Power Spectrum Optimization for Interference Mitigation via Iterative Function Evaluation

Hayssam Dahrouj[†], Wei Yu[†], Taiwen Tang[‡], and Steve Beaudin
[†]Electrical and Computer Engineering Dept., University of Toronto, Canada
[‡]BlinQ Networks Inc., Kanata, Ontario, Canada
Emails: {hayssam,weiyu}@comm.utoronto.ca, eric.tang@blinqnetworks.com, steve.beaudin2@yahoo.com

***Abstract*—This paper proposes practical methods for and examines the benefit of dynamic power spectrum optimization for interference mitigation in wireless networks. The paper envisions a distributed antenna system, deployed as a means to increase the network capacity for areas with high data traffic demand. The network comprises several access nodes (AN), each serving a fixed set of remote radio units called remote terminals (RT). The RTs belonging to each AN are separated from each other using orthogonal frequency division multiple access (OFDMA) over a fixed bandwidth, where only one RT is active at each frequency tone. The system performance is thus limited by internode interference solely, and no intranode interference. This paper proposes methods for power spectrum optimization based on the idea of iterative function evaluation. The proposed methods provide a significant improvement of the overall network throughput, as compared to conventional wireless networks with fixed transmit power spectrum. The proposed methods are computationally feasible and fast in convergence. They can be implemented in a distributed fashion across all access nodes, with reasonable amount of internode information exchange. Some of the proposed methods can be further implemented asynchronously at each AN, which makes them amenable to practical utilization.**

I. Introduction

Interference is a major bottleneck in wireless network design. Developing and optimizing advanced, yet practical, interference mitigation techniques is particularly important nowadays, due to the rapid pace of growth of wireless networks with enormous data usage, and the scarcity of the available radio resources, e.g. bandwidth and transmit power. Dynamic power spectrum optimization is an important class of interference mitigation methods that seek to increase the network capacity and reliability via power control. The present paper aims to develop novel, feasible, practical methods for power spectrum optimization.

Dynamic power spectrum optimization is especially applicable to distributed antenna system (DAS) where the base-station transmit capability is enhanced by adding multiple remote radio units. The setup under discussion assumes a cellular network comprising several remote terminals (RT), each covering a relatively small area as a means to increase the network capacity for areas with dense data traffic. The RTs are then connected to access nodes (ANs) via wireless links which are meant to replace the expensive optical fiber links. The ANs are responsible for the transmission strategies and radio resource management for the different RTs. From a design perspective, the interest of this paper is to mitigate the internode interference, thereby maximizing the aggregate data capacity of the RTs, via practical power spectrum optimization methods.

The main challenge in power control remains the problem of finding computationally efficient methods to allocate the power of the different transmitters across the different frequency tones. The power spectrum optimization problem is especially well studied in the literature for digital subscribers lines (DSL) [1], [2], [3], [4], [5]. For wireless networks, power control can be performed using the concept of interference price ([6], [7], [8], [9]) which quantifies the effect of interference between the multiple transmitter-receiver pairs in the wireless medium. The power control methods can also be incorporated with scheduling [10], [11], [12].

This goal of this paper is to study a class of *iterative function evaluation* based methods for power management. These methods have an advantage of low computational complexity, while showing a significant gain compared to the conventional maximum power transmission policy. Further, these methods lend themselves to distributed implementation with reasonable amount of inter-access-node information exchange. Some of the methods can also be implemented asynchronously, which makes them amenable to practical utilization.

The proposed methods make use of channel measurements done on a per-tone basis for every AN-RT pair. The measurements are subsequently provided to either a central server for further centralized processing, or to each of the several access nodes for distributed processing. These measurements are of particular interest in fixed deployment scenarios, where the channels are slow varying. Further, the measurement can be done periodically, thus allowing the adaptation of radio resource allocations with the dynamically changing environment. In this paper, the measurements are utilized for internode interference mitigation via joint dynamic power spectrum adaptation and scheduling.

II. System Model and Problem Formulation

A. System Model

Consider a wireless multicell network with L ANs and K RTs per AN, with single antennas at both the ANs and RTs. The RTs belonging to each AN are separated from each other using orthogonal frequency division multiple access (OFDMA) of N subcarriers over a fixed bandwidth, where only one RT is active at each frequency tone. This paper

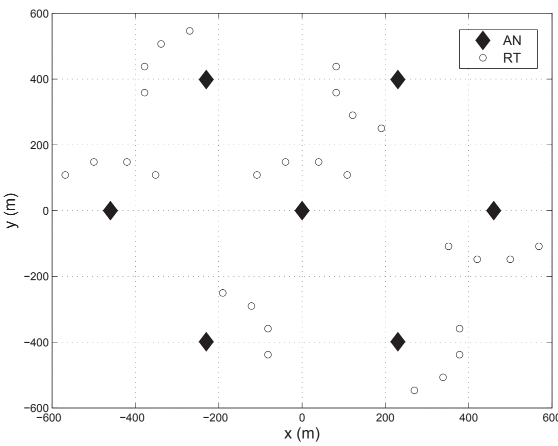

Fig. 1. A distributed antenna system with seven 7 ANs and 4 RTs per AN.

aims to use power management methods to alleviate internode interference. In particular, the lth AN may allocate its power P_l^n at each tone $n \in \{1, \cdots, N\}$, depending on the scheduling assignment of RTs and the channel gains between the AN-RT pairs. Let $k = f(l, n)$ and $k' = f(j, n)$ be the scheduled RTs of the lth AN and the jth RT respectively, both at the nth tone. The received signal at the kth RT is a summation of the intended signal and the internode interference:

$$y_l^n = h_{llk}^n x_l^n + \sum_{j \neq l} h_{jlk}^n x_j^n + z_l^n \tag{1}$$

where x_l^n is a complex scalar denoting the information signal for the kth RT, $h_{jlk}^n \in \mathbb{C}$ is the channel from the jth AN to the kth RT, and z_l^n is the additive white Gaussian noise with variance $\sigma^2/2$ on each of its real and imaginary components. Fig. 1 illustrates the system model for seven ANs and four RTs per AN.

B. Problem Formulation

For completeness, the overall network optimization problem can be stated as a maximization of the log utility function:

$$\begin{aligned} \max \quad & \sum_{l,k} \log\left(\bar{R}_{lk}\right) \\ \text{s.t.} \quad & R_{lk} = \sum_{\{n | k = f(l,n)\}} \log\left(1 + \text{SINR}_{lk}^n\right) \\ & 0 \leq P_l^n \leq S^{max} \qquad \forall l, n \\ & \text{SINR}_l^n = \frac{P_l^n |h_{llk}^n|^2}{\Gamma(\sigma^2 + \sum_{j \neq l} P_j^n |h_{jlk}^n|^2)} \end{aligned} \tag{2}$$

where $\bar{R}_{lk}$ is the long term average rate of the kth RT of the lth AN, S^{max} is the maximum power constraint imposed on each AN at each tone, and where the maximization is over the scheduling assignment $k = f(l, n)$, and the power spectral density levels P_l^n. The Γ is the signal-to-noise ratio (SNR) gap.

This paper adopts an iterative scheduling and power control policy similar to that in [10], [11], [12], in which the scheduling is done assuming fixed power, and power optimization is done assuming a fixed schedule. The focus of this paper is the power allocation step, which is essentially a weighted rate-sum maximization problem on a per-tone basis:

$$\begin{aligned} \max \quad & \sum_l w_{lk} r_{lk}^n \\ \text{s.t.} \quad & 0 \leq P_l^n \leq S^{max} \end{aligned} \tag{3}$$

where

$$r_{lk}^n = \log\left(1 + \frac{P_l^n |h_{llk}^n|^2}{\Gamma(\sigma^2 + \sum_{j \neq l} P_j^n |h_{jlk}^n|^2)}\right) \tag{4}$$

is the instantaneous rate of the scheduled kth RT for the lth AN at the nth tone, the weights comes from the scheduling objective (e.g. $w_{lk} = \frac{1}{\bar{R}_{lk}}$ for proportional fairness), and where the maximization is over the set of powers P_l^n. The rest of the paper examines practical numerical methods to solve (3) and quantifies their performance.

III. Power Spectrum Optimization

The weighted sum-rate maximization problem (3) is a non-convex optimization problem, whose global optimal solution is not easy to find. Like many previous approaches, this paper also aims at local optimal solutions, but with a focus on reduced computational complexity. In particular, the current paper considers a class of strategies based on an iterative function evaluation approach. The novel methods, described below, are simple to implement, fast in convergence, and similar in performance to traditional full-blown Newton's method. Some of the proposed methods can be implemented in a distributed fashion, and asynchronously at each AN, with no need for step size choices.

A. Iterative Function Evaluation Methods (IFEM)

1) Full-IFEM: We begin by taking the gradient of the objective function R of the problem (3) with respect to P_l^n:

$$\begin{aligned} \frac{\partial R}{\partial P_l^n} &= w_{lk} \frac{\partial r_{lk}^n}{\partial P_l^n} + \sum_{j \neq i} w_{jk'} \frac{\partial r_{jk'}^n}{\partial P_l^n} \\ &= \frac{w_{lk}}{P_l^n} \left(\frac{\text{SINR}_l^n}{1 + \text{SINR}_l^n}\right) - \\ & \quad \sum_{j \neq l} w_{jk'} \frac{|h_{ljk'}^n|^2}{\sigma^2 + \sum_{i \neq j} P_i^n |h_{ijk'}^n|^2} \frac{\text{SINR}_j^n}{1 + \text{SINR}_j^n} \end{aligned} \tag{5}$$

where SINR_j^n is defined as:

$$\text{SINR}_j^n = \frac{P_j^n |h_{jjk'}^n|^2}{\Gamma(\sigma^2 + \sum_{i \neq j} P_i^n |h_{ijk'}^n|^2)} \tag{6}$$

A local optimal solution must be such that the above gradient is zero. The main idea of the iterative function evaluation method (IFEM) is to rewrite the above as:

$$P_l^n = \frac{w_{lk} \frac{\text{SINR}_l^n}{1 + \text{SINR}_l^n}}{\sum_{j \neq l} w_{jk'} \frac{|h_{ljk'}^n|^2}{\sigma^2 + \sum_{i \neq j} P_i^n |h_{ijk'}^n|^2} \frac{\text{SINR}_j^n}{1 + \text{SINR}_j^n}} \tag{7}$$

It is now possible to compute all the terms on the right-hand-side of the equation using the current power allocation, and update the new power allocation as above. This step is a simple function evaluation, which can be done iteratively, hence this method is called Full-IFEM.

More formally, the power level of every AN at every tone, P_l^n, is updated from step t to $t+1$ according to the following:

$$P_l^n(t+1) = \left[\frac{w_{lk} \frac{\mathrm{SINR}_l^n(t)}{1+\mathrm{SINR}_l^n(t)}}{\sum_{j \neq l} w_{jk'} \frac{|h_{ljk'}^n|^2}{\sigma^2 + \sum_{i \neq j} P_i^n(t) |h_{ijk'}^n|^2} \frac{\mathrm{SINR}_j^n(t)}{1+\mathrm{SINR}_j^n(t)}} \right]_0^{S^{max}} \quad (8)$$

where the maximum power constraint is also taken into account.

2) θ-IFEM: The above full-IFEM algorithm requires finding the individual signal-to-interference-and-noise ratios (SINRs) at every iteration. To further simplify the computational complexity, we propose the following heuristic method that replaces the per-iteration SINR's with the values of SINR's calculated under the initial maximum power transmission strategy. Although this method does not guarantee local optimality, it has the advantage that its convergence is easy to prove. Further, it provides significant gain as compared to the maximum power transmission strategy, as the simulation results show. This method, called θ-IFEM, finds the power level P_l^n according to the following update equation:

$$P_l^n(t+1) = \left[\frac{w_{lk}}{\sum_{j \neq l} w_{jk'} \frac{|h_{ljk'}^n|^2}{\sigma^2 + \sum_{i \neq j} P_i^n(t) |h_{ijk'}^n|^2} \theta_{jl}^n} \right]_0^{S^{max}} \quad (9)$$

where

$$\theta_{jl}^n = \frac{\frac{\mathrm{SINR}_j^n}{1+\mathrm{SINR}_j^n}}{\frac{\mathrm{SINR}_l^n}{1+\mathrm{SINR}_l^n}} \quad (10)$$

is a fixed factor calculated from the maximum power transmission strategy.

3) IFEM: To further simplify the power update equations, one can set θ_{jl}^n to 1. This is in fact a high-SINR approximation of the problem. The resulting update equation becomes:

$$P_l^n(t+1) = \left[\frac{w_{lk}}{\sum_{j \neq l} w_{jk'} \frac{|h^n{}_{ljk'}|^2}{\sigma^2 + \sum_{i \neq j} P_i^n(t) |h^n{}_{ijk'}|^2}} \right]_0^{S^{max}} \quad (11)$$

For physical platforms that only permit each AN to allocate one value for the power across all tones, the power can be found by taking the average of the power values of IFEM. The resulting method is called AP IFEM.

B. Convergence Analysis

The convergence of full-IFEM is difficult to establish in full generality. But the following convergence result is available for both θ-IFEM and IFEM under both the synchronous and asynchronous models.

Proposition 1. *Starting from any initial $P_l^n(0)$, both θ-IFEM and IFEM algorithms converge to a unique fixed point. Furthermore, the convergence is still guaranteed under a totally asynchronous model.*

Proof: The proof is based on corollary 1 in [13], as both update equations of θ-IFEM and IFEM, written as $P_l^n(t+1) = \left[g\big(\mathbf{\Psi}^n(t)\big)\right]_0^{S^{max}}$, satisfy the following standard function properties:

1) If $P_l^n \geq 0 \ \forall l, n$, then $g\big(\mathbf{\Psi}^n\big) > 0$.
2) If $P_l^n \geq {P'}_l^n \ \forall l, n$, then $g\big(\mathbf{\Psi}^n\big) \geq g\big(\mathbf{\Psi}'^n\big)$.
3) For $\rho > 1$, we have $\rho g\big(\mathbf{\Psi}^n\big) > g(\rho \mathbf{\Psi^n}) \ \forall l, n$.

where the variables $P_l^n, \forall \quad l = 1, \cdots, L$, are stacked into one vector $\mathbf{\Psi}^n$. The convergence to the unique fixed point and the asynchronous convergence follow as a consequence. ■

C. Connection with SCALE ([3])

In [3], a power control algorithm named SCALE is proposed. The algorithm is motivated by geometric programming. SCALE is a two-stage algorithm, and in the notation of this paper can be thought of as having a θ-IFEM-like algorithm in the inner loop, and a θ_{jl}^n update in the outer loop. The SCALE algorithm defines $\alpha_j^n = \frac{\mathrm{SINR}_j^n}{1+\mathrm{SINR}_j^n}$, so that $\theta_{jl}^n = \alpha_j^n / \alpha_l^n$. It runs iterative function evaluation with fixed α_j^n in the inner loop, then update α_j^n based on the resulting SINRs in the outer loop. The full-IFEM proposed in this paper is essentially a simplification of SCALE. Instead of the two-stage process in which θ_{jl}^n is updated in an outer loop, full-IFEM implicitly updates the power vector and θ_{jl}^n at the same time.

In addition, as shown in the simulation results, the use of a single fixed θ_{jl}^n derived from the maximum transmit power level may already be near optimum (leading to θ-IFEM), thus θ_{jl}^n may not need to be updated at all. Further, at high SINR, θ_{jl}^n can be set to 1, leading to IFEM.

D. Comparison with Newton's Method

As a baseline comparison, we also describe the following Newton's method (NM) update equation as in [12]:

$$P_l^n(t+1) = \left[P_l^n(t) + \mu \Delta P_l^n(t)\right]_0^{S^{max}}, \quad (12)$$

where μ is the step size, and

$$\Delta P_l^n(t) = \frac{\frac{w_{lk}}{P_l^n(t)} \left(1 + \frac{1}{\mathrm{SINR}_l^n}\right)^{-1} - \sum_{j \neq l} \tau_{jl}^n}{\frac{w_{lk}}{(P_l^n(t))^2} \left(1 + \frac{1}{\mathrm{SINR}_l^n}\right)^{-2}} \quad (13)$$

where τ_{jl}^n is the interference price defined as

$$\tau_{jl}^n = w_{jk'} \frac{|h_{ljk'}^n|^2}{\sigma^2 + \sum_{i \neq j} P_i^n(t) |h_{ijk'}^n|^2} \frac{\mathrm{SINR}_j^n}{1+\mathrm{SINR}_j^n} \quad (14)$$

where $k' = f(j, n)$.

The main disadvantage of the Newton's method is that the choice of step size cannot be easily done in a distributed fashion, and certainly not asynchronously.

Cellular Layout	Hexagonal
Number of ANs	7
Frequency Reuse	1
Number of RTs per AN	4
AN-to-AN Distance	d_1
AN-to-RT Distance	d_2
Duplex	TDD
Channel Bandwidth	10 MHz
AN Max Tx Power per Subcarrier	-32.70 dBw
SINR Gap	12 dB
Total Noise Power Per Subcarrier	-158.61 dBw
Distance-dependent Path Loss	$128.1 + 37.6\log_{10}(d)$
Sampling Frequency	11.2 MHz
FFT Size	1024

TABLE I
SYSTEM MODEL PARAMETERS

To simplify the computations required at each iteration of Newton's method, we also propose a high-SINR Newton's method (HSNM) where the update equation (13) is approximated as

$$\Delta P_l^n(t) = \frac{\dfrac{w_{lk}}{P_l^n(t)} - \displaystyle\sum_{j\neq l} w_{jk'} \frac{|h_{ljk'}^n|^2}{\sigma^2 + \sum_{i\neq j} P_i^n(t)|h_{ijk'}^n|^2}}{\dfrac{w_{lk}}{(P_l^n(t))^2}} \quad (15)$$

IV. SIMULATIONS

This section evaluates the benefit of the proposed power spectrum optimization methods on a wireless network comprising seven ANs, and 4 RTs per AN, over 10MHz bandwidth. The transmission of each AN to its own RTs interferes with the other ANs' transmissions. RTs belonging to one AN are separated from each other using OFDMA with 1024 subcarriers, where only one RT is active at each frequency tone. The parameters used in simulation are as outlined in Table I. The AN-to-AN distance is set to d_1; the AN-to-RT distance is set to d_2. Both d_1 and d_2 vary so as to study the performance of the proposed methods for various topologies. For illustration purposes, the weighting factors w_{lk} in problem (3) are set to 1, $\forall(l,k)$, which allows a sum-rate comparison.

Fig. 2 shows the sum-rate performance over all ANs for a network with AN-to-AN distance $d_1 = 0.5$km and AN-to-RT distance $d_2 = 0.15$km over different realizations of the channel. Fig. 2 shows that there is a small performance loss due to the high SINR approximation (i.e. IFEM and HSNM have a lower performance as compared to full-IFEM). Nevertheless, IFEM and HSNM outperform the maximum power method significantly. We also observe that AP IFEM, which allocates one power value for each AN across all the tones, is always superior to the maximum power method.

Tables II, III, and IV illustrate the performance of IFEM for different network topologies. Table II considers the effects of cell sizes, and shows that the benefit of power optimization is more pronounced in a small-cell setting, where the interference level is higher. Tables III and IV examine the effect of RT

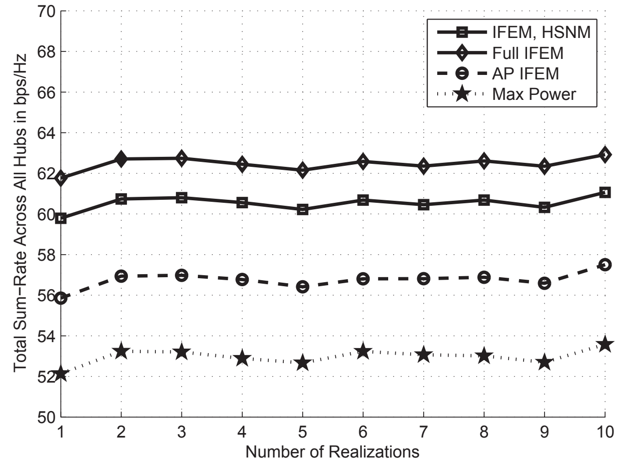

Fig. 2. Sum-rate in bps/Hz over 7 ANs, 4 RTs per AN. AN-to-AN distance is 0.5km. AN-to-RT distance is 150m.

Sum Rate (bps/Hz)	Small-cell ($d_1 = 0.5$km)	Large-cell ($d_1 = 1$km)
IFEM	60.68	91.33
HSNM	60.68	91.33
Full-IFEM	62.61	91.58
Max Power	53.01	86.22
Full-IFEM Gain	18.1%	6.2%

TABLE II
7 ANS, 4 RTS PER AN. d_1 IS THE AN-TO-AN DISTANCE. AN-TO-RT DISTANCE d_2 IS 150M.

locations within each cell. It is shown that the benefit of power optimization is noticeably higher for cell-edge users, where the interference is larger.

It can be observed from Tables II, III, and IV that IFEM and HSNM always have the same performance, as both employ a high-SINR appoximation. However, at the cell-edge of small cells, where the SINR level is not sufficiently large to justify the high-SINR approximation, IFEM and HSNM become inferior to full-IFEM. This is, however, not the case for cell-edge users of larger cells, i.e. $d_1 = 1$km, shown in Table IV, where SINR values are larger, and where IFEM, HSNM and full-IFEM again have similar performance. In all cases, IFEMs always remain superior to the maximum power method.

Figs. 3 and 4 compare the convergence performance of IFEM algorithms with the Newton's method. For fair comparison, the Newton's method is plotted here with a constant step size of 1. As seen in Fig. 3, because of the constant step size, the Newton's method has a poor performance initially, and IFEM converges faster overall. Note that the convergence speed comparison depends on the SINR. Fig. 3 corresponds to a high SINR situation, where IFEM outperforms the Newton's method. Fig. 4 shows an opposite situation, at a relatively low SINR, where the convergence of the Newton's method is faster than the full-IFEM. Note that at high SINR, the achievable sum-rate performances of IFEM, θ-IFEM, full-

Sum Rate (bps/Hz)	Cell-edge ($d_2 = 333$m)	Cell-center ($d_2 = 125$m)
IFEM	34.84	78.39
HSNM	34.84	78.39
Full-IFEM	41.11	78.77
Max Power	30.54	71.91
Full-IFEM Gain	34.6%	9.5%

TABLE III
7 ANs, 4 RTs PER AN. AN-TO-AN DISTANCE IS 0.5KM.

Sum Rate (bps/Hz)	Cell-edge ($d_2 = 667$m)	Cell-center ($d_2 = 250$m)
IFEM	44.86	83.55
HSNM	44.86	83.55
Full-IFEM	46.86	84.24
Max Power	41.49	80.18
Full-IFEM Gain	12.9%	5.1%

TABLE IV
7 ANs, 4 RTs PER AN. AN-TO-AN DISTANCE IS $d_1 = 1$KM.

IFEM and Newton's method are similar, while at low SINR, full-IFEM and Newton's method outperform θ-IFEM, which in term outperforms IFEM.

V. Conclusion

Given the scarcity of the available radio resources, the performance of future wireless networks is expected to widely depend on the feasibility of the dynamic power spectrum optimization methods. This paper presents novel and practical methods to manage interference in wireless systems. The proposed methods represent efficient ways of updating the power spectral density levels for all transmitters, based on the frequency domain channel measurements. The proposed methods, full-IFEM, θ-IFEM and IFEM, are simple methods, with low computational complexity and fast convergence. Their performance is similar to the full-blown Newton's method, but without the need for step size choices. They can also be implemented in a distributed fashion, and asynchronously at each transmitter, and are therefore excellent fits for practical applicability.

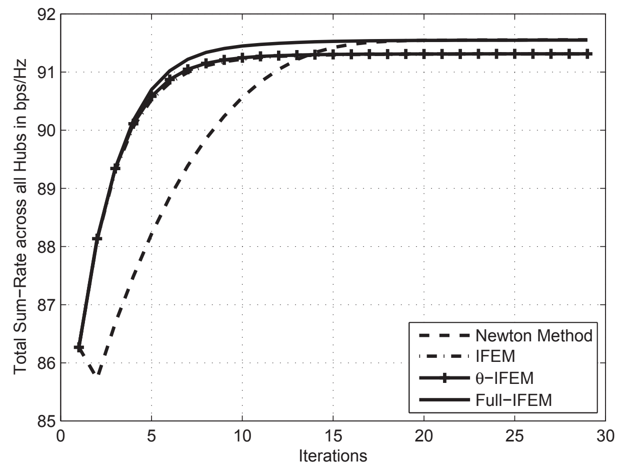

Fig. 3. Sum-rate in bps/Hz versus the number of iterations, over 7 ANs, 4 RTs per AN. AN-to-AN distance is 1km. AN-to-RT distance is 150m. It shows the convergence of the different methods at high SINR.

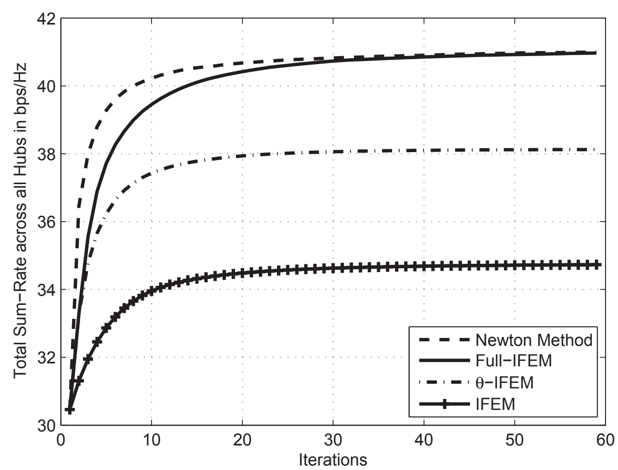

Fig. 4. Sum-rate in bps/Hz versus the number of iterations, over 7 ANs, 4 RTs per AN. AN-to-AN distance is 0.5km. AN-to-RT distance is 333m. It shows the convergence of the different methods at low SINR.

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