Multicell Interference Mitigation with Joint Beamforming and Common Message Decoding

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Abstract

Conventional wireless cellular systems treat out-of-cell interference as noise. This paper proposes methods and examines the benefit of designing decodable interference signals, whereby a transmitter may split its message into a common and a private part, and the common message may be decoded and subtracted by users in adjacent cells. This paper considers a downlink scenario, where the base-stations are equipped with multiple antennas, the mobile users are equipped with a single antenna, and multiple users are active simultaneously via spatial multiplexing. The network optimization problem consists of jointly determining the appropriate users in adjacent cells for rate splitting, the optimal transmit beamformers for common and private messages, and the optimal common-private rates to maximize the minimum achievable rate across the users. This paper shows that for fixed user selection and fixed common-private rate splitting, the optimization of transmit beamformers can be solved using a semidefinite programming (SDP) relaxation approach. Further, it is shown that for the case where the network consists of two message-splitting pairs, SDP relaxation is tight, i.e., beamforming is optimal. Finally, this paper proposes a heuristic user-selection and rate splitting strategy to characterize the performance improvement of for cell-edge users due to common-message decoding.

Index Terms

Beamforming, Han-Kobayashi strategy, interference channel, multiple-input multiple-output (MIMO), multiuser multiantenna multicell network, semidefinite programming (SDP)

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I. Introduction

In a conventional wireless cellular system, each base-station communicates with the mobile terminals independently; out-of-cell interference is treated as noise. Multiuser detection, while feasible for intracell users, e.g. [3] as in a code-division multiple access (CDMA) system, is difficult to implement for out-of-cell users, because the intercell interference is typically much weaker than the desired signal. Conventional cellular networks, however, are also typically designed to be interference-limited. This is especially so as networks are increasingly designed with full frequency reuse. Thus, although out-of-cell interference is weak, it can still be significantly above the background noise level. Further, in modern wireless networks, base-stations are often equipped with multiple antennas and typically have the ability to adapt their transmit powers and beamforming patterns, thereby influencing the effective strengths of the direct and interfering channels. These possibilities give rise to the potential for designing transmit signals for the purpose of multiuser detection at adjacent cells as means for interference mitigation.

The wireless multicell system can be modelled as an interference network. Unlike conventional multiuser multiantenna multicell systems where each base-station transmits an independent data stream to its respective users in each cell, this paper considers an approach inspired by the information theoretical study of the two-user interference channel due to Han and Kobayashi [4], where the transmit signals are explicitly designed so that they are partially decodable in adjacent cells. In the Han-Kobayashi strategy, each user's transmit signal is split into two parts: a private message to be decoded by the intended receiver only, and a common message to be decoded by both receivers for the sole purpose of interference mitigation. The Han-Kobayashi strategy gives the largest known achievable rate region for the two-user interference channel, and has been shown to achieve the capacity region of the two-user interference channel to within one bit [5].

The paper aims to take advantage of the insight offered by the Han-Kobayashi strategy to show that a common-private message splitting scheme can indeed bring a significant benefit to cell-edge users in a wireless cellular network. This paper goes beyond the simple two-user single-input single-output model in the information theory literature, and considers a multicell downlink system where the base-stations are equipped with multiple antennas, the remote receivers are equipped with a single antenna each, and multiple users may be active simultaneously in each cell and are separated via spatial multiplexing using downlink beamforming. In this case, the problem of designing the optimal common-private splitting scheme becomes intertwined with the selection of users for common message decoding and the design of their respective downlink beamformers across the cells.

Towards this problem, this paper first considers a design criterion of minimizing the total transmit power across all the base-stations subject to rate constraints for each user. It is shown that for fixed user selection and fixed common-private rate splitting, the problem of optimizing transmit beamformers for both the private and common data streams can be solved using a semidefinite programming (SDP) relaxation approach. Further, for the special case where the network consists of two message-splitting pairs, SDP relaxation is in fact tight, i.e., beamforming is optimal.

In the second part of this paper, we consider the maximization of minimum achievable rate across a multicell network with multiple users per cell. We propose a numerical algorithm for determining the most suitable out-of-cell users for common message decoding, the appropriate rate splitting levels, and the optimal beamforming vectors for both common and private messages at the base-stations. The proposed algorithm involves heuristic greedy discrete optimization and convex relaxation as its main components. The results of this paper show that common message decoding by the out-of-cell users can be quite effective in mitigating intercell interference, thereby improving the performance of cell-edge users.

A. Related Work

From an information theoretical perspective, the simple two-user single-input single-output interference channel has been studied extensively in the literature (see [5] and references therein); however, the full characterization of its capacity region remains open, except for the case of strong interference [6], and the case of sum capacity in a low interference regime [7], [8], [9]. The largest known achievable rate region for the two-user interference channel is due to Han and Kobayashi [4], [10]. Recently, Etkin, Tse and Wang [5] offered a key insight into the optimization of Han-Kobayashi strategy by showing that a simple scheme of setting the private message power at the opposite receiver to be at the background noise level achieves within one bit of the capacity region of the interference channel. Thus, the part of the out-of-cell interference signal which is above the background noise level should essentially be regarded as common message and be decoded.

This paper aims to utilize information theoretical concepts on realistic communication systems.

In this end, [11] considered the power control problem for the two-user fading interference channel. In a digital subscriber line setting, [12], [13] considered competitive optimum power allocation and the joint optimization of transmit spectra and common-private rate splitting, respectively. However, the above studies are restricted to the single-antenna channel model.

Most of the literature on the multi-antenna multicell interference environment focuses on the scenario where multiple base-stations may coordinate in their respective beamforming strategy to avoid excessive mutual interference. As shown in [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], such a beamforming level coordination can already provide considerable gain as compared to an uncoordinated system. The present paper goes one step further in pointing out that even larger performance gain is possible if the beamformers are explicitly designed to account for the possibility of interference subtraction.

This paper utilizes the technique of SDP relaxation for downlink beamforming design, which is pioneered in the work of Bergtsson and Ottersten [28] for the single-cell scenario, and subsequently extended for multicell systems [29], systems with interference constraints [30], and multicast systems [31]. In particular, we utilize a condition for the optimality of SDP relaxation based on the work of [32] to characterize a class of problems in which rank-one beamforming is optimal with common message decoding.

B. Organization

The remainder of the paper is organized as follows. Section II contains the system model and the problems formulation. Section III presents the SDP relaxation approach for solving the joint beamforming and common message decoding problem of minimizing the total transmit power subject to service rate requirements. Section IV presents the algorithm for characterizing the achievable rate improvement using common message decoding. Section V provides simulation results. Concluding remarks are made in Section VI.

The notations used in this paper are as follows. Lower case letters are used to denote scalars. Lower bold case letters are used to denote vectors. Upper bold case letters are used to denote matrices. tr(.) denotes the trace operation of a matrix. The operator $(.)^H$ denotes the Hermitian of a matrix. $\mathbb{C}$ denotes the complex space.

II. Problem Formulation

A. System Model

Consider a multicell multiuser spatial multiplex system with N cells and K users per cell with N_t antennas at each base-station and a single antenna at each remote user. Transmit beamforming is employed at the base-station to separate users within each cell. This paper proposes a joint beamforming and common message decoding scheme to alleviate intercell interference. In particular, the jth user in ith cell may split its data stream into two parts: $x^p_{i,j}$, which is a complex scalar denoting the private information with $\mathbf{w}^p_{i,j} \in \mathbb{C}^{N_t \times 1}$ as the associated beamforming vector, and $x^c_{i,j}$, which denotes the common information signal with $\mathbf{w}^c_{i,j} \in \mathbb{C}^{N_t \times 1}$ as the associated beamforming vector. The user (i,j)'s common message $x^c_{i,j}$ is intended to be decoded by both the user (i,j)'s own receiver and by one single out-of-cell lth user in the mth cell with $m \neq i$. The user (i,j)'s receiver, on the other hand, is designed to decode a common message from one single out-of-cell user $(\hat{i},\hat{j})$ with $\hat{i} \neq i$. In general, (m,l) does not need to be the same as $(\hat{i},\hat{j})$.

The channel model can be written down as follows:

$$y_{i,j} = \sum_{l} \mathbf{h}^H_{i,i,j} \left(\mathbf{w}^p_{i,l} x^p_{i,l} + \mathbf{w}^c_{i,l} x^c_{i,l} \right) + \sum_{m \neq i, n} \mathbf{h}^H_{m,i,j} \left(\mathbf{w}^p_{m,n} x^p_{m,n} + \mathbf{w}^c_{m,n} x^c_{m,n} \right) + z_{i,j} \tag{1}$$

where $y_{i,j} \in \mathbb{C}$ is the received signal at the jth user in the ith cell, $\mathbf{h}_{l,i,j} \in \mathbb{C}^{N_t \times 1}$ is the vector channel from the base-station of the lth cell to the jth user in the ith cell, and $z_{i,j}$ is the additive white Gaussian noise with power σ^2. Fig. 1 illustrates the system model for a network with three cells and three users per cell sectors.

B. Han-Kobayashi Decoding

The main idea of the Han-Kobayashi strategy is to design decodable common messages which can be subtracted by some out-of-cell user in order to reduce its interference. Suppose that the jth user in ith cell, denoted as the (i,j)th user, intends to subtract the common message from the $(\hat{i},\hat{j})$th user (with $i \neq \hat{i}$), it needs to successfully decode $x^c_{i,j}$, $x^p_{i,j}$, and $x^c_{\hat{i},\hat{j}}$. The decoding condition then amounts to that of a three-user multiple-access channel with inputs $x^p_{i,j}$, $x^c_{i,j}$ and $x^c_{\hat{i},\hat{j}}$, and output $y_{i,j}$.

We can succinctly write down the relevant mutual information expressions for the multiple-access channel in terms of signal-to-noise-and-interference ratios (SINRs) by first defining the power of the received signal at $y_{i,j}$ as follows

$$T_{i,j} = \sum_{m,n} |\mathbf{h}_{m,i,j}^H \mathbf{w}_{m,n}^p|^2 + \sum_{m,n} |\mathbf{h}_{m,i,j}^H \mathbf{w}_{m,n}^c|^2 + \sigma^2. \tag{2}$$

Further, let $S_{i,j} = T_{i,j} - |\mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j}^p|^2 - |\mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j}^c|^2 - |\mathbf{h}_{\hat{i},i,j}^H \mathbf{w}_{\hat{i},\hat{j}}^c|^2$. Then, the rate constraints of the multple-access channel can be expressed as SINR constraints for the individual and joint decodings of the three messages as follows:

$$\frac{|\mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j}^p|^2}{S_{i,j}} \geq 2^{R_{i,j}^p} - 1, \tag{3}$$

$$\frac{|\mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j}^c|^2}{S_{i,j}} \geq 2^{R_{i,j}^c} - 1, \tag{4}$$

$$\frac{|\mathbf{h}_{\hat{i},i,j}^H \mathbf{w}_{\hat{i},\hat{j}}^c|^2}{S_{i,j}} \geq 2^{R_{\hat{i},\hat{j}}^c} - 1, \tag{5}$$

$$\frac{|\mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j}^c|^2 + |\mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j}^p|^2}{S_{i,j}} \geq 2^{R_{i,j}^p + R_{i,j}^c} - 1, \tag{6}$$

$$\frac{|\mathbf{h}_{\hat{i},i,j}^H \mathbf{w}_{\hat{i},\hat{j}}^c|^2 + |\mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j}^p|^2}{S_{i,j}} \geq 2^{R_{i,j}^p + R_{\hat{i},\hat{j}}^c} - 1, \tag{7}$$

$$\frac{|\mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j}^c|^2 + |\mathbf{h}_{\hat{i},i,j}^H \mathbf{w}_{\hat{i},\hat{j}}^c|^2}{S_{i,j}} \geq 2^{R_{i,j}^c + R_{\hat{i},\hat{j}}^c} - 1, \tag{8}$$

$$\frac{|\mathbf{h}_{\hat{i},i,j}^H \mathbf{w}_{\hat{i},\hat{j}}^c|^2 + |\mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j}^c|^2 + |\mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j}^p|^2}{S_{i,j}} \geq 2^{R_{i,j}^p + R_{i,j}^c + R_{\hat{i},\hat{j}}^c} - 1 \tag{9}$$

where $R_{i,j}^p$, $R_{i,j}^c$ and $R_{\hat{i},\hat{j}}^c$ are the private and common message rates for user (i,j), and the common message rate for user $(\hat{i},\hat{j})$, respectively. Note that in a multicell network in which multiple receivers may implement the decoding of the common information, one set of seven constraints must be applied to each receiver.

The above formulation of the capacity region of the multiple-access channel implicitly assumes that the receiver has the ability to jointly decode the respective common and private data streams. The ability for joint decoding (as opposed to successive decoding) is crucial as the intersection of multiple multiple-access channel regions does not always necessarily occur at the corner points. In fact, even for the single-input single-output Gaussian interference channel, joint decoding is necessary to achieve within one bit of the capacity region [5].

However, joint decoding is also more complex to implement. In this paper, we propose an alternative where only successive decoding points are considered. Further, we restrict ourselves to a fixed order of decoding the common message from one's own transmitter first, then the common message from the out-of-cell transmitter, and finally the private message from its own transmitter. Although successive decoding with a fixed decoding order is not necessarily optimal from an information theoretic perspective, the above decoding order is reasonable for the following reason. The underlying interference channel typically has weaker interfering links as compared to direct links, so the common information rate is typically constrained by the interfering link. Hence, it is sensible to decode the common information from one's own transmitter first to help the decoding of common information from the other transmitter. Further, private message should be decoded last to take advantage of the reduced interference due to common message decoding.

With this fixed decoding order, we can write down alternative expressions for SINRs for the common and private messages for each user. Assume that the $\hat{j}$th user in the $\hat{i}$th cell shares its common information with the jth user in the ith cell. Let Γ^p_{ij}, $\Gamma^c_{ij,ij}$ and $\Gamma^c_{\hat{i}\hat{j},ij}$ denote the SINRs for the private message of the user (i,j), the common message from user (i,j)'s own transmitter, and the common message from the out-of-cell user $(\hat{i},\hat{j})$, respectively. Then, the decoding condition becomes

$$\Gamma^c_{ij,ij} = \frac{|\mathbf{h}^H_{i,i,j}\mathbf{w}^c_{i,j}|^2}{T_{i,j} - |\mathbf{h}^H_{i,i,j}\mathbf{w}^c_{i,j}|^2} \geq 2^{R^c_{i,j}} - 1 \tag{10}$$

$$\Gamma^c_{\hat{i}\hat{j},ij} = \frac{|\mathbf{h}^H_{\hat{i},i,j}\mathbf{w}^c_{\hat{i},\hat{j}}|^2}{T_{i,j} - |\mathbf{h}^H_{i,i,j}\mathbf{w}^c_{i,j}|^2 - |\mathbf{h}^H_{\hat{i},i,j}\mathbf{w}^c_{\hat{i},\hat{j}}|^2} \geq 2^{R^c_{\hat{i},\hat{j}}} - 1 \tag{11}$$

$$\Gamma^p_{ij} = \frac{|\mathbf{h}^H_{i,i,j}\mathbf{w}^p_{i,j}|^2}{T_{i,j} - |\mathbf{h}^H_{i,i,j}\mathbf{w}^c_{i,j}|^2 - |\mathbf{h}^H_{\hat{i},i,j}\mathbf{w}^c_{\hat{i},\hat{j}}|^2 - |\mathbf{h}^H_{i,i,j}\mathbf{w}^p_{i,j}|^2} \geq 2^{R^p_{i,j}} - 1 \tag{12}$$

Clearly, the rate region achievable with this successive decoding order is a subset of the rate region achievable with joint decoding. However, as will be shown later in the simulation section, restricting to successive decoding with this fixed decoding order gives negligible performance loss for the overall system. Thus, the rest of this paper assumes this simpler formulation.

C. *Transmit Power Minimization Problem*

To solve a network optimization problem with common information decoding, for each particular jth user in the ith cell, we need to choose an appropriate $\hat{j}$th user in the $\hat{i}$th cell whose common information user (i,j) will decode. We also need to choose the common and private beamforming vectors and the common-private rate splitting for each user to optimize the overall objective. Thus, the overall network optimization is quite complex.

Before dealing with the optimization problem for the entire network, in this section, we first formulate the following problem of minimizing the total transmit power across all the base-stations subject to fixed user pairing for common information decoding, fixed target rates, and fixed private-common splitting rates. This problem is a key component of the overall network optimization problem. Let $R_{i,j}$ be the fixed target rate for the jth user in the ith cell, which is split into a fixed private part $R^p_{i,j}$ and a fixed common part $R^c_{i,j}$ with $R^p_{i,j} + R^c_{i,j} = R_{i,j}$. Let user (i,j) decode common information from user $(\hat{i},\hat{j})$ with $i \neq \hat{i}$. Assuming successive decoding, the power minimization problem can be formulated as

$$\begin{aligned} \text{minimize} \quad & \sum_{i,j} ||\mathbf{w}^p_{i,j}||^2 + ||\mathbf{w}^c_{i,j}||^2 \\ \text{subject to} \quad & \Gamma^c_{ij,ij} \geq 2^{R^c_{i,j}} - 1, \\ & \Gamma^c_{\hat{i}\hat{j},ij} \geq 2^{R^c_{\hat{i},\hat{j}}} - 1, \\ & \Gamma^p_{i,j} \geq 2^{R^p_{i,j}} - 1, \\ & R^p_{i,j} + R^c_{i,j} = R_{i,j}, \qquad \forall i,j \end{aligned} \tag{13}$$

where the minimization is over $\mathbf{w}^p_{i,j}$, $\mathbf{w}^c_{i,j}$. Note that as mentioned earlier, each user (i,j) only decodes common message from at most one other $(\hat{i},\hat{j})$. Likewise, each $(\hat{i},\hat{j})$'s common message is intended for only one other (i,j). Also, if user (i,j) does not split its message into a common and a private part, then $R^c_{ij} = 0$ and the constraint for $\Gamma^c_{ij,ij}$ becomes redundant. Likewise, if user (i,j) does not decode common information from another user, this is equivalent to setting $R^c_{\hat{i},\hat{j}} = 0$ and letting the constraint for $\Gamma^c_{\hat{i}\hat{j},ij}$ be redundant. In this formulation, the target rates in the optimization problem (13) are always assumed to be feasible.

The problem formulation above aims to minimize the total transmit power across the base-stations. It is straightforward to generalize the above formulation to account for the minimization of per-base-station (or per-antenna) power by including appropriate weights in the power expression, and to adapt these weights in an outer optimization loop. For instance, we can weight the transmit power at the ith base-station by a factor α_i, and solve the resulting weighted power minimization problem. An outer loop can then control the per-base-station power by adjusting the weights: a higher α_i leads to a lower power at base-station i, and vice versa (see [33]).

D. Achievable Rate Characterization Problem

The formulation of the network optimization problem as a power minimization subject to fixed target rates is useful for constant bit-rate applications. However, one issue with such a formulation is that it is often not easy to determine a priori whether a set of R_{ij}'s is feasible. In fact, even for the single-cell system, it can be shown that although the condition for feasibility is trivial (i.e. any SINR constraints are always feasible) when the channel is full rank with the same number of users as the number of antennas, only a necessary feasibility condition is known when the channel is rank deficient [34]. For a multicell multiuser multiantenna system with both common and private messages, solving the feasbility problem exactly is equivalent to a complete characterization of Han-Kobayashi region for the multiantenna interference channel, which is not yet available.

Further, practical wireless systems are often rate adaptive. Thus, the problem of rate maximization subject to power constraints is practically more relevant, especially for variable-rate applications. In addition, the pairing of users for common information decoding and how the achievable rate is split between private and common parts are also important parameters for optimization. Toward these ends, this paper ultimately aims to examine the improvement in the maximum achievable rate across the network using common message decoding, subject to a total power constraint across the base-stations. This network optimization problem is formulated as follows (again assuming successive decoding):

$$\begin{aligned} \max \min\nolimits_{(i,j)} \quad & R_{i,j} \\ \text{subject to} \quad & \Gamma^c_{ij,ij} \geq 2^{R^c_{i,j}} - 1, \\ & \Gamma^c_{\hat{i}\hat{j},ij} \geq 2^{R^c_{i,\hat{j}}} - 1, \\ & \Gamma^p_{i,j} \geq 2^{R^p_{i,j}} - 1, \\ & R^p_{i,j} + R^c_{i,j} = R_{i,j}, \qquad \forall i, j \\ & \sum_{i,j} ||\mathbf{w}^p_{i,j}||^2 + ||\mathbf{w}^c_{i,j}||^2 \leq P_{max} \end{aligned} \tag{14}$$

where the maximization part of the optimization is now over not only the beamforming vectors $\mathbf{w}^p_{i,j}$, $\mathbf{w}^c_{i,j}$, but also all possible private-common rate splittings, and all possible pairings of users $(\hat{i}, \hat{j})$ and (i, j) for common message decoding. Note that if different service levels are required for different users, one can easily take this into account by maximizing the minimum weighted rates, i.e., $\max\min_{i,j} \alpha_{i,j} R_{i,j}$, instead. The rest of this paper first provides solution to (13), then utilizes the algorithm for solving (13) to solve (14).

III. SDP Relaxation for Power Minimization Subject to Rate Constraints

The joint beamforming, common-private rate splitting, and user selection problem is a mixed discrete and continuous optimization problem. Finding the global optimal solution for such a problem would likely require a combinatorial search with exponential complexity. Thus, instead of looking for global optimal solutions, this section first focuses on the power minimization component of the overall problelm, i.e. (13), for which efficient practical algorithm exists.

In particular, this paper makes an observation that if one fixes the user selection and the common-private rate splitting, (i.e., the assignment of user $(\hat{i}, \hat{j})$ whose common message user (i, j) will decode, and the associated $R^p_{i,j}$ and $R^c_{i,j}$), the optimization of beamforming vectors $\mathbf{w}^p_{i,j}$ and $\mathbf{w}^c_{i,j}$ can be handled by an SDP relaxation method, and can therefore be efficiently solved.

More specifically, let $\mathbf{V}^p_{i,j} = \mathbf{w}^p_{i,j}(\mathbf{w}^p_{i,j})^H$ and $\mathbf{V}^c_{i,j} = \mathbf{w}^c_{i,j}(\mathbf{w}^c_{i,j})^H$. The objective function of (13) can be reformulated as $\sum_{i,j} \mathrm{tr}(\mathbf{V}^p_{i,j}) + \mathrm{tr}(\mathbf{V}^c_{i,j})$. Also, let $\mathbf{H}_{m,i,j} = \mathbf{h}_{m,i,j}(\mathbf{h}_{m,i,j})^H$, one can rewrite $T_{i,j}$ defined earlier in (2) as

$$T_{i,j} = \sum_{m,n} \mathrm{tr}(\mathbf{H}_{m,i,j}\mathbf{V}^p_{m,n}) + \sum_{m,n} \mathrm{tr}(\mathbf{H}_{m,i,j}\mathbf{V}^c_{m,n}) + \sigma^2 \tag{15}$$

Then, for fixed $R^p_{i,j}$ and $R^c_{i,j}$ and for fixed $(\hat{i}, \hat{j})$ for each (i, j), (13) can be written as an SDP

$$\begin{aligned}
\text{minimize} \quad & \sum_{i,j} \mathrm{tr}(\mathbf{V}^p_{i,j}) + \mathrm{tr}(\mathbf{V}^c_{i,j}) \qquad (16)\\
\text{subject to} \quad & \left(\frac{1}{2^{R^c_{i,j}} - 1} + 1\right) \mathrm{tr}(\mathbf{H}^H_{i,i,j}\mathbf{V}^c_{i,j}) - T_{i,j} \geq 0 \\
& \left(\frac{1}{2^{R^c_{\hat{i},\hat{j}}} - 1} + 1\right) \mathrm{tr}(\mathbf{H}^H_{\hat{i},i,j}\mathbf{V}^c_{\hat{i},\hat{j}}) + \mathrm{tr}(\mathbf{H}^H_{i,i,j}\mathbf{V}^c_{i,j}) - T_{i,j} \geq 0 \\
& \left(\frac{1}{2^{R^p_{i,j}} - 1} + 1\right) \mathrm{tr}(\mathbf{H}^H_{i,i,j}\mathbf{V}^p_{i,j}) + \mathrm{tr}(\mathbf{H}^H_{i,i,j}\mathbf{V}^c_{i,j}) + \mathrm{tr}(\mathbf{H}^H_{\hat{i},i,j}\mathbf{V}^c_{\hat{i},\hat{j}}) - T_{i,j} \geq 0 \\
& \mathbf{V}^p_{i,j} \succeq 0, \mathbf{V}^c_{i,j} \succeq 0, \qquad \forall i, j
\end{aligned}$$

where the minimization is over the Hermitian positive semidefinite matrices $\mathbf{V}^p_{i,j}$ and $\mathbf{V}^c_{i,j}$. The above reformulation is a relaxation of (13) because the original problem requires the matrices $\mathbf{V}^p_{i,j}$ and $\mathbf{V}^c_{i,j}$ to be rank one, while the relaxation does not necessarily produce a rank-one solution.

Nevertheless, because SDP is a convex optimization problem for which efficient numerical algorithms are available, the SDP relaxation approach offers an efficient way of finding good solutions to the original problem (13). Note that implementing this SDP solution requires a central processor with the knowledge of the channels between every base-station and every user in the network (i.e. $\mathbf{H}_{m,i,j}$, $\forall m, i, j$).

The use of SDP relaxation for solving downlink beamforming problem is originally due to [28], where it is proved that for the single-cell system the relaxation actually admits a rank-one optimal solution to the original problem. The same is true for the multicell problem [16] if no common-private information splitting is employed. With common-private rate splitting, the SDP in general does not always admit a rank-one optimal solution. In this case, randomization techniques can be used to produce a rank-one matrix, which is often a good solution to the original optimization problem [31], [35]; see also [36]. More specifically, this paper proposes the following randomization approach. After obtaining a set of optimal solutions $\{\mathbf{V}_{i,j}^{c*}, \mathbf{V}_{i,j}^{p*}\}$ of (16), if it is not rank one, we generate a set of rank-one solution of (13) as follows:

Algorithm 1 (Obtaining rank-one solution from SDP relaxation):

1) Generate random vectors $\{\mathbf{v}_{i,j}^c, \mathbf{v}_{i,j}^p\}$ according to complex Gaussian distributions $\mathbf{v}_{i,j}^c \sim \mathcal{N}(0, \mathbf{V}_{i,j}^{c*})$ and $\mathbf{v}_{i,j}^p \sim \mathcal{N}(0, \mathbf{V}_{i,j}^{p*})$.
2) Substitute $\mathbf{w}_{i,j}^c = \sqrt{p_{i,j}^c}\mathbf{v}_{i,j}^c$ and $\mathbf{w}_{i,j}^p = \sqrt{p_{i,j}^p}\mathbf{v}_{i,j}^p$ into (13), then solve (13) with $\mathbf{v}_{i,j}^c$ and $\mathbf{v}_{i,j}^p$ fixed. The problem (13), which is now over $p_{i,j}^p$ and $p_{i,j}^c$, has linear objective and linear constraints. So, it is a linear programming problem.
3) Repeat the above steps many times. Choose the best solution among them.

Experimentally, this randomization technique works very well.

It should be noted that the solutions to the relaxed problem actually have a sensible interpretation even when they are not rank one. The mutual information expression for the multiple-input single-output channel $y = h\mathbf{x} + z$ is

$$\log\left(1 + \frac{h^H S_x h}{\sigma^2}\right) \tag{17}$$

where S_x is the transmit covariance matrix of $\mathbf{x}$. When minimizing the total transmit power subject to a rate constraint, the optimal S_x is in fact rank one for which beamforming is optimal. But when the above mutual information is optimized under a different constraint (such as the multicast constraint [31] or similarly the decodability condition under Han-Kobayashi coding), the optimal S_x can have higher rank. This means that beamforming is no longer optimal.

Nevertheless, the resulting rate is still achievable if one allows multiple data streams per common or private message. In this case, we can write $S_x = \sum_{d=1}^{D} v_d v_d^H$, i.e., the transmit signal consists of D data streams, each with its own beamforming vector v_d.

Interestingly, there is a special case in which even with private-common splitting the SDP relaxation (16) does admit an optimal rank-one solution. This happens when there are at most two pairs of information splittings in the network. The proof of this fact can be obtained from the result of [32], where it is shown that a high-rank solution of an SDP can be reduced to a lower rank-one solution if the size of the optimization problem satisfies certain condition. This condition is satisfied when the number of common information splittings in (16) is no more than two as stated in the following theorem.

Theorem 1: In a multicell network with multiple users per cell, if at most two pairs of users participate in private-common information splitting and common information decoding with a fixed decoding order, then for the fixed user pairing and fixed feasible private and common message rates $R^c_{i,j}, R^p_{i,j} > 0$, the SDP relaxation of (16) admits an optimal rank-one solution.

Proof: Consider a complex-valued SDP problem with N_v variables and N_c SDP constraints. Suppose that the SDP problem is primal and dual feasible, and that any optimal solution does not have a zero matrix component. Then, a result due to [32] states that if $N_c \leq N_v + 2$, then the SDP problem has an optimal solution where each of the optimal matrix variables is rank one.

For a conventional multicell spatial multiplex system with multiple users per cell, the transmit beamforming optimization problem with private information only involves equal number of variables as number of constraints. However, whenever a transmitter implements private common information splitting, it introduces one extra variable, i.e., the common message, but two extra constraints, i.e. the common message must be decodable at both the intended receiver and by some out-of-cell user. Thus, the condition of [32] for the existence of an optimal rank-one solution of (13) is satisfied whenever there are at most two pairs of transmitters engaging in common information decoding. Note that the condition that any optimal solution does not have a zero matrix component is always satisfied as long as $R^c_{i,j}, R^p_{i,j} > 0$. ■

The technique of [32] gives not only an existence proof for a rank-one optimal solution, but also a practical algorithm for constructing a rank-one solution from an initial high-rank solution. Further, we observe that the crucial part of the above argument is the number of active constraints in the system. As each common information data stream needs to satisfy two constraints, it is quite possible for one of the two constraints to be inactive at the optimal solution. As inactive constraints can be removed without affecting the optimal solution, this implies that condition of [32] is satisifed as long as there are at most two common data streams with two simultaneous active constraints. We state this result in the following theorem.

Theorem 2: In a multicell network with multiple users per cell implementing common-private information splitting with fixed user pairing, fixed feasible private and common message rates $R^c_{i,j}, R^p_{i,j} > 0$, and fixed decoding order, if the number of common information streams in the entire network for which the decodability constraints by the intended and the out-of-cell users are simultaneously active is at most two, then the SDP relaxation of (16) admits an optimal rank-one solution.

In practical implementation, we found that in a majority of cases, only one of two constraints for each common message is tight at the optimal point. Typically, the common information rate is constrained by the channel to the out-of-cell decoder only. The decodability constraint for the common information within the cell is typically not tight. Thus, in majority of cases, a rank-one solution exists for the optimization problem (16). Note that Theorems 1 and 2 can be thought of as a generalization of corresponding results in [28], [16], where the optimality of beamforming with private information only is established for the single-cell and multicell networks, respectively.

IV. Joint Beamforming, Rate Splitting and User Selection

We now move beyond the power minimization problem and consider the the maximization of the minimum achievable rate problem stated in (14). A brute force approach to solving the joint optimization problem would involve searching over all possible user decoding-pair combinations, and all possible rate targets and common-private rate splittings. For each combination, the SDP relaxation approach in the previous section can be used to find the beamforming vectors and the resulting total power. The optimal decoding-pair and rate splitting is the combination that gives the minimal overall power. This exhaustive search strategy is clearly infeasible for any reasonably sized network, as the search space is exponentially large.

This paper proposes a heuristic approach in which the user pairings are selected according to the interference-to-noise ratios (INR). The intuition is based on the approximate characterization of the capacity region of the two-user interference channel due to [5], which showed that to achieve within one bit of the capacity region, any interference above the noise level should be regarded as common information. Thus, the INR of user $(\hat{i}, \hat{j})$ at user (i, j) gives an indication as to whether common-message splitting at user $(\hat{i}, \hat{j})$ is worthwhile for user (i, j).

Let $(\hat{i}, \hat{j}) \rightarrow (i, j)$ denote that user (i, j) decodes common message from user $(\hat{i}, \hat{j})$. The idea is to search through all possible user pairings, and select the one with the highest INR as the best candidate for common-private message splitting, (while satisfying the condition that each user splits rate for only one other user and decodes common message from only one other user). Define the INR of user $(\hat{i}, \hat{j})$ at user (i, j) as $\text{INR}_{(i,j)\rightarrow(\hat{i},\hat{j})}$ defined as:

$$\text{INR}_{(i,j)\rightarrow(\hat{i},\hat{j})} = \frac{|\mathbf{h}_{i,\hat{i},\hat{j}}^H \mathbf{w}_{i,j}^p|^2}{S_{\hat{i},\hat{j}} - |\mathbf{h}_{i,\hat{i},\hat{j}}^H \mathbf{w}_{i,j}^p|^2} \tag{18}$$

In a N-cell network with K users per cell, there are $(N-1)K$ such INR entries for each receiver, and the network has $(N-1)NK^2$ such INRs in total. Note that in practice, the INRs can be estimated using pilot signals.

To calculate the initial set of INRs, this paper proposes to start with the maximum achievable rate $\hat{R}^{(0)}$ corresponding to private messages transmission only. We then use the obtained INRs for pairing users for common message decoding by adding one pair at the time. Each additional pair of users for common message decoding allows us to increase the minimum achievable rate for all users.

Finally, we also need to determine the optimal splitting of common and private rates in the entire process. For the newly added user pair, we propose to linearly search for the optimal common-private rate splitting. To increase the target rate of users that already have common-private rate splitting, we increase their common rates. For users that have private rates only, we increase their private rates. This approach is heuristic, as the common-private rate splittings of the user pairs are in theory interdependent. Nevertheless, this approach is found to work fairly well. The proposed algorithm is summarized below:

Algorithm 2: (Joint beamforming and common-message decoding for maximizing the minimum achievable rate)

1) Find the maximum achievable rate $\hat{R}^{(0)}$ with private information only. This is obtained by linearly increasing the target rate, then solving an SDP-relaxation problem in each step, eventually stopping at the last feasible point.

2) Form a sorted list with M entries of $\text{INR}_{(i,j)\rightarrow(\hat{i},\hat{j})}$ with $i \neq \hat{i}$, i.e. the INR due to the interference from user (i, j) seen at user $(\hat{i}, \hat{j})$ by doing the following. First, sort all $(N-1)NK^2$ INRs from the largest entry to the smallest entry. Then, whenever a user appears either on the right-hand side of the "$\rightarrow$" or the left-hand side of the "$\rightarrow$" in the subscripts of INR's, remove subsequent entries where the user appears on the same side to ensure that each user splits only one common message and decodes only one common message stream.
3) Initialize $L = 1$.
4) Consider the L^{th} pair $(\hat{i}, \hat{j}) \rightarrow (i, j)$ on the INR list. Split the rate of user $(\hat{i}, \hat{j})$ as follows:
   a) Initialize $R_{\hat{i},\hat{j}} = \hat{R}^{(L-1)}$.
   b) Gradually increase $R_{\hat{i},\hat{j}}$ by increasing the common rates of the first $(L-1)$ users that are already involved in rate splitting (while fixing their private rates), and by setting the remaining $(NK - L)$ users' private rates to be equal to $R_{\hat{i},\hat{j}}$.
   c) For the fixed value of $R_{\hat{i},\hat{j}}$, find the optimal rate splitting for the user $(\hat{i}, \hat{j})$ through a linear search by calling the SDP relaxation routine for different values of $0 \leq R^p_{\hat{i},\hat{j}} \leq R_{\hat{i},\hat{j}}$. If the SDP relaxation problem satisifes the conditions of either Theorem 1 or Theorem 2, use the rank reduction technique of [32] to find an optimal rank-one solution; otherwise, use Algorithm 1 to find a rank-one solution. Call the optimal private rate $\hat{R}^p_{\hat{i},\hat{j}}$.
   d) Go to step (b) and stop at the largest feasible value of $R_{\hat{i},\hat{j}}$ subject to the power constraint. Call it $\hat{R}^{(L)}$.
5) Increment L. Go to step 4) and repeat for up to M pairs.

Step 4(b) can be further improved by doing an additional optimization on the splitting of the private and common rates of the $(L-1)$ users that are already involved in common information decoding. Such an optimization, however, requires additional exhaustive searches with considerable complexity. Step 4(b) above uses a simple approach of increasing the common rates of the first $(L-1)$ users, while increasing the private rates of the remaining $(NK - L)$ users each time the target rate $R_{\hat{i},\hat{j}}$ is updated. The rationale behind this heuristics is the fact that each of the first $(L-1)$ users has already qualified as a good candidate for rate splitting. As the target rate increases, these users are expected to allocate a larger proportion of their data rates for the common part. Finally, the above algorithm may also be improved by updating the INR list as soon as a rate splitting is determined, at additional complexity. Although the proposed algorithm does not guarantee global optimality, it nevertheless provides significant gain as the simulation section of this paper shows.

The complexity of the algorithm depends on the total number of decoding pairs M, the step size in the rate search, the number of antennas N_t, the number of cells N, and the number of users per cell K. Consider the Lth decoding pair, and let N_L be the number of target rate points searched. For each fixed target rate point R_L^i where $i \in \{1, \cdots, N_L\}$, the optimal rate splitting is found using one-dimensional linear search with step size ΔR_L^i, for a total of $\frac{R_L^i}{\Delta R_L^i}$ searches. Each of these searches requires an SDP call. The SDP problem for the Lth decoding pair has $(NK + L)$ matrix variables, where every matrix is of dimension $N_t \times N_t$. The complexity of the SDP solution, assuming an interior-point implementation, is in the order of the cube of the number of variables, so it is $O((NK + L)^3 N_t^6)$ per path-following interior-point iteration, multiplied by the number of iterations (which is not a strong function of the problem size) [36], [37]. So, the overall complexity is

$$\sum_{L=1}^{M} \sum_{i=1}^{N_L} \frac{R_L^i}{\Delta R_L^i} O\big((NK + L)^3 N_t^6\big) \tag{19}$$

which is polynomial in the problem size.

V. SIMULATIONS

This section illustrates the benefit of joint beamforming and common information decoding on a multicell networks by simulation. Consider first a 2-cell network with 4 users per cell as shown in Fig. 2 where common message decoding is performed only for the two users situated directly between the two base-stations. The base-stations are equipped with 4 antennas each. Realistic channel models are used in the simulation: the noise power spectral density is set to -162 dBm/Hz; the channel vectors are chosen according to a distance-dependent path loss $L = 128.1 + 37.6 \log_{10}(d)$, where d is the distance in kilometers, with log-normal shadowing with 8dB standard deviation, and a Rayleigh fading component. The distance between neighboring base-stations is 1.4km; an antenna gain of 15dBi is assumed.

Fig. 3 shows the minimum total transmit power as a function of different common-private rate splittings when every user in the 2-cell network is assigned a target rate of 2 bits/sec/Hz. The two users with common-message decoding are situated at distances $d_1 = d_2 = d$ away from their respective base-stations. (The other users are located randomly within each cell.) The minimum total transmit power is plotted as a function of d for various common-private rate splittings. For example, the line marked with $(R^p_{1,1}, R^p_{2,1}) = (2,2)$ represents the case of private message only—this is actually optimal when d is less than about 0.4 km. As the users move closer to the cell edge, assigning $(R^p_{1,1}, R^p_{2,1}) = (0,2)$ becomes optimal. The minimum transmit power over all possible common-private rate splittings is the lower envelope of all these curves. Fig. 3 shows that the benefit of common message decoding in term of total transmit power reduction at the base-stations is substantial. It can be up to 12dB when the users are at the cell edge, where out-of-cell interference is the largest. Fig. 3 also shows that determining the appropriate rate splitting is crucial, and that the optimal rate splitting is channel dependent. In this two-cell two-user case, it is observed that the optimal splitting always occurs at the rate boundary (i.e. $(R^p_{i,j}$ is either 0 or 2). But, this is not necessarily the case when more than two users are involved in common message decoding.

Fig. 4 and Fig. 5 show the benefit of common message decoding on the improvement of the achievable rate for various values of the SINR gap under different topologies. The two users with common message decoding are within distances d_1 and d_2 from their respective base-stations as shown in Fig. 2. It is clear from the figures that the gain for users at the cell edges is substantial when the achievable rate with common message decoding is compared to the feasible rate with private information transmission only. The gain decreases as the users get closer to the cell center where the interference is typically limited. Fig. 6 shows a similar plot with inter-base-station distance of 2.8km. It is seen that the rate improvement in this case only occurs at much higher transmit power values.

To account for practical coding and modulation, the above figures also include the effect of SINR gap on the achievable rates with common-message decoding, i.e., the achievable rate is related to SINR as

$$\mathrm{Rate} = \log\left(1 + \frac{\mathrm{SINR}}{\Gamma}\right) \tag{20}$$

where Γ depends on the coding and modulation schemes. For fixed locations of the users, it is observed that in general, the achievable rate gain decreases somewhat when the SINR gap increases.

Table I tabulates the improvement in the minimum achievable rate using common message decoding with a total transmit power constraint of 46dBm for the 2-cell 4-user per cell network with base-station distance of 1.4km, and with various values of SINR gaps. It is interesting to note that although the absolute feasible rate gain is smaller with a larger gap, the percentage gain, which ranges from 40% to 50%, is actually larger.

The simulation results above are all obtained under a fixed decoding order of decoding the common message from one's own transmitter first, then the common message from the out-of-cell user next, and finally the private message from one's own transmitter last. Further, randomization technique is used to produce a rank-one solution. Fig. 7 shows that restricting the decoding order and restricting to rank-one solution are without loss of optimality. Fig. 7 plots the total transmit power vs. target rates for various topologies with either the full Han-Kobayashi region with joint decoding, i.e. the constraints (3)-(9) with the full-rank solution, or with the fixed decoding order, i.e. the constraints (10)-(12) with a rank-one solution. There is virtually no difference in the minimum transmit power for fixed target rates in each case.

Finally, we simulate a 3-cell network with 3 users per cell sector shown in Fig. 1 to illustrate the improvement in the minimum achievable rate with common message decoding. Again, realistic channel models are used. In addition, the antenna element responses here also include a directional component due to sectorization. As the proposed joint beamforming and common message decoding is expected to bring the largest benefit to users experiencing the most interference, the simulation specifically places mobile users at the cell edge to illustrate this effect.

To illustrate the behavior of Algorithm 2, which maximizes the minimum achievable rate improvement over users pairing, common-message decoding and beamforming, we plot the minimal total transmit power as a function of target achievable rate in Fig. 9 for each additional common-message decoding pair. The pairing of users for common-message decoding is shown in Fig. 8. As the plot illustrates, with common-message splitting of multiple user pairs, the feasible rate improves from 2 bits/sec/Hz to 6 bits/sec/Hz, which is quite substantial.

VI. CONCLUDING REMARKS

Information theoretical studies have long suggested that common-private message splitting at the transmitters and common-message decoding at the receivers have the potential to significantly improve the achievable rate region of the interference channel. This paper is an effort toward making the information theoretical insight practical for a realistic multiuser multiantenna multicell network. By taking advantage of a semidefinite relaxation approach for transmit beamforming design, and by incorporating the additional components such as optimal rate splitting and user selection across the multiple cells, this paper shows that common-message decoding can indeed bring substantial benefit to cell-edge users in a practical multicell network in terms of improvement in maximum minimal achievable rate.

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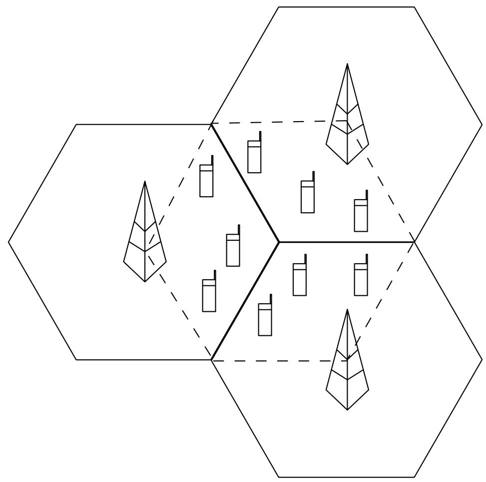

Fig. 1. A wireless network with three base-stations and three users per cell sectors.

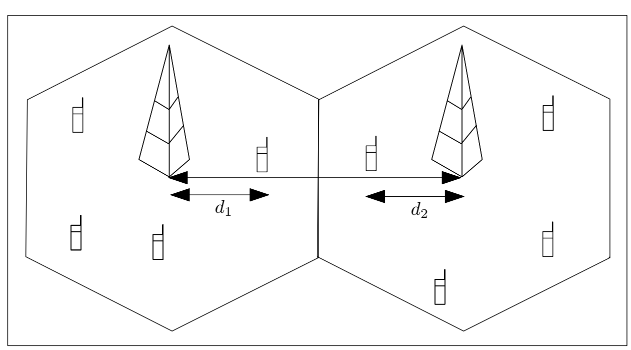

Fig. 2. A two-cell four-user per cell configuration with two users located between two base-stations at distance d.

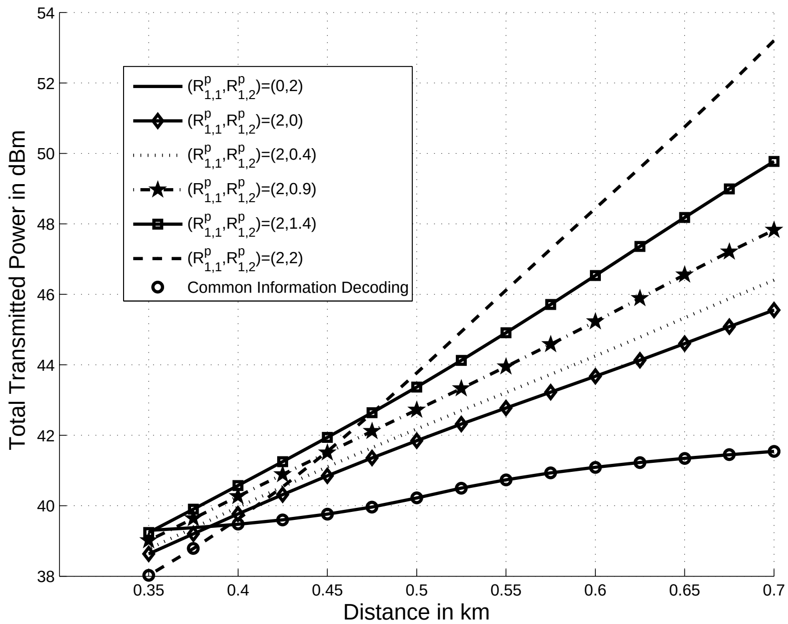

Fig. 3. Total transmitted power versus distance in km for different rate splits for two-cell network with four users per cell.

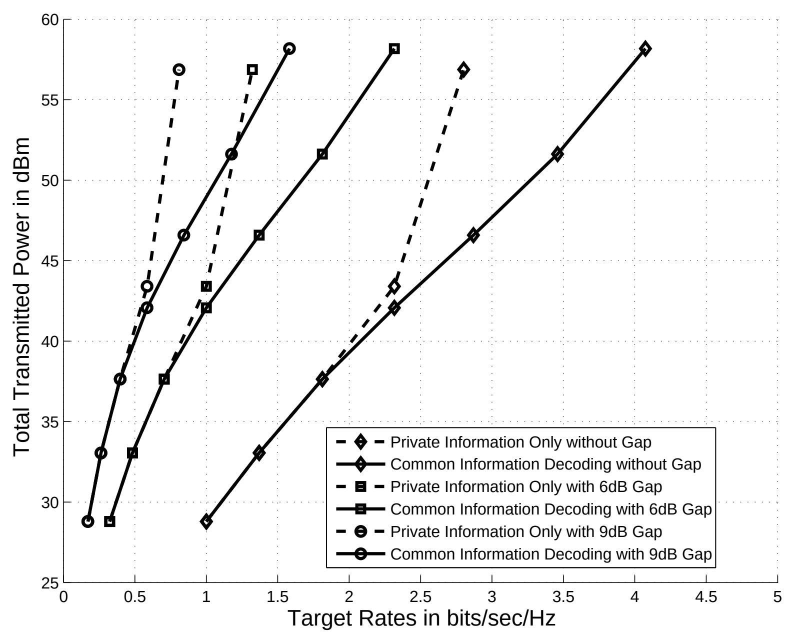

Fig. 4. Total transmitted power versus the rate targets for both the case of private-message only and the case of common-message decoding in a two-cell network with four users per cell for various gap values and $d_1 = d_2 = 0.4km$ with inter-base-station distance of $1.4km$.

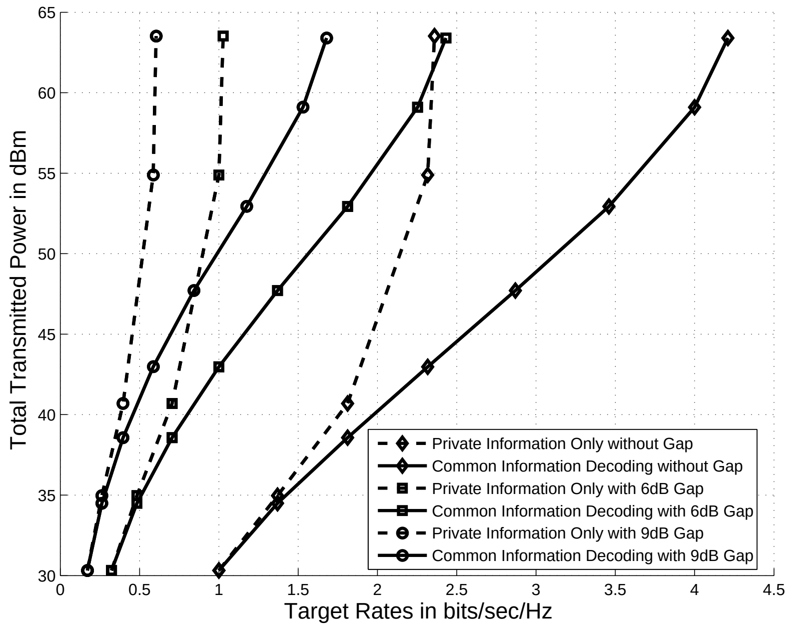

Fig. 5. Total transmitted power versus the rate targets for both the case of private-message only and the case of common-message decoding in a two-cell network with four users per cell for various gap values and $d_1 = d_2 = 0.5km$ with inter-base-station distance of $1.4km$.

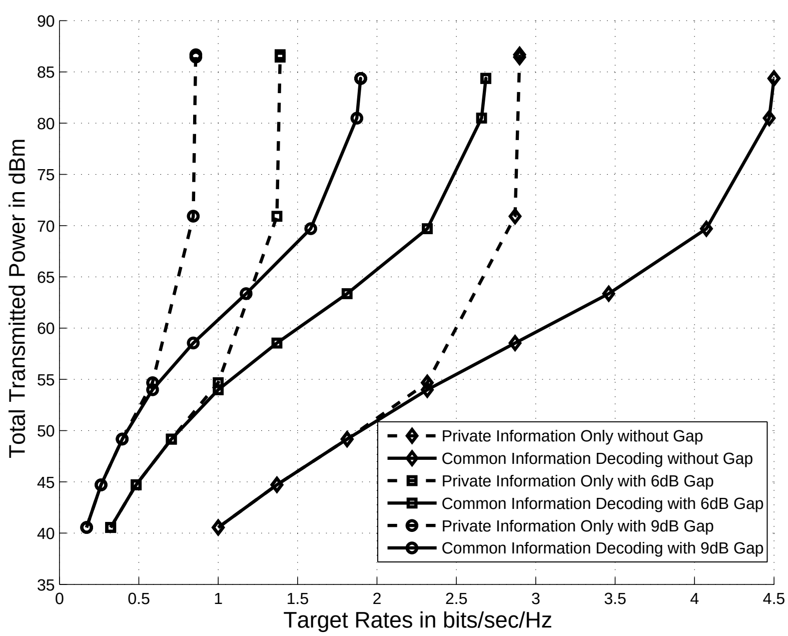

Fig. 6. Total transmitted power versus the rate targets for both the case of private-message only and the case of common-message decoding in a two-cell network with four users per cell for various gap values and $d_1 = d_2 = 1km$ with inter-base-station distance of $2.8km$.

TABLE I

FEASIBILITY GAIN RESULTS FOR A SUM-POWER CONSTRAINTS OF 46DBM AND INTER-BASE-STATION DISTANCE OF $1.4km$

d_1 in km	d_2 in km	R_{max}, private information only in bps/Hz	R_{max}, common information dec. in bps/Hz	Percentage gain
Gap=0dB				
0.4	0.4	2.48	2.80	13%
0.5	0.5	2.10	2.66	26%
Gap=6dB				
0.4	0.4	1.11	1.32	19%
0.5	0.5	0.87	1.22	40%
Gap=9dB				
0.4	0.4	0.66	0.81	23%
0.5	0.5	0.50	0.74	47%

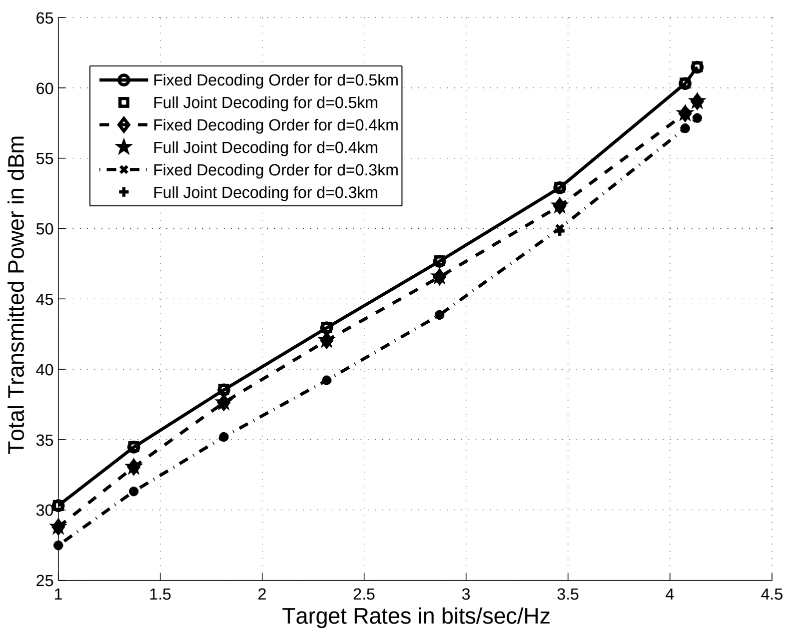

Fig. 7. Fixed decoding order vs. full Han-Kobayashi region for a two-cell network with four users per cell.

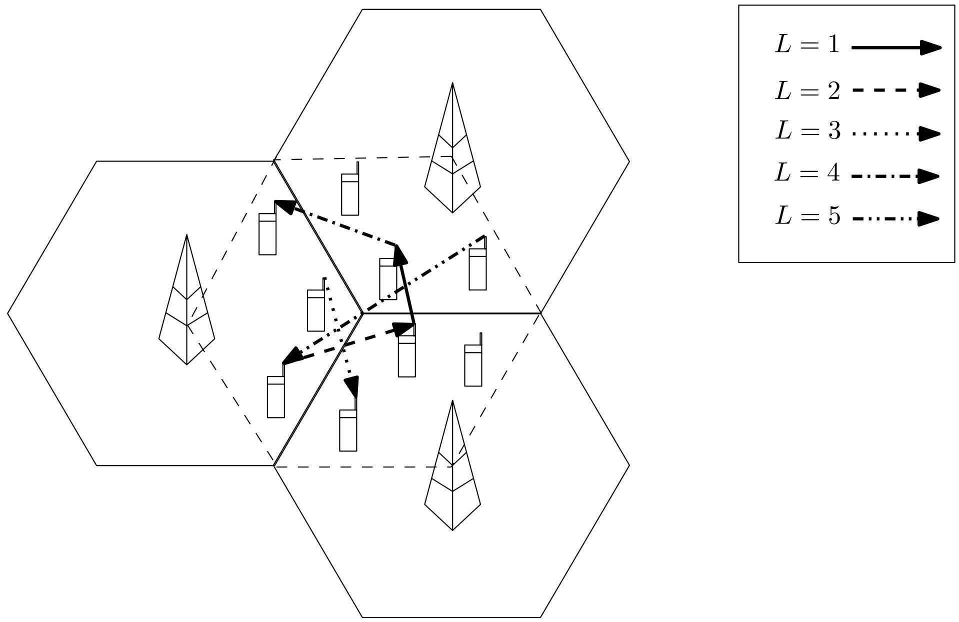

Fig. 8. Pairing of users in the three-cell, three-user-per-cell network.

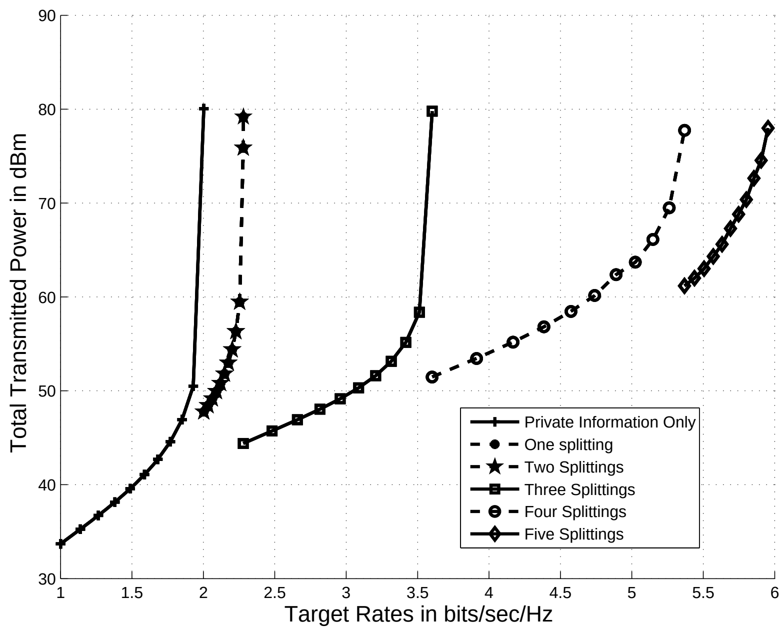

Fig. 9. Total transmitted power versus the target rates for both the case of private-message only and the case of common-message decoding in a three-cell network with three users per cell.