# SPARSE ACTIVITY DETECTION FOR MASSIVE CONNECTIVITY IN CELLULAR NETWORKS: MULTI-CELL COOPERATION VS LARGE-SCALE ANTENNA ARRAYS

*Zhilin Chen, Foad Sohrabi, and Wei Yu*

Department of Electrical and Computer Engineering
University of Toronto, Toronto, ON, Canada M5S 3G4
Emails: {zchen, fsohrabi, weiyu}@comm.utoronto.ca

## ABSTRACT

Sparse device activity detection for machine-type communications has attracted increasing attention in recent studies. However, most of the previous works focus on the single-cell case. This paper studies the impact of the inter-cell interference on the device activity detection problem with non-orthogonal signatures in multi-cell systems by employing the computationally efficient approximate message passing algorithm (AMP). Specifically, this paper studies the impact of the inter-cell interference by either treating it as noise or recovering it, showing that it is always beneficial to recover the interference at each base station (BS). Two network architectures, namely BSs with large antenna arrays and network with multi-cell cooperation, are compared in terms of their effectiveness in overcoming inter-cell interference. This paper provides an analytical characterization of probabilities of false alarm and missed detection. Simulation results show that large-scale antenna array is effective in improving the performance of all users whereas cooperation is effective in improving the performance of cell-edge users. In terms of the detection performance of the 95-percentile users, simulation results under a typical network setting show that having twice as many antennas provides almost the same benefit as multi-cell cooperation.

***Index Terms—*** Device activity detection, approximate message passing, machine-type communications (MTC)

## 1. INTRODUCTION

Massive machine-type communications (mMTC) aim to meet the demand for wireless connectivity to tens of millions of devices with event-driven traffic in application domains such as smart city by using the future fifth generation (5G) cellular infrastructure. A main challenge of mMTC is scalable and efficient random access design in the uplink for a large number of devices. This paper studies the sporadic user activity detection problem for random access.

This paper considers a pilot-based random access protocol with non-orthogonal signature sequence for each user transmitted synchronously in a multi-cell system. We exploit the sparse activity pattern of the devices by adopting the low-complexity approximate message passing (AMP) algorithm with Bayesian denoiser for device activity detection and channel estimation. Our main contributions include an analysis of the impact of the inter-cell interference at each BS under two scenarios: either treating the interference as noise or recovering the interference, and a comparison of equipping a large-scale antenna array at each base station (BS) versus cooperation among adjacent BSs for overcoming inter-cell interference. Our aims are to quantify (i) the benefit of exploiting the structure of interference, as compared to treating interference as noise, and (ii) the benefit of large-scale antenna arrays versus cooperation in terms of cumulative distribution of device activity detection performance.

Related works on massive random access and device activity detection include [1–9]. In conventional cellular system without considering the effect of inter-cell interference, [1, 2] propose the use of compressed sensing technique for joint user activity detection with data detection or channel estimation. By further exploiting channel statistics, [3] adopts the AMP algorithm with Bayesian denoiser, and characterizes the detection performance. The user activity detection is also studied in a cloud radio access network (C-RAN) in [4, 5], where [4] proposes a modified Bayesian compressed sensing algorithm via joint processing at the cloud, and [5] compares quantize-and-forward and detect-and-forward under fronthaul constraint. In the context of massive multiple-input and multiple-output (MIMO), [6] designs a collision resolution protocol with uncoordinated orthogonal pilot sequences in a multi-cell scenario, whereas [7] studies the asymptotic detection performance with non-orthogonal sequences in a single cell scenario via AMP. Besides the aforementioned works on design and analysis for practical systems, the massive random access is also studied from information theoretical perspectives in [8, 9]. This paper differs from previous works in aiming to study the effect of inter-cell interference on user activity detection for multi-cell systems via AMP, and to investigate the potential of implementing large-scale antenna arrays or BS cooperation for enhanced detection in terms of the probability of missed detection (PM) and the probability of false alarm (PF).

## 2. SPARSE ACTIVITY DETECTION PROBLEM

Consider a hexagonal cellular network with $B$ cells indexed by $1, 2, \cdots, B$. Each cell contains one BS equipped with $M$ antennas at the center, and $N$ uniformly randomly distributed single-antenna users. Assume that only a small subset of the users are active in each coherence block. Let $a_{bn} \in \{1, 0\}$ indicate whether or not user $n$ in cell $b$ is active. For user identification and channel estimation, each user is assigned a unique signature sequence $\mathbf{s}_{bn} = [s_{bn1}, s_{bn2}, \cdots, s_{bnL}] \in \mathbb{C}^{1\times L}$, where $L$ is the pilot length. Assuming that the channel is static in each coherence block, and all users transmit their signature sequences with the same power, the received signal $\mathbf{Y}_b \in \mathbb{C}^{L\times M}$ at BS $b$ can be modeled as

$$\begin{aligned}\mathbf{Y}_b &= \sum_{n=1}^{N} a_{bn}\mathbf{s}_{bn}^T\mathbf{h}_{bbn} + \sum_{j\neq b}\sum_{n=1}^{N} a_{jn}\mathbf{s}_{jn}^T\mathbf{h}_{bjn} + \mathbf{W}_b \\ &= \mathbf{S}_b\mathbf{X}_{bb} + \sum_{j\neq b}\mathbf{S}_j\mathbf{X}_{bj} + \mathbf{W}_b, \qquad (1)\end{aligned}$$

where $\mathbf{h}_{bjn} \in \mathbb{C}^{1\times M}$ is the channel from user $n$ in cell $j$ to BS $b$, $\mathbf{W}_b \in \mathbb{C}^{M\times L}$ is the effective independent and identically distributed (i.i.d.) Gaussian noise whose variance $\sigma_{\mathrm{w}}^2$ depends on the signal-to-noise ratio (SNR) at the BS, $\mathbf{S}_j \triangleq [\mathbf{s}_{j1}^T, \cdots, \mathbf{s}_{jN}^T] \in \mathbb{C}^{L\times N}$ is the signature matrix of all users in cell $j$, and $\mathbf{X}_{bj} \triangleq [\mathbf{x}_{bj1}^T, \cdots, \mathbf{x}_{bjN}^T]^T \in \mathbb{C}^{N\times M}$, where $\mathbf{x}_{bjn} \triangleq a_{jn}\mathbf{h}_{bjn} \in \mathbb{C}^{1\times M}$ is the row vector of $\mathbf{X}_{bj}$. The second term in (1) is the inter-cell interference.

This paper aims to study the recovery of $\mathbf{X}_{bb}$ for user activity detection under inter-cell interference. We focus on the regime where $N \gg L$ so that the signature sequences cannot be mutually orthogonal. But since only a small number of users are active in each block, $\mathbf{X}_{bb}$ exhibits a row-sparse structure, which allows the use of compressed sensing techniques for recovery. This paper assumes that each signature sequence $\mathbf{s}_{jn}$ in the system is generated according to i.i.d. complex Gaussian distribution with zero mean and variance $1/L$ such that each sequence has unit power.

To exploit the sparsity in $\mathbf{X}_{bb}$, we employ AMP with Bayesian denoiser, which accounts for the statistical information of $\mathbf{X}_{bj}, \forall j$. We assume that each user is active with a small probability $\Pr(a_{bn} = 1) = \lambda$ in an i.i.d. fashion. The channel is modeled as $\mathbf{h}_{bjn} = g_{bjn}\bar{\mathbf{h}}_{bjn}$, where $g_{bjn}$ is the large-scale fading component assumed to be known to the BSs, and $\bar{\mathbf{h}}_{bjn}$ is Rayleigh fading following $\mathcal{CN}(0, \mathbf{I})$. Each row of $\mathbf{X}_{bj}$ then follows a mixed Bernoulli-Gaussian distribution as $(1-\lambda)\delta_{\mathbf{0}} + \lambda\mathcal{CN}(0, g_{bjn}^2\mathbf{I})$ parameterized by the large-scale fading $g_{bjn}$, where $\delta_{\mathbf{0}}$ is the point mass at $\mathbf{0}$.

## 3. AMP BASED USER ACTIVITY DETECTION

### 3.1. AMP with Bayesian Denoiser

AMP is an iterative algorithm originally proposed in [10] and extended for different sparse signal recovery problems [11–14]. In this paper, we adopt the algorithm in [3,7,12]. Consider a general model $\mathbf{Y} = \mathbf{S}\mathbf{X}+\mathbf{W}$ without interference, where $\mathbf{S} \in \mathbb{C}^{L\times N}$ is a Gaussian matrix, $\mathbf{X} \in \mathbb{C}^{N\times M}$ is row sparse, and $\mathbf{W} \in \mathbb{C}^{L\times M}$ is Gaussian noise with i.i.d. entries following $\mathcal{CN}(0, \sigma_{\mathrm{w}}^2)$. Starting with $\mathbf{X}^0 = \mathbf{0}$ and $\mathbf{Z}^0 = \mathbf{Y}$, AMP proceeds at each iteration as

$$\mathbf{X}^{t+1} = \eta_t(\mathbf{S}^*\mathbf{Z}^t + \mathbf{X}^t), \tag{2}$$

$$\mathbf{Z}^{t+1} = \mathbf{Y} - \mathbf{S}\mathbf{X}^{t+1} + NL^{-1}\mathbf{Z}^t\langle\eta_t'(\mathbf{S}^*\mathbf{Z}^t + \mathbf{X}^t)\rangle, \tag{3}$$

where $t = 0, 1, \cdots$ is iteration index, $\mathbf{X}^t$ is the estimate of $\mathbf{X}$ at iteration $t$, $\mathbf{Z}^t$ is residual, $\eta_t(\cdot) \triangleq [\eta_t(\cdot, g_1), \cdots, \eta_t(\cdot, g_N)]^T$ with $\eta_t(\cdot, g_n) : \mathbb{C}^{1\times M} \to \mathbb{C}^{1\times M}$ being an appropriately designed nonlinear function known as *denoiser* that operates on the $n$th row vector of $\mathbf{S}^*\mathbf{Z}^t + \mathbf{X}^t$ parameterized by $g_n$ drawing from some distribution $p_G$, $(\cdot)^*$ is conjugate transpose, $\eta_t'(\cdot) \triangleq [\eta_t'(\cdot, g_1), \cdots, \eta_t'(\cdot, g_N)]^T$ with $\eta_t'(\cdot, g_n)$ being the first order derivative of $\eta_t(\cdot, g_n)$, and $\langle\cdot\rangle$ is the average of all derivatives through 1 to $N$. The third term in the right hand side of (3) is the correction term known as "Onsager term".

A useful property of AMP is that the matched filtered output $\tilde{\mathbf{X}}^t \triangleq \mathbf{S}^*\mathbf{Z}^t + \mathbf{X}^t$ in (2) can be modeled as signal plus noise-and-multiuser-interference, i.e., $\tilde{\mathbf{X}}^t = \mathbf{X} + \mathbf{V}^t$, where $\mathbf{V}^t$ is Gaussian due to the correction term.

The performance of AMP can be analyzed in the regime $L, N \to \infty$ with fixed $L/N$ via the state evolution, which predicts the covariance matrix of the row vectors of $\mathbf{V}^t$ as

$$\mathbf{\Sigma}_{t+1} = \sigma_w^2\mathbf{I} + NL^{-1}\mathbb{E}\big[\mathbf{D}^t(\mathbf{D}^t)^*\big], \tag{4}$$

where $\mathbf{D}^t \triangleq \big(\eta_t(\mathbf{R}+\mathbf{U}^t, G) - \mathbf{R}\big)^T \in \mathbb{C}^{M\times 1}$ with random vectors $\mathbf{R}$ and $\mathbf{U}^t$ capturing the distributions of the row vectors of $\mathbf{X}$ and $\mathbf{V}^t$, respectively, and $\mathbf{U}^t \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}_t)$, and random variable $G$ capturing the distribution of $g_n$. The expectation is taken over $\mathbf{R}$, $\mathbf{U}^t$, and $G$.

Suppose that the row vectors of $\mathbf{X}$ are drawn from the same distribution as those of $\mathbf{X}_{bb}$ in (1), which is Bernoulli-Gaussian parameterized by the large-scale fading $g_{bbn}$, the Bayesian denoiser that minimizes the mean square error of each entry is given by the conditional mean $\mathbb{E}\big[\mathbf{R}|\tilde{\mathbf{R}}^t, G\big]$, where $\tilde{\mathbf{R}}^t \triangleq \mathbf{R}+\mathbf{U}^t$ and $G$ captures the distribution of large-scale fading $g_{bbn}$. We express $\mathbb{E}\big[\mathbf{D}^t(\mathbf{D}^t)^*\big]$ as

$$\mathbb{E}\big[\mathbf{D}^t(\mathbf{D}^t)^*\big] = \mathbb{E}\big[\mathbb{E}\big[\mathbf{C}^t\big|G\big]\big], \tag{5}$$

where $\mathbf{C}^t \in \mathbb{C}^{M\times M}$ is the covariance matrix at iteration $t$ as $\mathbf{C}^t = \mathbb{E}\big[\big(\mathbb{E}[\mathbf{R}|\tilde{\mathbf{R}}^t, G] - \mathbf{R}\big)^T\big((\mathbb{E}[\mathbf{R}|\tilde{\mathbf{R}}^t, G] - \mathbf{R})^T\big)^*\big|\tilde{\mathbf{R}}^t, G\big]$.

It has been shown in [3] that $\mathbb{E}\big[\mathbf{C}^t\big|G\big]$, $\mathbb{E}\big[\mathbb{E}\big[\mathbf{C}^t\big|G\big]\big]$ in (5) and $\mathbf{\Sigma}_{t+1}$ in (4) are all diagonal matrices with identical entries. By expressing $\mathbf{\Sigma}_t$ as $\mathbf{\Sigma}_t = \tau_t^2\mathbf{I}$, the Bayesian denoiser $\eta_t(\cdot, g_{bbn})$ can be simplified as [3]

$$\eta_t(\tilde{\mathbf{x}}_n^t, g_{bbn}) = \frac{g_{bbn}^2(g_{bbn}^2 + \tau_t^2)^{-1}\tilde{\mathbf{x}}_n^t}{1 + \frac{1-\lambda}{\lambda}\big(\frac{g_{bbn}^2+\tau_t^2}{\tau_t^2}\big)^M \exp(-\Delta\|\tilde{\mathbf{x}}_n^t\|_2^2)}, \tag{6}$$

where $\tilde{\mathbf{x}}_n^t$ is the $n$th row vector of the matched filtered output in (2), and $\Delta \triangleq \tau_t^{-2} - (g_{bbn}^2+\tau_t^2)^{-1}$. Note that the value of $\tau_t^2$ is needed in the denoiser expression, which can be empirically estimated at each iteration via an similar approach in [15].

### 3.2. AMP with Inter-cell Interference

To implement AMP for the signal model (1), we need to decide how to deal with the inter-cell interference. We consider two possibilities: either treat the interference as noise or seek to recover the interference. For the first approach, since the interference is a sum of a large number of independent random signals, we approximate it by Gaussian random variable with covariance matrix as

$$\begin{aligned}\mathbf{C}_{\text{inf}} =&\mathbb{E}\left[\text{vec}\left(\sum_{j\neq b}\mathbf{S}_j\mathbf{X}_{bj}\right)\text{vec}\left(\sum_{j\neq b}\mathbf{S}_j\mathbf{X}_{bj}\right)^*\right]\\ =&\lambda L^{-1}\mathbb{E}\left[\sum_{j\neq b}\sum_{n=1}^{N} g_{bjn}^2\right]\mathbf{I},\end{aligned} \tag{7}$$

from which we observe that the inter-cell interference can be merged into the white Gaussian noise as $\mathbf{W}' \triangleq \sum_{j\neq b}\mathbf{S}_j\mathbf{X}_{bj} + \mathbf{W}$ with variance $\lambda L^{-1}\mathbb{E}[\sum_{j\neq b}\sum_n g_{bjn}^2] + \sigma_w^2$ for each entry.

For the second approach, to recover the signals of users in other cells, we reformulate the system model in (1) as

$$\mathbf{Y}_b = \begin{bmatrix} \mathbf{S}_1, & \cdots, & \mathbf{S}_B \end{bmatrix}\begin{bmatrix} \mathbf{X}_{1b} \\ \vdots \\ \mathbf{X}_{Bb}\end{bmatrix} + \mathbf{W}_b, \tag{8}$$

which corresponds to an "interference-free" system. Note that the signature and the large-scale fading of every user are assumed to be known at BS $b$. This approach turns the inter-cell interference into useful information at the cost of increasing the size of the signal to be recovered from $N\times M$ to $NB\times M$, which increases the computational complexity. We compare the two schemes in the following proposition.

**Proposition 1.** *Consider the sparse signal recovery in multi-cell systems in* (1) *via AMP with Bayesian denoiser. Let* $\Sigma_\infty^{\mathrm{TIN}}$ *and* $\Sigma_\infty^{\mathrm{EST}}$ *denote the converged covariance matrices from the state evolution* (4) *in the asymptotic regime* $L, N \to \infty$ *with fixed* $L/N$ *respectively for the case where the inter-cell interference is treated as white Gaussian noise, and the case where the interference is recovered. We have* $\Sigma_\infty^{\mathrm{EST}} \preceq \Sigma_\infty^{\mathrm{TIN}}$.

An intuitive explanation of the proposition is as follows: considering the signal model in (8), suppose that we set the denoiser that operates on the rows of the inter-cell interference signal $\mathbf{X}_{jb}, j \neq b$ as $\eta_t(\cdot,\cdot) = \mathbf{0}$. Then the resulting mean square error of denoising the interference would just be equal to the power of the interference, which has an effect similar to treating the interference as noise on the recovery. However, by exploiting the statistical information of the interference to design the Bayesian denoiser in the second approach, the mean square error can be reduced and the recovery performance can be improved.

Although Proposition 1 indicates that recovering the inter-cell interference is better than ignoring the inter-cell interference, it is also worth noting that in practice the BSs may not want to detect all the devices in the network due to the computational complexity involved and the need to store the signature sequences and to estimate the large-scale fading of all devices. A good strategy is that each BS detects the users from its own cell and a few neighboring cells while treating the rest of the inter-cell interference as noise. Simulation results of this paper show that this strategy brings improvement over treating all inter-cell interference as noise when the number of antennas is large.

### 3.3. User Activity Detection

In either the case of treating the interference as noise or the case of recovering the interference, we adopt likelihood ratio (LLR) test to detect user activity based on $\tilde{\mathbf{X}}_{bb}^t$, which denotes the matched filtered output of AMP at iteration $t$. For user $n$ in cell $b$, when the user is active ($H_1$), i.e., $a_{bn} = 1$, the $n$th row vector of $\tilde{\mathbf{X}}_{bb}^t$ can be modeled as the following Gaussian distribution

$$p(\tilde{\mathbf{x}}_{bbn}^t | a_{bn} = 1) = \frac{\exp\left(-\|\tilde{\mathbf{x}}_{bbn}^t\|_2^2 (\tau_{tb}^2 + g_{bbn}^2)^{-1}\right)}{\pi^M (\tau_{tb}^2 + g_{bbn}^2)^M}, \tag{9}$$

where $\tau_{tb}^2$ is the diagonal entry of $\mathbf{\Sigma}_t^{\mathrm{TIN}}$ or $\Sigma_t^{\mathrm{EST}}$, and the additional subscript $b$ indicates that the result is obtained at BS $b$. When the user is inactive ($H_0$), the row vector follows the distribution

$$p(\tilde{\mathbf{x}}_{bbn}^t | a_{bn} = 0) = \frac{\exp\left(-\|\tilde{\mathbf{x}}_{bbn}^t\|_2^2 \tau_{tb}^{-2}\right)}{\pi^M \tau_t^{2M}}. \tag{10}$$

The LLR at BS $b$ for user $n$ in cell $b$ is

$$\begin{aligned}\mathrm{LLR}_{bbn} &= \log\left(\frac{p(\tilde{\mathbf{x}}_{bbn}^t | a_{bn} = 1)}{p(\tilde{\mathbf{x}}_{bbn}^t | a_{bn} = 0)}\right) \\ &= \log\left(\frac{\tau_{tb}^{2M}}{(\tau_{tb}^2 + g_{bbn}^2)^M} \exp\left(\|\tilde{\mathbf{x}}_{bbn}^t\|_2^2 \Delta\right)\right). \end{aligned} \tag{11}$$

By observing that $\mathrm{LLR}_{bbn}$ is monotonic in $\|\tilde{\mathbf{x}}_{bbn}^t\|_2^2$, we can set a threshold $l_{bn}$ on $\|\tilde{\mathbf{x}}_{bbn}^t\|_2^2$ to perform the detection.

## 4. ENHANCED USER ACTIVITY DETECTION AND PERFORMANCE ANALYSIS

We now consider two strategies for overcoming the inter-cell interference for enhanced device activity detection, we consider two schemes: equipping each BS with a large-scale antenna array or allowing cooperation among BSs.

### 4.1. Non-cooperative Detection with Large Antenna Arrays

By using a large-scale antenna array, each BS receives multiple observations about the user activity from the antennas, which improves the reliability of detection. Based on (9) and (10), PM and PF are computed, respectively, as follows,

$$\begin{aligned}P_M^{\mathrm{NC},bn} &= \int_{\mathcal{D}^{\mathrm{NC}}} \frac{\exp\left(-\|\tilde{\mathbf{x}}_{bbn}^t\|_2^2 (\tau_{tb}^2 + g_{bbn}^2)^{-1}\right)}{\pi^M (\tau_{tb}^2 + g_{bbn}^2)^M} d\tilde{\mathbf{x}}_{bbn} \\ &= \Gamma^{-1}(M) \cdot \bar{\gamma}\left(M, l_{bn}^{\mathrm{NC}} (g_{bbn}^2 + \tau_{tb}^2)^{-1}\right), \end{aligned} \tag{12}$$

$$\begin{aligned}P_F^{\mathrm{NC},bn} &= \int_{/\mathcal{D}^{\mathrm{NC}}} \frac{\exp\left(-\|\tilde{\mathbf{x}}_{bbn}^t\|_2^2 \tau_{tb}^{-2}\right)}{\pi^M \tau_{tb}^{2M}} d\tilde{\mathbf{x}}_{bbn} \\ &= 1 - \Gamma^{-1}(M) \cdot \bar{\gamma}\left(M, l_{bn}^{\mathrm{NC}} \tau_{tb}^{-2}\right), \end{aligned} \tag{13}$$

where $\Gamma(\cdot)$ is the Gamma function, $\bar{\gamma}(\cdot,\cdot)$ is the lower incomplete Gamma function, $\mathcal{D}^{\mathrm{NC}} \triangleq \{\|\tilde{\mathbf{x}}_{bbn}^t\|_2^2 < l_{bn}^{\mathrm{NC}}\}$ with threshold $l_{bn}^{\mathrm{NC}}$, and $/\mathcal{D}^{\mathrm{NC}}$ is the complementary of $\mathcal{D}^{\mathrm{NC}}$.

Similar to [7] that considers the single-cell activity detection using AMP with a large number of antennas, we also have $P_M^{\mathrm{NC},bn}, P_F^{\mathrm{NC},bn} \to 0$ when $M \to \infty$ (in the asymptotic regime where $L, N, K \to \infty$ with their ratios fixed) for the multi-cell case, indicating perfect detection under a infinite number of antennas. The difference with single-cell case is that by including inter-cell interference, the resulting $\tau_{tb}^2$ will be considerably larger. Nevertheless, simulations show that PF and PM drop rapidly as $M$ increases, and a good performance can be achieved even without a very large $M$.

### 4.2. Cooperative Detection

We now consider the exchange of LLRs between the neighboring BSs for improving the detection performance. Assume that multiple BSs, denoted as $\mathcal{B}_{bn} \subseteq \{1, \cdots, B\}$, all individually compute the LLR of user $n$ in cell $b$ and send the LLRs to BS $b$. We treat the inter-cell interference at each BS as independent for the ease of derivation, so that the collected results can be regarded as samples from independent noise, and the aggregated LLR at BS $b$ can be expressed as

$$\begin{aligned}\mathrm{LLR}_{bn}^{\mathrm{AG}} &= \log\left(\prod_{j\in\mathcal{B}_{bn}} \frac{p(\tilde{\mathbf{x}}_{jbn}^t | a_{bn} = 1)}{p(\tilde{\mathbf{x}}_{jbn}^t | a_{bn} = 0)}\right) \\ &= \sum_{j\in\mathcal{B}_{bn}} \mathrm{LLR}_{jbn}, \end{aligned} \tag{14}$$

where $\mathrm{LLR}_{jbn}$ is the LLR obtained at BS $j$ for user $n$ in cell $b$ similar to (11). By plugging $\mathrm{LLR}_{jbn}$ into (14), we obtain

$$\mathrm{LLR}_{bn}^{\mathrm{AG}} = \log\left(\frac{\exp\left(\sum_{j\in\mathcal{B}_{bn}} \frac{g_{jbn}^2 \|\tilde{\mathbf{x}}_{jbn}\|_2^2}{\tau_{tj}^2 (g_{jbn}^2 + \tau_{tj}^2)}\right)}{\prod_{j\in\mathcal{B}_{bn}} (\tau_{tj}^2 + g_{jbn}^2)^M \tau_{tj}^{-2M}}\right). \tag{15}$$

We observe that $\mathrm{LLR}_{bn}^{\mathrm{AG}}$ is monotonic in $\sum_{j\in\mathcal{B}_{bn}} \frac{g_{jbn}^2 \|\tilde{\mathbf{x}}_{jbn}\|_2^2}{\tau_{tj}^2 (g_{jbn}^2 + \tau_{tj}^2)}$, which is a weighted sum of $\|\tilde{\mathbf{x}}_{jbn}\|_2^2$. Compare with the non-cooperative case in (11) that performs the detection on $\|\tilde{\mathbf{x}}_{bbn}\|_2^2$, cooperative detection can be performed on the weighted sum of $\|\tilde{\mathbf{x}}_{jbn}\|_2^2, j \in \mathcal{B}_{bn}$, where the weight depends on the link strength $g_{jbn}^2$ to BS $j$ and $\tau_{tj}^2$ at BS $j$.

By concatenating all $\tilde{\mathbf{x}}_{jbn}, j \in \mathcal{B}_{bn}$ as $\tilde{\mathbf{x}}_{bn}$, the PM for user $n$ in cell $b$ can be expressed as

$$P_M^{\mathrm{CO},bn} = \int_{\mathcal{D}^{\mathrm{CO}}} p(\tilde{\mathbf{x}}_{bn}|a_{bn}=1)d\tilde{\mathbf{x}}_{bn}$$
$$= \int_{\mathcal{D}^{\mathrm{CO}}} \frac{\exp\left(\sum_{j\in\mathcal{B}_{bn}} \frac{-\|\tilde{\mathbf{x}}_{jbn}\|_2^2}{g_{jbn}^2+\tau_{tj}^2}\right)}{\prod_{j\in\mathcal{B}_{bn}} \pi^M(\tau_{tj}^2+g_{jbn}^2)^M} d\tilde{\mathbf{x}}_{bn}, \tag{16}$$

where the decision region $\mathcal{D}^{\mathrm{CO}} \triangleq \left\{\sum_{j\in\mathcal{B}_{bn}} \frac{g_{jbn}^2\|\tilde{\mathbf{x}}_{jbn}\|_2^2}{\tau_{tj}^2(g_{jbn}^2+\tau_{tj}^2)} < l_{bn}^{\mathrm{CO}}\right\}$ with threshold $l_{bn}^{\mathrm{CO}}$. Similarly, PF is given as

$$P_F^{\mathrm{CO},bn} = \int_{/\mathcal{D}^{\mathrm{CO}}} \frac{\exp\left(\sum_{j\in\mathcal{B}_{bn}} -\frac{\|\tilde{\mathbf{x}}_{jbn}\|_2^2}{\tau_{tj}^2}\right)}{(\pi^M\tau_{tj}^{2M})^{|\mathcal{B}_{bn}|}} d\tilde{\mathbf{x}}_{bn}, \tag{17}$$

where $/\mathcal{D}^{\mathrm{CO}}$ is the complementary region of $\mathcal{D}^{\mathrm{CO}}$ with respect to the whole space, and $|\mathcal{B}_{bn}|$ is the cardinality of $\mathcal{B}_{bn}$.

It is worth noting that by integrating in spherical coordinates instead of Cartesian coordinates, closed-form expressions of both (16) and (17) can be obtained and numerically evaluated.

### 4.3. Comparison and Implementation Issues

We first compare the computational complexities of cooperative detection and non-cooperative detection with large-scale antenna arrays. Suppose that the denoiser function is pre-computed and stored as table lookup, then for the AMP algorithm the complexity mainly lies in the matched filtering and residual calculation, which depend on the problem size as $O(B'NLM)$ per iteration, where $B'$ is the number of cells from which the signal should be recovered at each BS. For non-cooperative activity detection, since it is not necessary to recover the signal from other cells, although the performance can be improved by recovering the inter-cell interference, the complexity can be as small as $O(NLM)$ when each BS only recovers the signal from its own cell, i.e., $B'=1$. However, for cooperative detection, since BSs with cooperation should exchange LLRs of users in neighboring cells, it is necessary that $B'>1$ and the value of $B'$ depends on the size of cooperation cluster.

We then discuss the prior information required in both scenarios. For non-cooperative case, at each BS the information on signature and large-scale fading of each user in its own cell is necessary to implement AMP and LLR test.[1] If extra information about the users in neighboring cells is available, the BS can seek to recover the inter-cell interference for better performance. For cooperative case, each BS should know the signature and the large-scale fading of each user in its own cell and other neighboring cells for the purpose of computing and exchanging LLRs.

## 5. SIMULATION RESULTS

The performances of non-cooperative detection with large-scale antenna arrays and cooperative detection are compared in a 19-cell system with 2000 users per cell among which 100 are active. The BS-to-BS distance is 2000m. The path-loss is modeled as $15.3 + 37.6\log_{10}(d)$ where $d$ is BS-user distance measured in meter, and shadowing fading in dB is Gaussian with zero mean and standard deviation 8. The transmit power of each user is 23dBm, and the background noise is $-169$dBm/Hz over 10MHz.

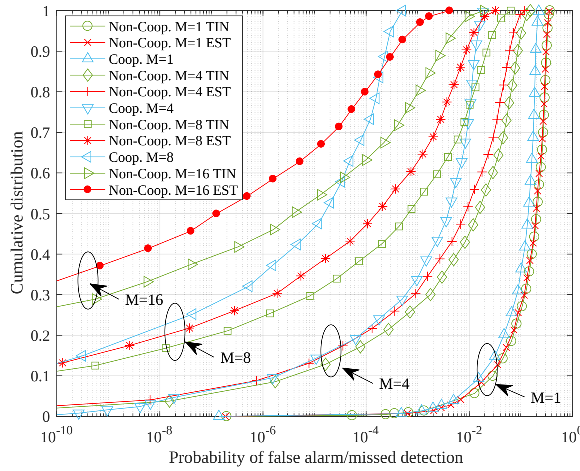


**Fig. 1**. Comparison of non-cooperative detection with large antenna array and cooperative detection.

Fig. 1 shows the cumulative distributions of the PM/PF of the users in the center cell, where thresholds $l_{bn}^{\mathrm{CO}}$ and $l_{bn}^{\mathrm{NC}}$ are properly chosen such that PM and PF are equal. We compare three scenarios: (i) each BS only recovers the signals of the users in its own cell with treating all inter-cell interference as noise, and performs the activity detection on its own users (with legend "Non-Coop. TIN"); (ii) each BS recovers the signal from its own cell as well as from other six neighboring cells with treating the rest of the inter-cell interference as noise, and performs the activity detection only on the users in its own cell (with legend "Non-Coop. EST"); (iii) each BS recovers the signal from its own cell as well as from other six neighboring cells while treating the rest of the inter-cell interference as noise, computes LLRs of the users in all seven cells, and exchanges the LLRs among cooperative BSs to help each other to perform the activity detection based on the aggregated LLR (with legend "Coop."). The first two scenarios correspond to the non-cooperative detection, and the third corresponds to the cooperative detection. We observe that the detection performance is already satisfactory when the number of antennas is 16. For non-cooperative detection, by recovering the strong interference from neighboring cells there is a performance improvement compared to treating all interference as noise, especially when the number of antennas is large. For cooperative detection, with the same number of antennas at the BSs, cooperation brings substantial improvement for the cell-edge users. Under this network setting, having twice as many antennas in the non-cooperative case achieves about the same performance as the cooperative case in terms of the 95-percentile user detection error.

## 6. CONCLUSION

This paper characterizes the performance of AMP based user activity detection for multi-cell systems using AMP, and compares two schemes to combat inter-cell interference. Results show that it is always beneficial to recover the inter-cell interference at each BS, and BS with large antenna array effectively improves the performances of all users whereas cooperation among BSs mainly improves the performances of cell-edge users.

[1] It is also possible to design the minimum mean-square error denoiser for AMP without assuming the knowledge of large-scale fading [3]. However, the calculations of PF/PM would be more involved.

## 7. REFERENCES


[1] F. Monsees, M. Woltering, C. Bockelmann, and A. Dekorsy, "Compressive sensing multi-user detection for multicarrier systems in sporadic machine type communication," in *IEEE Veh. Technol. Conf. (VTC Spring)*, May 2015, pp. 1–5.

[2] G. Wunder, H. Boche, T. Strohmer, and P. Jung, "Sparse signal processing concepts for efficient 5G system design," *IEEE Access*, vol. 3, pp. 195–208, Feb. 2015.

[3] Z. Chen, F. Sohrabi, and W. Yu, "Sparse activity detection for massive connectivity," *to appear in IEEE Trans. Signal Process.*, 2018.

[4] X. Xu, X. Rao, and V. K. N. Lau, "Active user detection and channel estimation in uplink CRAN systems," in *IEEE Inter. Conf. Commun. (ICC)*, London, UK, June 2015, pp. 2727–2732.

[5] Z. Utkovski, O. Simeone, T. Dimitrova, and P. Popovski, "Random access in C-RAN for user activity detection with limited-capacity fronthaul," *IEEE Signal Process. Lett.*, vol. 24, no. 1, pp. 17–21, Jan. 2017.

[6] E. Björnson, E. de Carvalho, J. H. Sørensen, E. G. Larsson, and P. Popovski, "A random access protocol for pilot allocation in crowded massive MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 16, no. 4, pp. 2220–2234, Apr. 2017.

[7] L. Liu and W. Yu, "Massive device connectivity with massive MIMO," in *IEEE Int. Symp. Information Theory (ISIT)*, June 2017, pp. 1072–1076.

[8] X. Chen, T.-Y. Chen, and D. Guo, "Capacity of Gaussian many-access channels," *IEEE Trans. Inf. Theory*, vol. 63, no. 6, pp. 3516–3539, June 2017.

[9] W. Yu, "On the fundamental limits of massive connectivity," in *Information Theory and Application (ITA) Workshop*, San Diego, CA, USA, Feb. 2017.

[10] D. Donoho, A. Maleki, and A. Montanari, "Message-passing algorithms for compressed sensing," *Proc. Nat. Acad. Sci.*, vol. 106, no. 45, pp. 18914–18919, Nov. 2009.

[11] S. Rangan, "Generalized approximate message passing for estimation with random linear mixing," in *IEEE Inter. Symp. Inf. Theory (ISIT)*, St. Petersburg, Russia, July 2011, pp. 2168–2172.

[12] J. Kim, W. Chang, B. Jung, D. Baron, and J. C. Ye, "Belief propagation for joint sparse recovery," *[Online] available: http://arxiv.org/abs/1102.3289v1*, 2011.

[13] A. Maleki, L. Anitori, Z. Yang, and R. G. Baraniuk, "Asymptotic analysis of complex LASSO via complex approximate message passing (CAMP)," *IEEE Trans. Inf. Theory*, vol. 59, no. 7, pp. 4290–4308, July 2013.

[14] J. Ziniel and P. Schniter, "Efficient high-dimensional inference in the multiple measurement vector problem," *IEEE Trans. Signal Process.*, vol. 61, no. 2, pp. 340–354, Jan. 2013.

[15] A. Montanari, "Graphical models concepts in compressed sensing," in *Compressed Sensing: Theory and Applications*, Y. C. Eldar and G. Kutyniok, Eds., chapter 9, pp. 394–438. Cambridge University Press, New York, 2012.