions provide the globally optimal solution for this subproblem. The first order optimality condition

$$\nabla_{V_{b,k}} \left(\mathcal{L}_{eq} - \gamma_b \, \text{trace}(V_{b,k} V_{b,k}^\dagger) \right) = \mathbf{0},$$

yields

$$V_{b,k}^*(\gamma_b) = \alpha_{b,k} \left(M_b + \gamma_b \mathbf{I}_s \right)^{-1} \bar{H}_{b,k,b}^\dagger U_{b,k} W_{b,k}, \tag{27}$$

where $M_b = \sum_{m,j} \alpha_{m,j} \bar{H}_{m,j,b}^\dagger U_{m,j} W_{m,j} U_{m,j}^\dagger \bar{H}_{m,j,b} + \mu H_{r,b}^\dagger A A^\dagger H_{r,b}$. The optimal dual variable γ_b^* is found through bisection. The following proposition states the convergence of Algorithm 1 formally.

Proposition 1 *The inner loop optimization procedure in Algorithm 1 converges to a stationary point of Lagrangian maximization (14). Further, the optimal μ is one that satisfies (15). Such a μ leads to a KKT point of the joint BF and AF relaying optimization (9).*

Proof: For a given μ, by lemma 1 the coordinate ascent inner loop of Algorithm 1 converges to a stationary point of Lagrangian (14). This together with a μ that results in (15) satisfy the KKT conditions for problem (9). $\square$

V. SIMULATION RESULTS

We now evaluate the improvement in downlink throughput of a cellular network due to deploying an AF relay. Consider a cluster of $B = 3$ BSs in a pico-cell environment, each BS serves $K = 30$ users in a $120°$ sector, as depicted in Fig. 1. Each BS has $s = 4$ antennas and sends $D = 1$ data stream to each of its associated users. Relay and receivers observe uncoordinated interference from 9 surrounding BSs, each generating a rank-4 interference signal, i.e., $t = 36$ and $\text{rank}(S_{\mathbf{X}_t}) = 36$. Each receiver is equipped with $d = 1$ antennas. The relay node receives signal with $r_i = 16$ antennas and transmits by $r_o = 16$ antennas over an orthogonal BC to users with $d' = 1$ receive antenna. We assumed 6 nearby 16-antenna transmitters that interfere with transmission of the relay, i.e., $t' = 96$.
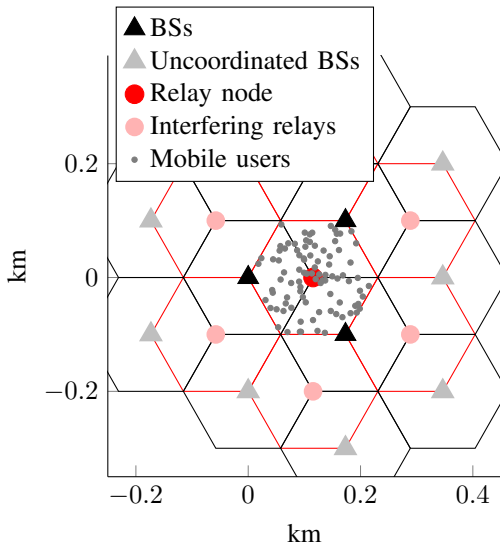
Fig. 1. Network topology used in simulations: a cluster of $B = 3$ BS's with $K = 30$ users associated with each, surrounded by 9 BSs causing uncoordinated interference. The relay node serves all the users in a separate frequency band with uncoordinated interference from 6 surrounding relays.
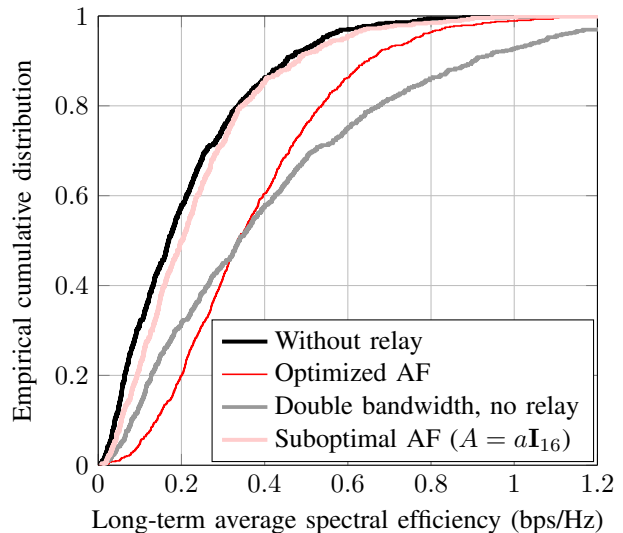

Fig. 2. The improvement in empirical cumulative distribution of user spectral efficiency due to AF relaying vs. doubling bandwidth. The relay node is equipped with 16 receive and transmit antennas.

In our simulations, both BSs and relays are located on hexagonal grids, with minimum distance of 200m, as depicted in Fig. 1. Both the BSs and the relays transmit at a maximum power of 1Watt over 10MHz. All the background AWGN's in the network have power spectral density of -170dBm/Hz. The path loss exponent is set to 3.76 and variance of the shadowing term is 8dB.

In simulations, the obtained transmit power of the relay by the inner loop of Algorithm 1, i.e.,

$$P_r^*(\mu) = \text{trace}(A^\dagger S_{\mathbf{Y}_r} A)\big|_{\left(A^*(\mu), V_{1,1}^*(\mu), \ldots, V_{B,K}^*(\mu)\right)}$$

is observed to be decreasing in dual variable μ. We find the optimal μ using the bisection method. In case of a discontinuity at a point μ_0 with $P_r^*(\mu_0^+) < P_r < P_r^*(\mu_0^-)$, we select the rate of user (b, k) as

$$R_{b,k}^* =$$
$$R_{b,k}^*(\mu_0^-) + \frac{R_{b,k}^*(\mu_0^-) - R_{b,k}^*(\mu_0^+)}{P_r^*(\mu_0^-) - P_r^*(\mu_0^+)} \left(P_r - P_r^*(\mu_0^-)\right).$$

Fig. 2 illustrates various empirical cumulative distributions of long-term average user rates under proportional fair scheduling. We observe that up to the median of user rates, AF relaying optimized by Algorithm 1 outperforms doubling the bandwidth of the network without relaying. Although doubling the bandwidth results in largest improvement in the overall throughput of the network, deploying the relay node favors the weaker cell-edge users and brings in a more uniform throughput improvement in the network. We also observe that restricting the relay combining matrix to the simple choice of $A = a\mathbf{I}_{16}$, where a is selected to satisfy the relay power constraint, leads to a poor performance.

VI. CONCLUSION

This paper proposes deploying multi-antenna AF relay nodes at the cell edge in a wireless cellular network to help mitigate interference signals at multiple remote terminals at the same time. We propose an iterative optimization procedure based on the WMMSE algorithm for maximizing users' WSR over transmit BF and relay combining matrices.

In the simulations, we observe that deploying a relay node in the network improves the throughput of all users in a more uniform fashion. Under proportional fair scheduling, the optimized AF relaying outperforms doubling the bandwidth of the network without deploying the relay, up to almost the median of rates.

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