

# Achievable Rate Improvement Using Common Message Decoding for Multicell Networks

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**Abstract**—The performance of a multicell wireless network is often limited by intercell interference. This paper considers the use of common-private message splitting at the transmitter and common-message decoding at the receiver for intercell interference mitigation. For a downlink multicell system with multiple antennas at the base-stations and single antenna at the mobiles, we utilize a heuristic SDP-relaxation-based strategy proposed in our previous work to determine the optimal beamforming vectors for both common and private messages, and propose a new algorithm to numerically characterize the improvement in the achievable rates with common-message decoding. Simulation results show that common message decoding can significantly improve the minimum achievable rate for cell-edge users when base-stations are close in distance to each other.

## I. INTRODUCTION

The performance of wireless cellular networks is often limited by inter-cell interference. This is especially so for networks with aggressive frequency reuse in which the direct and interference signal levels at the cell edge can be of comparable strength—both being well above the background noise level. A promising idea in this situation is the use of interference detection and cancellation techniques. In particular, it is possible to utilize the information theoretic Han-Kobayashi coding strategy [1] for the interference channel where the transmitted signals at the base-stations are designed as superpositions of common and private messages. The common messages are to be decoded both at the intended and at the interfered receivers, and the private message is to be decoded at the intended receiver only. The partial detection of interference increases the effective signal-to-interference-and-noise ratio (SINR) at the intended receiver, thereby improving the overall performance of the cellular network.

The Han-Kobayashi strategy can be further combined with beamforming. As shown in our previous work [2], for a downlink multicell system with multiple antennas at the base-stations and single antenna at the mobiles, it is possible to solve a network optimization problem using the idea of semidefinite programming (SDP) relaxation for beamformer design, combined with a heuristic search to jointly determine the appropriate users in adjacent cells for rate splitting and the optimal common-private rates. However, the problem formulation in [2] is restricted to that of minimizing the total transmit power across the base-stations subject to SINR constraints at the mobile users. It does not deal with the rate maximization problem or the feasibility of the SINR constraints.

This paper utilizes the heuristic SDP-relaxation-based strategy proposed in [2], and develops a numerical algorithm to characterize the improvement in the achievable rates with common-private message splitting. The proposed algorithm finds the maximum minimum achievable rate for the users across the different cells by successively tuning the common and private information rates in each splitting. A contribution of this paper is a set of simulation results that quantifies the benefit of common information decoding for cell-edge users.

Efficient characterization of the feasible rate region for spatial multiplex systems is in general a difficult problem, even for the single-cell case. In the single-cell case, the feasibility condition is trivial (i.e. any SINR constraints are always feasible) when the channel is full rank with the same number of users as the number of antennas, but only a necessary feasibility condition is known when the channel is rank deficient [3], [4], [5]. The system setup in this paper differs in that we deal with a multicell multiuser multiantenna scenario and consider the transmission of both common and private messages. In this realm, solving the feasibility problem exactly is equivalent to a complete characterization of Han-Kobayashi region for the multiantenna interference channel, which is not yet available. Thus, this paper focuses on heuristic approaches to quantify the improvement in achievable rate.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

### A. Channel Model

This paper considers a multicell multiuser spatial multiplex system with  $N$  cells and  $K$  users per cell with  $N_t$  antennas at each base-station and a single antenna at each remote user as in [2]. The  $j$ th user in the  $i$ th cell splits its data stream into a private part  $x_{i,j}^p$  and a common part  $x_{i,j}^c$ . The common message  $x_{i,j}^c$  is intended to be decoded by both the user  $(i, j)$ 's own receiver and by some out-of-cell user. Likewise, user  $(i, j)$ 's receiver is designed to decode a common message from some (possibly different) out-of-cell user  $(\hat{i}, \hat{j})$  with  $\hat{i} \neq i$ .

Let  $\mathbf{w}_{i,j}^p, \mathbf{w}_{i,j}^c \in \mathbb{C}^{N_t \times 1}$  be the associated beamforming vectors for the common and private messages for user  $(i, j)$ , respectively. The received signal is modeled as

$$y_{i,j} = \sum_l \mathbf{h}_{i,i,j}^H \left( \mathbf{w}_{i,l}^p x_{i,l}^p + \mathbf{w}_{i,l}^c x_{i,l}^c \right) + \sum_{m \neq i,n} \mathbf{h}_{m,i,j}^H \left( \mathbf{w}_{m,n}^p x_{m,n}^p + \mathbf{w}_{m,n}^c x_{m,n}^c \right) + z_{i,j} \quad (1)$$

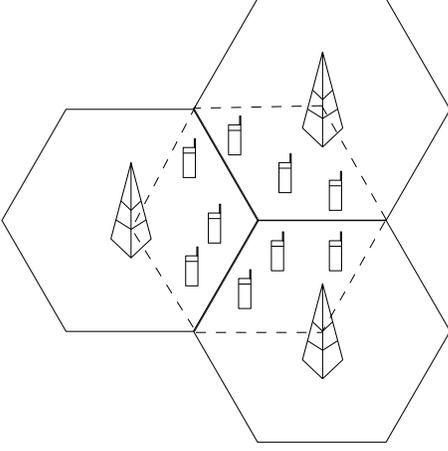


Fig. 1. A wireless network with three base-stations and three users per cell sectors.

where  $\mathbf{h}_{l,i,j} \in \mathbb{C}^{N_t \times 1}$  is the channel from the base-station of the  $l$ th cell to the  $j$ th user in the  $i$ th cell, and  $z_{i,j}$  is the additive white Gaussian noise. Fig. 1 illustrates the system model for a network with three cells and three users per cell sectors.

### B. Power Minimization Subject to Rate Target

Our previous work [2] dealt with a problem of minimizing the total transmit power across the base-stations subject to fixed target rate constraints  $R_{i,j}$ :

$$\begin{aligned} & \text{minimize} && \sum_{i,j} \|\mathbf{w}_{i,j}^p\|^2 + \|\mathbf{w}_{i,j}^c\|^2 && (2) \\ & \text{subject to} && \Gamma_{i,j}^p \geq 2^{R_{i,j}^p} - 1, \\ & && \Gamma_{ij,ij}^c \geq 2^{R_{ij,ij}^c} - 1, \\ & && \Gamma_{\hat{i},\hat{j},ij}^c \geq 2^{R_{\hat{i},\hat{j},ij}^c} - 1, \\ & && R_{i,j}^p + R_{i,j}^c = R_{i,j}. \quad \forall i, j \end{aligned}$$

where  $R_{i,j}^p$  and  $R_{i,j}^c$  are the private and common rates, and  $\Gamma_{i,j}^p$ ,  $\Gamma_{ij,ij}^c$  and  $\Gamma_{\hat{i},\hat{j},ij}^c$  are the private and common SINRs at user  $(i, j)$  assuming a natural decoding order, i.e. the common message at one's own transmitter is decoded first, the common message at the interfering transmitter is decoded next, and the private message at one's own transmitter is decoded last. The strategy proposed in [2] for solving (2) involves a SDP relaxation for beamformer design, along with a heuristic user pairing strategy based on the interference-to-noise ratios (INRs), and a search strategy for the optimal splitting of the common and private rates. In this paper, we utilize the algorithm proposed in [2] to solve a related and more practically relevant problem of maximizing the minimum service rate across the mobile users.

### C. Rate Maximization Subject to Power Constraint

One issue with the power-minimization formulation stated in the previous section is that it is often not easy to determine a priori whether a set of  $R_{i,j}$ 's is feasible. Further, practical wireless systems are often rate adaptive, thus the problem

of rate maximization subject to power constraints is more relevant. In this paper, we study the solution to the following problem

$$\begin{aligned} & \max \min_{(i,j)} R_{i,j} && (3) \\ & \text{subject to} && \Gamma_{i,j}^p \geq 2^{R_{i,j}^p} - 1, \\ & && \Gamma_{ij,ij}^c \geq 2^{R_{ij,ij}^c} - 1, \\ & && \Gamma_{\hat{i},\hat{j},ij}^c \geq 2^{R_{\hat{i},\hat{j},ij}^c} - 1, \\ & && R_{i,j}^p + R_{i,j}^c = R_{i,j}, \quad \forall i, j \\ & && \sum_{i,j} \|\mathbf{w}_{i,j}^p\|^2 + \|\mathbf{w}_{i,j}^c\|^2 \leq P_{max} \end{aligned}$$

where the maximization part of the optimization is over  $\mathbf{w}_{i,j}^p$ ,  $\mathbf{w}_{i,j}^c$ , all possible private-common rate splittings, and all possible pairings of users  $(\hat{i}, \hat{j})$  and  $(i, j)$  for common message decoding. Note that as mentioned earlier, each user  $(i, j)$  can only decode common message from one single interferer  $(\hat{i}, \hat{j})$ .

## III. ACHIEVABLE RATE IMPROVEMENT ALGORITHM

The SDP-relaxation-based algorithm proposed in [2] for solving (2) works as follows. Fixing  $R_{i,j}$ , the algorithm first finds the minimum transmit power with private information only using an SDP-relaxation-based technique. It then uses an heuristic approach to find the pairing of users for common information decoding based on the INRs obtained in the first step, and to optimize the splitting between the common and private rates.

The algorithm proposed in [2] works well if  $R_{i,j}$  is already feasible with private information only in the first step. The algorithm cannot be started, however, if the rates are not initially feasible. This means that the algorithm proposed in [2] is capable of minimizing the total transmit power for modest rates of  $R_{i,j}$ , but it is not able to characterize the improvement of the rate region beyond what is already feasible without common message decoding.

This paper proposes a numerical algorithm to address the issue above. The idea is to start with the maximum minimum achievable rate  $\hat{R}^{(0)}$  corresponding to private messages transmission only, then to use the obtained INRs for pairing users for common message decoding. Then, we gradually increase the target rate for all users, while searching for the optimal private-common rate splitting, one pair at a time. To increase the target rate of users that already have common-private rate splitting, we increase their common rates. For users that have private rates only, we increase their private rates. The proposed algorithm is summarized below:

- 1) Find the maximum minimum achievable rate  $\hat{R}^{(0)}$  with private information only. This is obtained by linearly increasing the target rate, then solving an SDP-relaxation problem in each step, eventually stopping at the last feasible point.
- 2) Form a list with  $(N-1)NK^2$  entries of  $\text{INR}_{(i,j) \rightarrow (\hat{i}, \hat{j})}$  with  $i \neq \hat{i}$ , i.e. the INR due to the interference from user  $(i, j)$  seen at user  $(\hat{i}, \hat{j})$ . Sort the list from the largest entry to the smallest entry.

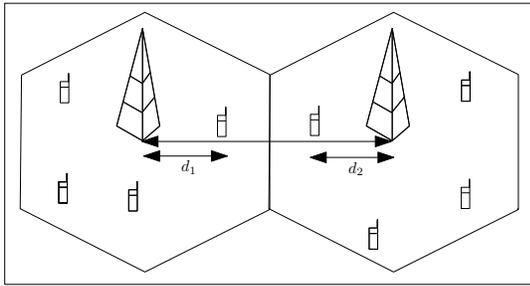


Fig. 2. A two-cell four-user per cell configuration with two users located between two base-stations at distances  $d_1$  and  $d_2$ .

- 3) Initialize  $L = 1$ .
- 4) Consider the  $L^{\text{th}}$  pair  $(\hat{i}, \hat{j}) \rightarrow (i, j)$  on the INR list. Split the rate of user  $(\hat{i}, \hat{j})$  as follows:
  - a) Initialize  $R_{\hat{i}, \hat{j}} = \hat{R}^{(L-1)}$ .
  - b) Gradually increase  $R_{\hat{i}, \hat{j}}$  by increasing the common rates of the first  $(L - 1)$  users that are already involved in rate splitting (while fixing their private rates), and by setting the remaining  $(NK - L)$  users' private rates to be equal to  $R_{\hat{i}, \hat{j}}$ .
  - c) For the fixed value of  $R_{\hat{i}, \hat{j}}$ , find the optimal rate splitting for the user  $(\hat{i}, \hat{j})$  through a linear search by calling the SDP relaxation routine for all possible values of  $0 \leq R_{\hat{i}, \hat{j}}^p \leq R_{\hat{i}, \hat{j}}$ . Call the optimal private rate  $\hat{R}_{\hat{i}, \hat{j}}^p$ .
  - d) Go to step (b) and stop at the largest feasible value of  $R_{\hat{i}, \hat{j}}$  subject to the power constraint. Call it  $\hat{R}^{(L)}$ .
- 5) Increment  $L$ . Go to step 4) and repeat for up to  $M$  pairs.

Step 4(b) can be further improved by doing an additional optimization on the splitting of the private and common rates of the  $(L - 1)$  users that are already involved in common information decoding. Such an optimization, however, requires additional exhaustive searches with considerable complexity. Step 4(b) above uses a simple approach of increasing the common rates of the first  $(L - 1)$  users, while increasing the private rates of the remaining  $(NK - L)$  users each time the target rate  $R_{\hat{i}, \hat{j}}$  is updated. The rationale behind this heuristic is the fact that each of the first  $(L - 1)$  users has already qualified as a good candidate for rate splitting. As the target rate increases, these users are expected to allocate a larger proportion of their data rates for the common part. Although the proposed algorithm does not guarantee global optimality, it nevertheless provides significant gain as the simulation section of this paper shows.

#### IV. SIMULATIONS

Consider first a 2-cell network with 4 users per cell as shown in Fig. 2 where common message decoding is performed only for the two users situated directly between the two base-stations. The base-stations are equipped with 4 antennas each. Realistic channel models are used in the simulation: the noise power spectral density is set to  $-162$  dBm/Hz; the channel

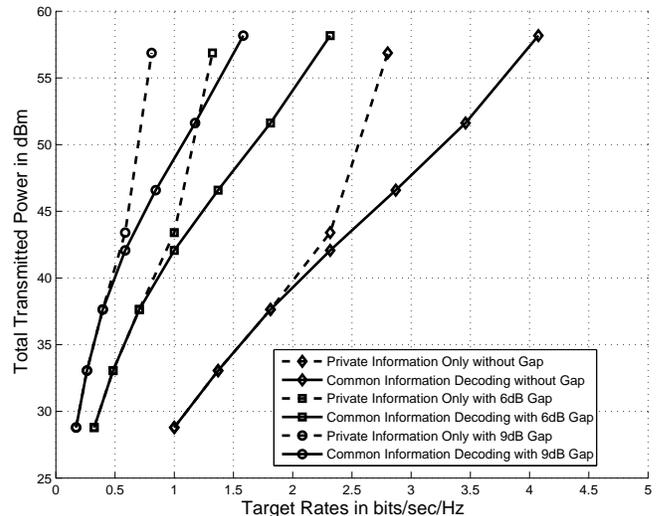


Fig. 3. Total transmitted power versus the rate targets for both the case of private-message only and the case of common-message decoding in a two-cell network with four users per cell for various gap values and  $d_1 = d_2 = 0.4$  km. Inter-base-station distance is  $1.4$  km.

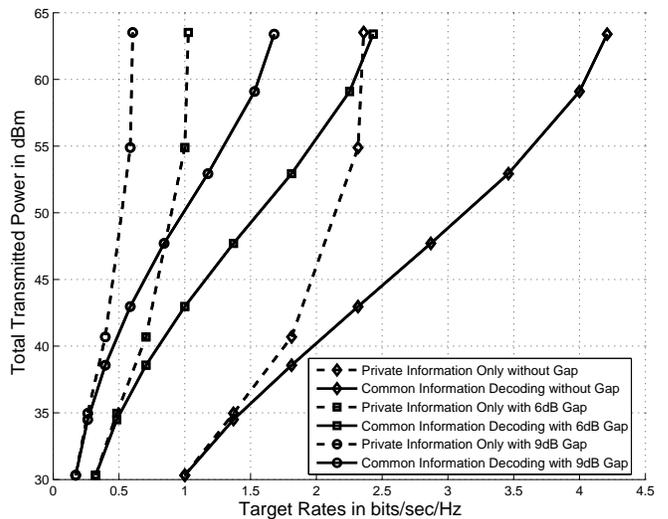


Fig. 4. Total transmitted power versus the rate targets for both the case of private-message only and the case of common-message decoding in a two-cell network with four users per cell for various gap values and  $d_1 = d_2 = 0.5$  km. Inter-base-station distance is  $1.4$  km.

vectors are chosen according to a distance-dependent path loss  $L = 128.1 + 37.6 \log_{10}(d)$  where  $d$  is the distance in kilometers, log-normal shadowing with 8dB variance, and a Rayleigh component. The distance between neighboring base-stations is set to be  $1.4$  km and an antenna gain of 15dBi is assumed.

Fig. 3 and Fig. 4 show the benefit of common message decoding on the improvement of the achievable rate for various values of the SINR gap under different topologies. The two users with common message decoding are at distances  $d_1$  and  $d_2$  from their respective base-stations as shown in Fig. 2. It

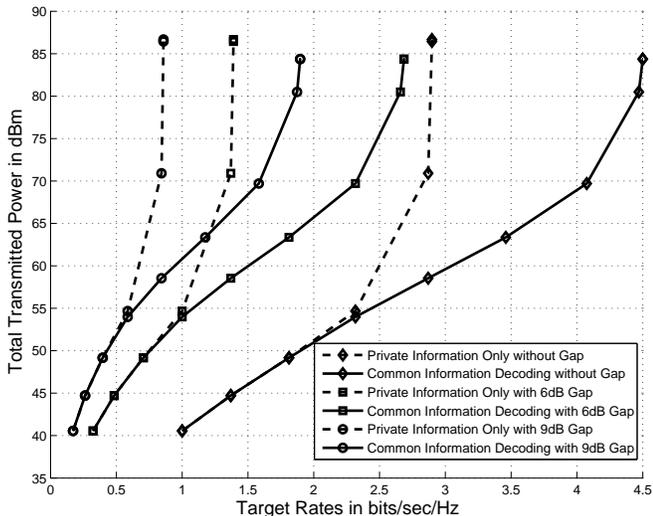


Fig. 5. Total transmitted power versus the rate targets for both the case of private-message only and the case of common-message decoding in a two-cell network with four users per cell for various gap values and  $d_1 = d_2 = 1\text{km}$ . Inter-base-station distance is  $2.8\text{km}$ .

TABLE I  
FEASIBILITY GAIN RESULTS FOR A SUM-POWER CONSTRAINTS OF 46DBM AND INTER-BASE-STATION DISTANCE OF 1.4km

$d_1$ in km	$d_2$ in km	$R_{max}$ , private information only in bps/Hz	$R_{max}$ , common information dec. in bps/Hz	Percentage gain
<b>Gap=0dB</b>				
0.4	0.4	2.48	2.80	13%
0.5	0.5	2.10	2.66	26%
<b>Gap=6dB</b>				
0.4	0.4	1.11	1.32	19%
0.5	0.5	0.87	1.22	40%
<b>Gap=9dB</b>				
0.4	0.4	0.66	0.81	23%
0.5	0.5	0.50	0.74	47%

is clear from the figures that the gain for the users at the cell edges is substantial when comparing the achievable rate of the common message decoding to the achievable rate of private information transmission only. The gain decreases as the users get closer to the cell center where the interference is typically limited. The figures also show how the SINR gap affects the feasibility gain. The achievable rate gain is smaller with a larger value of SINR gap. Fig. 5 shows a similar plot with inter-base-station distance of  $2.8\text{km}$ . It is seen that the rate improvement in this case only occurs at much higher transmit power values.

Table I shows the improvement of the minimum achievable rate using common message decoding with a total transmit power constraint of  $46\text{dBm}$  with base-station distance of  $1.4\text{km}$ . It is interesting to note that although the absolute achievable rate gain is smaller with a larger gap, the percentage gain, which ranges from 40% to 50%, is actually larger.

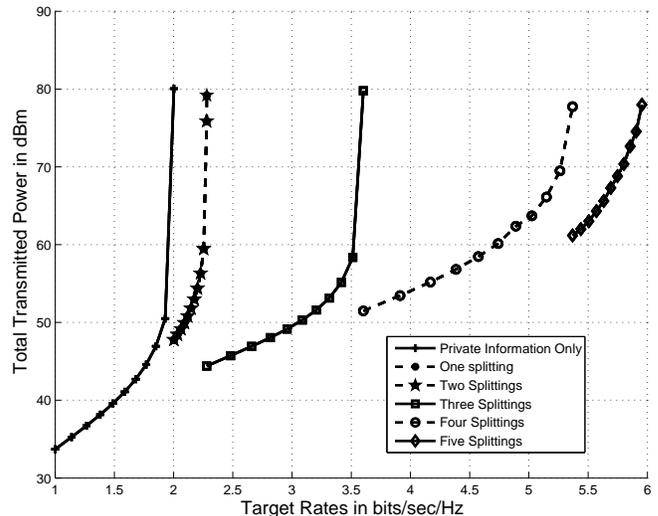


Fig. 6. Total transmitted power versus the target rates for both the case of private-message only and the case of common-message decoding in a three-cell network with three users per cell.

To illustrate the behavior of the proposed algorithm in a 3-cell setting, Fig. 6 shows the total transmit power as a function of target rates in a 3-cell network with 3 users per cell sector as shown in Fig. 1 and with neighboring base-station distance of  $1.4\text{km}$ . In addition, the antenna element responses here also include a directional component due to sectorization. The users are randomly distributed close to the cell edge. As the plot illustrates, the feasible rate gain is quite substantial with common-message splitting of multiple user pairs.

## V. CONCLUSION

This paper considers the benefit of common message decoding on improving the achievable rate of a multicell multiuser multi-antenna environment with spatial multiplexing. We propose a heuristic SDP-relaxation-based algorithm to quantify the rate gain due to the design of decodable interference signals. The algorithm is shown to provide substantial achievable rate gain for cell-edge users especially when base-stations are close to each other in distance.

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