Gaussian Z-Interference Channel with a Relay Link: Type II Channel and Sum Capacity Bound

Wei Yu and Lei Zhou
Department of Electrical and Computer Engineering
University of Toronto
Toronto, Ontario, Canada M5S 3G4
Email: weiyu@comm.utoronto.ca

***Abstract*— This paper studies the Gaussian Z-interference channel with a rate-limited digital relay link from one receiver to the other receiver. In a companion paper, we dealt with the Type I channel, where the relay link goes from the interference-free receiver to the interfered receiver. It was shown that in the weak interference regime, each relay bit can improve the sum capacity by up to one bit asymptotically in the high signal-to-noise-ratio and interference-to-noise-ratio limit. In this paper, we study the Type II channel where the relay link goes from the interfered receiver to the interference-free receiver. The capacity region for such a channel is established in the strong interference regime; achievable rate regions are established in the moderately strong and weak interference regimes. In the weak interference regime, we show that in contrast to the Type I channel, the sum capacity improvement due to relaying for the Type II channel is upper bounded by at most half a bit, even as the relay link rate goes to infinity.**

I. Introduction

This paper explores the use of relay techniques for interference mitigation in an interference channel, where the classical two-user interference channel is augmented by a noiseless relay link between the two receivers. We are motivated to study such a relay interference channel, because in many practical communication situations (e.g. a wireless cellular system with remote users at the cell edge) the receivers are often close to each other geographically and are capable of establishing an independent communication link for the purpose of interference mitigation. This paper focuses on the simplest Gaussian Z-interference channel (also known as the one-sided interference channel), in which one of the receivers gets an interference-free signal, the other receiver gets a combination of the intended and the interfering signals, and the channel is equipped with a noiseless link of fixed capacity from one receiver to the other. Our goal is to characterize the sum capacity improvements due to the use of relay strategies.

Depending on the direction of the noiseless link, the proposed model is named the Type I or the Type II Gaussian Z-relay-interference channel, as shown in Fig. 1. In a companion paper [1], we dealt with the Type I channel, where the digital relay link of finite capacity goes from the interference-free receiver to the interfered receiver. It was shown that a binning and partial interference forwarding strategy is asymptotically sum-capacity achieving in the weak interference regime. In fact, every bit in relay link rate increases the sum rate by exactly one bit in the high signal-to-noise-ratio (SNR) and high interference-to-noise-ratio (INR) limit. The digital relay essentially achieves the cut-set bound for sum capacity.

This paper deals with the Type II channel where the situation is quite different. In a Type II Gaussian Z-relay-interference channel, the digital relay link goes from the interfered receiver to the interference-free receiver. Our main coding strategy for the Type II channel is based on a combination of the Han-Kobayashi common-private information splitting scheme and two relaying strategies: decode-and-forward and quantize-and-forward. In the proposed scheme, the interfered receiver, which decodes the common message and observes a noisy version of the neighbor's private message, describes the common message with a bin index and describes the neighbor's private message using a quantization scheme. It is shown that, in the strong interference regime, a special form of the proposed relaying scheme, which deploys the decode-and-forward strategy only, is capacity achieving. On the other hand, in the weak interference regime, the largest rate region is achieved with pure quantize-and-forward. Further, in the weak interference regime, the sum capacity gain due to the digital link for the Type II channel is upper bounded by half a bit. This is in direct contrast to the Type I channel, in which each relay bit can be worth up to one bit in sum capacity.

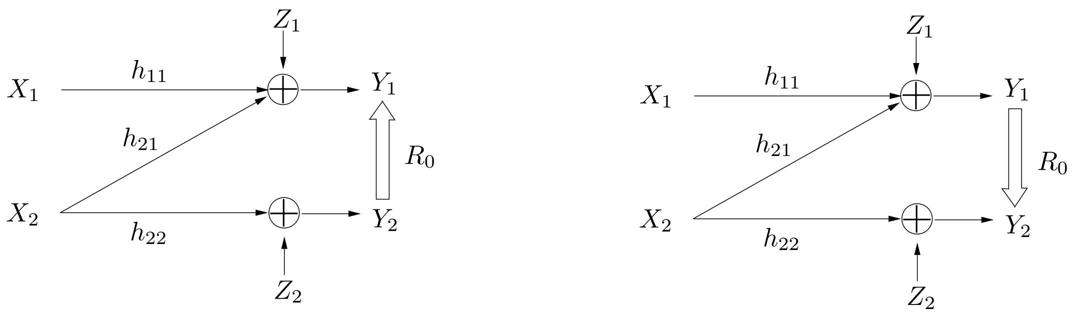
This paper is related to the recent work of Sahin and Erkip [2], [3], Marić et al. [4], Dabora et al. [5], where the achievable rate regions and relay strategies are studied for a related but different channel model of an interference channel with an additional relay node. This paper is also related to the work of Simeone et al. [6], where a one-dimensional cellular model is considered in the presence of uni- or bi-directional relay links. A common theme in all of these studies is the importance of forwarding interference (via either binning or quantization) for the purpose of interference subtraction.

II. Gaussian Z-Interference Channel with a Relay Link

A. Channel Model

Consider the Type II Gaussian Z-relay-interference channel, where the relay link goes from the interfered receiver to the interference-free receiver as shown in Fig. 1(b):

$$\begin{cases} Y_1 = h_{11}X_1 + h_{21}X_2 + Z_1 \\ Y_2 = h_{22}X_2 + Z_2 \end{cases} \tag{1}$$



(a) Type I (b) Type II

Fig. 1. Gaussian Z-interference channel with a relay link: (a) Type I; (b) Type II.

where X_1 and X_2 are the transmit signals with power constraints P_1 and P_2 respectively, h_{ij} represents the channel gain from transmitter i to receiver j, and Z_1, Z_2 are independent additive white Gaussian noises (AWGN) with power N. In addition, the Type II Gaussian Z-relay-interference channel is equipped with a digital noiseless link of fixed capacity R_0 from the receiver 1 to the receiver 2.

In the Type II channel, the relay (Y_1) observes a noisy version of the intended signal at the relay destination (Y_2). Thus, Y_1 is capable of using decode-and-forward and quantize-and-forward [7] relay strategies to help the decoding at Y_2. Intuitively, when the interference link is weak, the digital link may not be very efficient, because receiver 1's knowledge of X_2 is inferior to that of the receiver 2. On the other hand, when the interference link is very strong, receiver 1 becomes a better receiver for X_2 than receiver 2, in which case the digital link is capable of increasing user 2's rate by as much as R_0. This paper makes these intuitions precise by deriving an achievability theorem for the weak and moderately strong interference regimes, and a capacity theorem for the strong and very strong interference regimes.

To simplify the notation, the following definitions are used throughout this paper:

$$\mathsf{SNR}_1 = \frac{|h_{11}|^2 P_1}{N} \qquad \mathsf{SNR}_2 = \frac{|h_{22}|^2 P_2}{N}$$
$$\mathsf{INR}_2 = \frac{|h_{21}|^2 P_2}{N} \qquad \gamma(x) = \frac{1}{2}\log(1+x)$$

where $\log(\cdot)$ is base 2. In addition, denote $\overline{\beta} = 1-\beta$.

B. Achievable Rate Region

Theorem 1: For the Type II Gaussian Z-interference channel with a digital relay link of limited rate R_0 from the interfered receiver to the interference-free receiver as shown in Fig. 1(b), in the weak interference regime defined by $\mathsf{INR}_2 \le \mathsf{SNR}_2$, the following rate region is achievable

$$\bigcup_{0\le\beta\le 1} \left\{ (R_1,R_2) \middle| R_1 \le \gamma\left(\frac{\mathsf{SNR}_1}{1+\beta\mathsf{INR}_2}\right), \right.$$
$$\left. R_2 \le \gamma(\beta\mathsf{SNR}_2) + \gamma\left(\frac{\overline{\beta}\mathsf{INR}_2}{1+\mathsf{SNR}_1+\beta\mathsf{INR}_2}\right) + \delta(\beta,R_0) \right\}, \tag{2}$$

where

$$\delta(\beta,R_0) = \gamma\left(\frac{\beta(2^{2R_0}-1)\mathsf{INR}_2}{2^{2R_0}(1+\beta\mathsf{SNR}_2)+\beta\mathsf{INR}_2}\right). \tag{3}$$

In the moderately strong interference regime, defined by

$$\mathsf{SNR}_2 \le \mathsf{INR}_2 \le 2^{2R_0}(1+\mathsf{SNR}_2) - 1 \triangleq \mathsf{INR}_2^{\dagger}, \tag{4}$$

the following rate region is achievable:

$$\mathrm{co}\left\{ \bigcup_{\alpha\in\mathbb{R},0\le\beta\le1,\ R_a+R_b\le R_0} \mathcal{R}_{\alpha,\beta}(R_a,R_b) \right\} \tag{5}$$

where "co" denotes convex hull and $\mathcal{R}_{\alpha,\beta}(R_a,R_b)$ is a pentagon region given by

$$\left\{ (R_1,R_2) \middle| \begin{array}{l} R_1 \le \gamma(\frac{\mathsf{SNR}_1}{1+\beta\mathsf{INR}_2}) \\ R_2 \le \min\{\gamma(\mathsf{SNR}_2) + R_b + \eta(\alpha,\beta,R_a), \\ \qquad \gamma(\beta\mathsf{SNR}_2) + \gamma\left(\frac{\overline{\beta}\mathsf{INR}_2}{1+\beta\mathsf{INR}_2}\right) \\ \qquad +\zeta(\alpha,\beta,R_a)\} \\ R_1+R_2 \le \gamma(\beta\mathsf{SNR}_2) + \gamma\left(\frac{\mathsf{SNR}_1+\overline{\beta}\mathsf{INR}_2}{1+\beta\mathsf{INR}_2}\right) \\ \qquad +\zeta(\alpha,\beta,R_a) \end{array} \right\}. \tag{6}$$

where

$$\zeta(\alpha,\beta,R_a) = \gamma\left(\frac{\beta\mathsf{INR}_2}{(1+\beta\mathsf{SNR}_2)(1+\frac{\sigma^2}{N})}\right), \tag{7}$$

and

$$\eta(\alpha,\beta,R_a) = \gamma\left(\frac{(1+2\alpha\overline{\beta}+\alpha^2\overline{\beta})\mathsf{INR}_2 + \beta\overline{\beta}\alpha^2\mathsf{INR}_2\mathsf{SNR}_2}{(1+\mathsf{SNR}_2)(1+\frac{\sigma^2}{N})}\right) \tag{8}$$

with

$$\frac{\sigma^2}{N} = \frac{1+\mathsf{SNR}_2+(1+2\alpha\overline{\beta}+\alpha^2\overline{\beta})\mathsf{INR}_2+\beta\overline{\beta}\alpha^2\mathsf{INR}_2\mathsf{SNR}_2}{(2^{2R_a}-1)(1+\mathsf{SNR}_2)}. \tag{9}$$

In the strong interference regime defined by

$$\mathsf{INR}_2^{\dagger} \le \mathsf{INR}_2 \le (1+\mathsf{SNR}_1)\mathsf{INR}_2^{\dagger} \triangleq \mathsf{INR}_2^{\ddagger}, \tag{10}$$

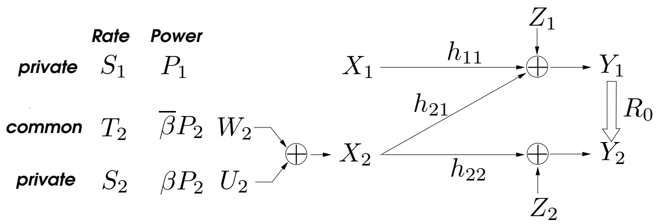


Fig. 2. Common-private information splitting for Type II channel.

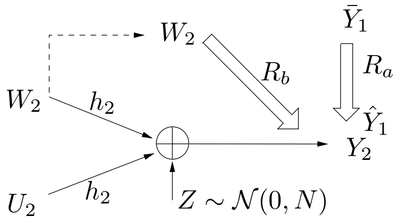


Fig. 3. Gaussian multiple-access channel with two digital relay links.

the capacity region is given by

$$\left\{ (R_1, R_2) \middle| \begin{array}{rcl} R_1 & \leq & \gamma(\mathsf{SNR}_1) \\ R_2 & \leq & \gamma(\mathsf{SNR}_2) + R_0 \\ R_1 + R_2 & \leq & \gamma(\mathsf{SNR}_1 + \mathsf{INR}_2) \end{array} \right\}. \tag{11}$$

In the very strong interference regime defined by

$$\mathsf{INR}_2 \geq \mathsf{INR}_2^{\ddagger}, \tag{12}$$

the capacity region is given by

$$\left\{ (R_1, R_2) \middle| \begin{array}{l} R_1 \leq \gamma(\mathsf{SNR}_1) \\ R_2 \leq \gamma(\mathsf{SNR}_2) + R_0 \end{array} \right\}. \tag{13}$$

Proof: We first prove the achievability of the rate region given in (5). It will be shown that this is an achievable rate region not only for the moderately strong interference regime, but also for any INR and SNR. We then show that (5) reduces to (2) in the weak interference regime, and reduces to (11) and (13) in the strong and very strong interference regimes, respectively.

The achievability of (5) is based on the Han-Kobayashi common-private information splitting scheme [8], as shown in Fig. 2.

A two-step decoding procedure is used. Consider first the decoding of (X_1, W_2) at Y_1. The achievable set of (S_1, T_2) is the capacity region of a multiple-access channel, denoted by $\mathcal{C}_1$, where

$$\left\{ \begin{array}{rcl} S_1 & \leq & \gamma\left(\dfrac{\mathsf{SNR}_1}{1+\beta\mathsf{INR}_2}\right) \\ T_2 & \leq & \gamma\left(\dfrac{\overline{\beta}\mathsf{INR}_2}{1+\beta\mathsf{INR}_2}\right) \\ S_1 + T_2 & \leq & \gamma\left(\dfrac{\mathsf{SNR}_1 + \overline{\beta}\mathsf{INR}_2}{1+\beta\mathsf{INR}_2}\right). \end{array} \right. \tag{14}$$

Next, consider the decoding of (W_2, U_2) at Y_2 with the help of a digital relay link of rate R_0. This is a multiple-access channel with a rate-limited relay, where the relay has complete knowledge of W_2 and a noisy observation $h_{21}U_2 + Z_1$, obtained by subtracting X_1 and W_2 from the received signal at Y_1. Each of these two pieces of information is useful for decoding (W_2, U_2) at Y_2.

We now consider a relay scheme which splits the digital link in two parts: R_a bits of the digital link for describing W_2, and R_b for describing U_2, where $R_a + R_b = R_0$. Since W_2 is perfectly known at the relay, a straightforward binning technique of decode-and-forward can be used to describe W_2. It can be shown that a relay link of rate R_b enlarges the capacity region of the multiple-access channel by increasing the rate for W_2 and the sum rate by exactly R_b.

On the other hand, since only a noisy version of U_2 is available at the relay, a quantize-and-forward strategy using Wyner-Ziv coding ([9], [7]) may be used for describing U_2. A straightforward application of quantize-and-forward would be to quantize $h_{21}U_2 + Z_1$ with Y_2 used as the decoder side information. However, the presence of W_2 offers other possibilities. First, the decoder of the multiple-access channel may choose to decode W_2 before decoding U_2, in which case W_2 becomes an additional decoder side information for Wyner-Ziv coding. Second, instead of quantizing $h_{21}U_2 + Z_1$ with W_2 completely subtracted from the relay's observation, the relay may choose to subtract W_2 partially—doing so can benefit the Wyner-Ziv rate. This second approach is adopted for the rest of the proof. Interestingly, the second approach turns out to include the first approach as a special case.

More specifically, let the relay form the following fictitious signal

$$\bar{Y}_1 = h_{21}(U_2 + W_2) + \alpha h_{21} W_2 + Z_1 \tag{15}$$

for some $\alpha \in \mathbb{R}$. The proposed relay scheme, which combines the decode-and-forward technique and the quantize-and-forward technique, is now illustrated in Fig. 3, where W_2 and U_2 are the inputs of the multiple-access channel, $(Y_2, \hat{Y}_1)$ is the output, and $\hat{Y}_1$ is a quantized version of $\bar{Y}_1$. With complete knowledge of W_2 at the relay, the capacity of this multiple-access relay channel, denoted by $\mathcal{C}_2$, is given by the set of rates (S_2, T_2) where

$$\left\{ \begin{array}{rcl} S_2 & \leq & I(U_2; Y_2, \hat{Y}_1 | W_2) \\ T_2 & \leq & I(W_2; Y_2, \hat{Y}_1 | U_2) + R_b \\ S_2 + T_2 & \leq & I(U_2, W_2; Y_2, \hat{Y}_1) + R_b \end{array} \right. \tag{16}$$

We now adopt a Gaussian quantization scheme to quantize $\bar{Y}_1$:

$$\hat{Y}_1 = \bar{Y}_1 + e \tag{17}$$

where e is a Gaussian random variable independent of $\bar{Y}_1$, with a distribution $\mathcal{N}(0, \sigma^2)$. With the encoder side information W_2 at the input of the relay link and the decoder side information Y_2 at the output of the relay link, the Wyner-Ziv coding rate for quantizing $\bar{Y}_1$ into $\hat{Y}_1$ is given by [10] [7]

$$I(\hat{Y}_1; W_2, \bar{Y}_1) - I(\hat{Y}_1; Y_2) \leq R_a. \tag{18}$$

But

$$\begin{aligned}
& I(\hat{Y}_1; W_2, \bar{Y}_1) - I(\hat{Y}_1; Y_2) \\
&\quad = \quad I(\hat{Y}_1; \bar{Y}_1) + I(\hat{Y}_1; W_2|\bar{Y}_1) - I(\hat{Y}_1; Y_2) \\
&\quad \stackrel{(a)}{=} \quad I(\hat{Y}_1; \bar{Y}_1) - I(\hat{Y}_1; Y_2) \\
&\quad \stackrel{(b)}{=} \quad I(\hat{Y}_1; \bar{Y}_1|Y_2)
\end{aligned} \tag{19}$$

where both (a) and (b) come from the fact that $\hat{Y}_1 = \bar{Y}_1 + e$ and e is independent of W_2 or Y_2. Thus, the rate constraint becomes

$$I(\hat{Y}_1; \bar{Y}_1|Y_2) \leq R_a. \tag{20}$$

To fully utilize the channel, we set $\hat{Y}_1$ to be such that $I(\hat{Y}_1; \bar{Y}_1|Y_2)$ is equal to R_a. To find σ^2, note that

$$\begin{aligned}
R_a &= I(\hat{Y}_1; \bar{Y}_1|Y_2) \\
&= h(\hat{Y}_1|Y_2) - h(\hat{Y}_1|\bar{Y}_1, Y_2) \\
&= \frac{1}{2}\log(2\pi e \sigma^2_{\hat{Y}_1|Y_2}) - \frac{1}{2}\log(2\pi e \sigma^2) \\
&= \frac{1}{2}\log\left(\frac{\sigma^2_{\hat{Y}_1|Y_2}}{\sigma^2}\right)
\end{aligned} \tag{21}$$

where $\sigma^2_{\hat{Y}_1|Y_2}$, the conditional minimum mean-squared error (MMSE) of $\hat{Y}_1$ given Y_2, can be calculated as

$$\begin{aligned}
\sigma^2_{\hat{Y}_1|Y_2} &= \sigma^2_{\hat{Y}_1} - \sigma_{\hat{Y}_1, Y_2}(\sigma^2_{Y_2})^{-1}\sigma_{Y_2, \hat{Y}_1} \\
&= (\beta\overline{\beta}\alpha^2|h_{21}|^2|h_{22}|^2P_2^2 + (|h_{22}|^2P_2 + N)(N + \sigma^2) \\
&\quad + (1 + 2\alpha\overline{\beta} + \alpha^2\overline{\beta})P_2|h_{21}|^2N)/(|h_{22}|^2P_2 + N).
\end{aligned}$$

Substituting the above $\sigma^2_{\hat{Y}_1|Y_2}$ into (21), we obtain (9).

Now, we evaluate the multiple-access relay channel capacity region $\mathcal{C}_2$ in (16). Define

$$\left\{\begin{aligned}
I(U_2; \hat{Y}_1|Y_2, W_2) &\stackrel{\triangle}{=} \zeta(\alpha, \beta, R_a) \\
I(W_2; \hat{Y}_1|Y_2, U_2) &\stackrel{\triangle}{=} \xi(\alpha, \beta, R_a) \\
I(W_2, U_2; \hat{Y}_1|Y_2) &\stackrel{\triangle}{=} \eta(\alpha, \beta, R_a).
\end{aligned}\right. \tag{22}$$

Applying Gaussian distributions $W_2 \sim \mathcal{N}(0, \overline{\beta}P_2)$ and $U_2 \sim \mathcal{N}(0, \beta P_2)$, $\mathcal{C}_2$ becomes

$$\left\{\begin{aligned}
S_2 &\leq \gamma(\beta\mathsf{SNR}_2) + \zeta(\alpha, \beta, R_a) \\
T_2 &\leq \gamma(\overline{\beta}\mathsf{SNR}_2) + \xi(\alpha, \beta, R_a) + R_b \\
S_2 + T_2 &\leq \gamma(\mathsf{SNR}_2) + \eta(\alpha, \beta, R_a) + R_b.
\end{aligned}\right. \tag{23}$$

The computations of $\zeta(\alpha, \beta, R_a)$, $\xi(\alpha, \beta, R_a)$ and $\eta(\alpha, \beta, R_a)$ again involve Gaussian MMSE expressions. First,

$$\begin{aligned}
\eta(\alpha, \beta, R_a) &= h(\hat{Y}_1|Y_2) - h(\hat{Y}_1|U_2, W_2, Y_2) \\
&= \frac{1}{2}\log\left(\frac{\sigma^2_{\hat{Y}_1|Y_2}}{N + \sigma^2}\right).
\end{aligned} \tag{24}$$

Substituting (22) into the above, we obtain (8). Likewise,

$$\begin{aligned}
\zeta(\alpha, \beta, R_a) &= h(\hat{Y}_1|Y_2, W_2) - h(\hat{Y}_1|U_2, W_2, Y_2) \\
&= \frac{1}{2}\log\left(\frac{\sigma^2_{\hat{Y}_1|Y_2, W_2}}{N + \sigma^2}\right).
\end{aligned} \tag{25}$$

A similar computation leads to (7). The computation of $\xi(\alpha, \beta, R_a)$ can be done in a similar fashion, but the detailed expression does not affect our final result.

Finally, an achievable rate region for the Gaussian Z-interference channel with a relay link is a set of (R_1, R_2) with $R_1 = S_1$ and $R_2 = S_2 + T_2$, for which $(S_1, T_2) \in \mathcal{C}_1$ and $(S_2, T_2) \in \mathcal{C}_2$. Combining the $\mathcal{C}_1$ region (14) and the $\mathcal{C}_2$ region (23), we obtain a pentagon achievable rate region $\mathcal{R}_{\alpha,\beta}(R_a, R_b)$ for each fixed α, $0 \leq \beta \leq 1$ and $R_a + R_b = R_0$:

$$\left\{ (R_1, R_2) \,\middle|\, \begin{aligned}
& R_1 \leq \gamma\left(\tfrac{\mathsf{SNR}_1}{1+\beta\mathsf{INR}_2}\right) \\
& R_2 \leq \min\{\gamma(\mathsf{SNR}_2) + R_b + \eta(\alpha, \beta, R_a), \\
& \qquad \gamma(\beta\mathsf{SNR}_2) + \gamma\left(\tfrac{\overline{\beta}\mathsf{INR}_2}{1+\beta\mathsf{INR}_2}\right) \\
& \qquad + \zeta(\alpha, \beta, R_a)\} \\
& R_1 + R_2 \leq \gamma(\beta\mathsf{SNR}_2) + \gamma\left(\tfrac{\mathsf{SNR}_1 + \overline{\beta}\mathsf{INR}_2}{1+\beta\mathsf{INR}_2}\right) \\
& \qquad + \zeta(\alpha, \beta, R_a)
\end{aligned} \right\}. \tag{26}$$

Therefore, the overall achievable rate region is

$$\mathrm{co}\left\{ \bigcup_{\alpha\in\mathbb{R}, 0\leq\beta\leq 1,\ R_a + R_b \leq R_0} \mathcal{R}_{\alpha,\beta}(R_a, R_b) \right\} \tag{27}$$

This concludes the proof of the general achievability region (5).

In the following, we show that (2), (11) and (13) are all included in the above achievable rate region.

First, consider the weak interference regime, where $\mathsf{INR}_2 \leq \mathsf{SNR}_2$. For any nonnegative R_b and when $\mathsf{INR}_2 \leq \mathsf{SNR}_2$, it is easy to verify the following two inequalities:

$$\gamma(\beta\mathsf{SNR}_2) + \gamma\left(\frac{\overline{\beta}\mathsf{INR}_2}{1 + \beta\mathsf{INR}_2}\right) \leq \gamma(\mathsf{SNR}_2) + R_b \tag{28}$$

and

$$\zeta(\alpha, \beta, R_a) \leq \eta(\alpha, \beta, R_a). \tag{29}$$

Thus, the second term in the minimization in the expression of R_2 in (26) is always less than the first term. As a result, $\mathcal{R}_{\alpha,\beta}(R_a, R_b)$ simplifies to

$$\left\{ (R_1, R_2) \,\middle|\, \begin{aligned}
& R_1 \leq \gamma\left(\tfrac{\mathsf{SNR}_1}{1+\beta\mathsf{INR}_2}\right) \\
& R_2 \leq \gamma(\beta\mathsf{SNR}_2) + \gamma\left(\tfrac{\overline{\beta}\mathsf{INR}_2}{1+\beta\mathsf{INR}_2}\right) \\
& \qquad + \zeta(\alpha, \beta, R_a) \\
& R_1 + R_2 \leq \gamma(\beta\mathsf{SNR}_2) + \gamma\left(\tfrac{\mathsf{SNR}_1 + \overline{\beta}\mathsf{INR}_2}{1+\beta\mathsf{INR}_2}\right) \\
& \qquad + \zeta(\alpha, \beta, R_a)
\end{aligned} \right\}. \tag{30}$$

Inspecting the above region, we see that R_a enters the above expression only through $\zeta(\alpha, \beta, R_a)$. It is easy to verify that $\zeta(\alpha, \beta, R_a)$ is a monotonically nondecreasing function of R_a. Thus, the maximum achievability region is obtained for $R_a = R_0$ and $R_b = 0$. This means that a pure quantization scheme is optimal in the weak interference regime.

Further, α enters the rate region expression also only through $\zeta(\alpha, \beta, R_0)$. Thus, we can choose α to maximize $\zeta(\alpha, \beta, R_0)$, or equivalently, to minimize σ^2 in (9). Taking the derivative of (9) with respect to α and setting it to zero, the optimal α is

$$\alpha^* = -\frac{1}{1+\beta\mathsf{SNR}_2}. \tag{31}$$

Substituting α^* into (9), we obtain

$$\frac{\sigma^2}{N} = \frac{1}{2^{2R_0}-1}\left(1+\frac{\beta\mathsf{INR}_2}{1+\beta\mathsf{SNR}_2}\right), \tag{32}$$

which gives a derivation of (3):

$$\zeta(\alpha^*,\beta,R_0) = \gamma\left(\frac{\beta(2^{2R_0}-1)\mathsf{INR}_2}{2^{2R_0}(1+\beta\mathsf{SNR}_2)+\beta\mathsf{INR}_2}\right) \triangleq \delta(\beta,R_0). \tag{33}$$

Finally, we take the union of all $\mathcal{R}_{\alpha^*,\beta}(R_0,0)$. Substitute the above $\delta(\beta,R_0)$ into (30) and denote the rate constraints of the pentagon as

$$\begin{aligned} f_1(\beta) &= \gamma\left(\frac{\mathsf{SNR}_1}{1+\beta\mathsf{INR}_2}\right) \\ f_2(\beta) &= \gamma\left(\beta\mathsf{SNR}_2\right)+\gamma\left(\frac{\overline{\beta}\mathsf{INR}_2}{1+\beta\mathsf{INR}_2}\right)+\delta(\beta,R_0) \\ f_3(\beta) &= \gamma\left(\beta\mathsf{SNR}_2\right)+\gamma\left(\frac{\mathsf{SNR}_1+\overline{\beta}\mathsf{INR}_2}{1+\beta\mathsf{INR}_2}\right)+\delta(\beta,R_0). \end{aligned}$$

It is easy to check that, first, the pentagon reduces to a rectangle when $\beta=1$. Second, $f_2(\beta)-f_3(\beta)$ is a constant. Third, when $\mathsf{INR}_2 \le \mathsf{SNR}_2$, we have $f_1^{'}(\beta)<0$ and $f_2^{'}(\beta)=f_3^{'}(\beta)\ge 0$. Therefore, as β decreases from 1 to 0, $f_1(\beta)$ is monotonically increasing, while both $f_2(\beta)$ and $f_3(\beta)$ are monotonically decreasing. Consequently, the union of achievable pentagons, $\bigcup_{0\le\beta\le1}\mathcal{R}_{\alpha^*,\beta}(R_0,0)$ is defined by $R_1 \le \gamma(\mathsf{SNR}_1)$, $R_2 \le \gamma(\mathsf{SNR}_2)+\delta(\beta,R_0)$, and lower-right corner points of the pentagons

$$\begin{cases} R_1 = \gamma\left(\dfrac{\mathsf{SNR}_1}{1+\beta\mathsf{INR}_2}\right) \\ R_2 = \gamma(\beta\mathsf{SNR}_2)+\gamma\left(\dfrac{\overline{\beta}\mathsf{INR}_2}{1+\mathsf{SNR}_1+\beta\mathsf{INR}_2}\right)+\delta(\beta,R_0). \end{cases} \tag{34}$$

It can be verified that this region is convex when $\mathsf{INR}_2 \le \mathsf{SNR}_2$, and therefore convex hull is not needed. This establishes the rate region (2) for the weak interference regime.

In the moderately strong interference regime, where $\mathsf{SNR}_2 \le \mathsf{INR}_2 \le \mathsf{INR}_2^{\dagger}$, the achievability of (5) follows directly from the general achievability region. Note that, in this regime, the rate region is achieved by a mixed scheme, which include both decode-and-forward and quantize-and-forward strategies.

Finally, consider the strong interference regime, where $\mathsf{INR}_2 \ge \mathsf{INR}_2^{\dagger}$ and the very strong interference regime, where $\mathsf{INR}_2 \ge \mathsf{INR}_2^{\ddagger}$. We show that (11) and (13) are the capacity regions, respectively.

First, by setting[1] $R_b = R_0$, $R_a = 0$ and $\beta = 0$, the achievable rate region $\mathcal{R}_{\alpha,\beta}(R_a,R_b)$ in (26) reduces to

$$\left\{(R_1,R_2)\left|\begin{array}{rcl} R_1 &\le& \gamma(\mathsf{SNR}_1) \\ R_2 &\le& \min\{\gamma(\mathsf{SNR}_2)+R_0,\gamma(\mathsf{INR}_2)\} \\ R_1+R_2 &\le& \gamma(\mathsf{SNR}_1+\mathsf{INR}_2)\end{array}\right.\right\}. \tag{35}$$

This rate region reduces to (11) in the strong interference regime, because

$$\gamma(\mathsf{SNR}_2)+R_0 \le \gamma(\mathsf{INR}_2) \tag{36}$$

when $\mathsf{INR}_2 \ge \mathsf{INR}_2^{\dagger}$. Thus, (11) is achievable.

Further, in the very strong interference regime, where $\mathsf{INR}_2 \ge \mathsf{INR}_2^{\ddagger}$, the constraint on R_1+R_2 in (11) becomes redundant. Thus, the rate region reduces to (13).

Next, we give a converse proof to show that (11) and (13) are indeed the capacity regions in the strong and very strong interference regimes, respectively. The idea is to show that if (R_1,R_2) is in the achievable rate region for the Type II Gaussian Z-relay-interference channel, i.e., X_1 can be reliably decoded at Y_1 at rate R_1, and X_2 can be reliably decoded at Y_2 at rate R_2, then X_2 must also be decodable at the Y_1.

First, observe that by the cut-set upper bound [11], reliable decoding of X_2 at Y_2 requires

$$R_2 \le \gamma(\mathsf{SNR}_2)+R_0. \tag{37}$$

To show that X_2 must be decodable at Y_1, note that after the decoding of X_1 at Y_1, X_1 can be subtracted from the received signal to form

$$\tilde{Y}_1 = h_{21}X_2+Z_1. \tag{38}$$

The capacity of this channel is $\gamma(\mathsf{INR}_2)$. On the other hand, R_2 is bounded by $\gamma(\mathsf{SNR}_2)+R_0$, which is less than $\gamma(\mathsf{INR}_2)$ when $\mathsf{INR}_2 \ge \mathsf{INR}_2^{\dagger}$. So, X_2 is always decodable based on $\tilde{Y}_1$.

Now, since both X_1 and X_2 are decodable at Y_1 in the strong interference regime, the achievable rate region of the Gaussian Z-relay-interference channel in the strong interference regime must be a subset of the capacity region of a Gaussian multiple-access channel with X_1, X_2 as inputs and Y_1 as output, which is

$$\left\{(R_1,R_2)\left|\begin{array}{rcl} R_1 &\le& \gamma(\mathsf{SNR}_1) \\ R_2 &\le& \gamma(\mathsf{INR}_2) \\ R_1+R_2 &\le& \gamma(\mathsf{SNR}_1+\mathsf{INR}_2)\end{array}\right.\right\}. \tag{39}$$

Combining (37), (39), and observing that

$$\gamma(\mathsf{SNR}_2)+R_0 \le \gamma(\mathsf{INR}_2) \tag{40}$$

when $\mathsf{INR}_2 \ge \mathsf{INR}_2^{\dagger}$, we proved that the achievable rate region of the Gaussian Z-relay-interference channel must be bounded by (11) when $\mathsf{INR}_2 \ge \mathsf{INR}_2^{\dagger}$, which reduces to (13) when $\mathsf{INR}_2 \ge \mathsf{INR}_2^{\ddagger}$. ∎

The achievability proof for the weak interference case in Theorem 1 is based on a quantize-and-forward scheme in which Y_1 quantizes a fictitious signal $\bar{Y}_1 = h_{21}(U_2+W_2)+\alpha^* h_{21}W_2+Z_1$, which is a linear combination of its own received signal and the decoded W_2, with α^* *optimized* for

[1] The value of α does not affect $\mathcal{R}_{\alpha,\beta}(R_a,R_b)$ when $R_a=0$.

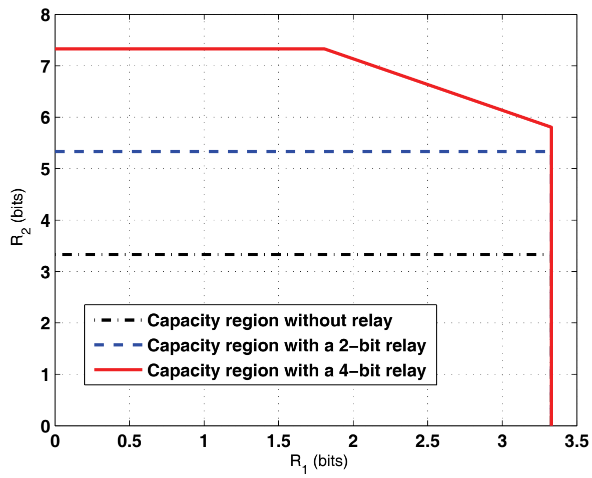


Fig. 4. Capacity region of the Gaussian Z-interference channel in the strong interference regime with and without a digital relay link of Type II.

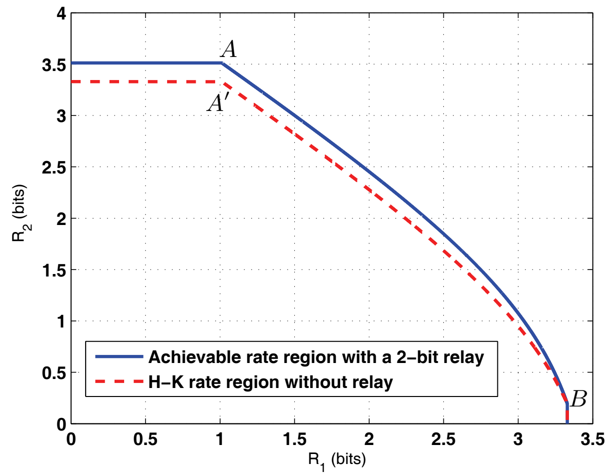


Fig. 5. Capacity region of the Gaussian Z-interference channel in the weak interference regime with and without a digital relay link of Type II.

maximum overall achievable rate region. The quantized $\bar{Y}_1$ is decoded with $Y_2 = h_{22}(U_2 + W_2) + Z_2$ as decoder side information in establishing the multiple-access relay channel capacity region $\mathcal{C}_2$ with Wyner-Ziv coding.

As mentioned earlier, another possible quantize-and-forward strategy is to restrict the decoding order for the multiple-access channel $\mathcal{C}_2$ to be that of decoding W_2 first, then U_2. In this case, W_2 would be known at *both* the input and the output of the relay link when decoding U_2. Thus, the relay only has to quantize $h_{21}U_2 + Z_1$ with $h_{22}U_2 + Z_2$ as decoder side information in Wyner-Ziv coding. Surprisingly, it can be shown that these two different strategies give the exact same achievable rate region in the weak-interference regime. Such an approach can be used to give an alternative proof for the weak-interference result in Theorem 1.

C. Numerical Examples

The achievable rate region for the Type II Gaussian Z-relay-interference channel is structured as follows. In addition to the weak, strong and very strong interference regimes, there is also a new "moderately strong" interference regime, where a combination of decode-and-forward and quantize-and-forward strategies may be needed.

In the strong and very strong interference regimes, the relay expands the capacity region by decoding X_2 and forwarding its bin index to help Y_2 decode X_2. The boundaries of the strong and very strong regimes depend on the relay link rate. In these two regimes, the entire X_2 is common information. Due to the strong interference link, this common message X_2 is guaranteed to be decodable at Y_1.

As a numerical example, Fig. 4 shows how the capacity region of a Gaussian Z-interference channel in the strong and very strong interference regimes is expanded by the digital relay link from the interfered receiver to the interference-free receiver. The channel parameters are set to be $\mathsf{SNR}_1 = \mathsf{SNR}_2 = 20$ dB and $\mathsf{INR}_2 = 55$ dB. Without the digital link, this is a classic Gaussian Z-interference channel in the very strong interference regime, where $\mathsf{INR}_2 \geq \mathsf{SNR}_2(1 + \mathsf{SNR}_1)$. In this regime, the capacity region is a rectangle [12], as depicted by the dash-dotted region in Fig.4. With a 2-bit digital link, the Gaussian Z-interference channel remains in the very strong interference regime. The capacity region, given by (13) is depicted by the dashed rectangular region in Fig. 4. It represents a rate increase in R_2 of exactly 2 bits. When $R_0 = 4$ bits, the Gaussian Z-interference channel falls into the strong interference regime. The capacity region, given by (11), becomes the solid pentagon region. In this regime, for some R_1 lower than a certain threshold, the rate increase in R_2 is exactly 4 bits. For other values of R_1, the rate increase in R_2 is less than 4 bits. Further increase in the rate of the digital link would increase the rate constraint on R_2, but not the sum rate.

In the weak interference regime, where $\mathsf{INR}_2 \leq \mathsf{SNR}_2$, Theorem 1 shows that a pure quantize-and-forward of the private message should be used for relaying. Intuitively, this is because when the interference link is weak the common message rate is limited by the interference link, which cannot be helped by relaying. Thus, the digital link ought to focus on helping the decoding of private information at Y_2 by quantize-and-forward.

As a numerical example, Fig. 5 shows the achievable rate region of a Gaussian Z-interference channel with $\mathsf{SNR}_1 = \mathsf{SNR}_2 = 20$ dB and $\mathsf{INR}_2 = 15$ dB with and without the relay link, which falls into the weak interference regime. The dashed region denoted by points A' and B represents the rate region achieved without the digital link. The solid rate region denoted by points A and B is with a 2-bit digital link.

In Fig. 5, points A and A' both correspond to $\beta = 1$, where the entire X_2 is the private message. As β decreases from 1 to 0, the rate pair moves from point A (or A') to point B, which corresponds to $\beta = 0$, where the entire X_2 is common message. From the rate pair expression (2), the effect of the digital link is to shift the rate region of the channel without the relay upward by $\delta(\beta, R_0)$ bits. Since $\delta(\beta, R_0)$ is monotonically decreasing as β decreases from 1 to 0, for fixed R_1, the largest increase in R_2 corresponds to $\delta(1, R_0)$, i.e. the increase from point A' to A. Note that A and A' are the maximum sum-rate points of the Type II Gaussian Z-interference channel with and without the relay. They correspond to all-private message transmission, which is in contrast to the Type I case where the maximum sum rate is achieved with some $\beta^* \neq 1$. Finally, we note that the relay does not affect point B, which corresponds to $\beta = 0$, because $\delta(0, R_0) = 0$.

D. Sum Capacity Upper Bound

By Theorem 1, an achievable sum rate of the Type II Gaussian Z-interference channel with a relay link of rate R_0 in the weak interference regime is

$$R_{sum} = \gamma\left(\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\right) + \gamma(\mathsf{SNR}_2) + \delta(1, R_0) \tag{41}$$

which is obtained by setting $\beta = 1$ in (2). Comparing with the sum capacity of the Gaussian Z-interference channel without the relay in the weak interference regime, the sum-rate increase using the relay scheme of Theorem 1 is upper bounded by

$$\begin{aligned}\delta(1, R_0) &= \frac{1}{2}\log\left(\frac{1+\mathsf{SNR}_2+\mathsf{INR}_2}{1+\mathsf{SNR}_2+2^{-2R_0}\mathsf{INR}_2}\right) \\ &\leq \gamma\left(\frac{\mathsf{INR}_2}{1+\mathsf{SNR}_2}\right) \\ &\leq \frac{1}{2}\end{aligned} \tag{42}$$

where $\mathsf{INR}_2 \leq \mathsf{SNR}_2$ is used in the last inequality. This is illustrated by an example in Fig. 5, where the rate increase from point A' to point A is about 0.2 bits, which is less than 1/2 bits and is a fraction of the 2-bit capacity of the digital link. This is in stark contrast to the Type I channel, where each bit of the relay capacity can increase the overall sum rate by one bit in the high INR/SNR limit. The half bit upper bound is in fact general, as shown in the following theorem.

Theorem 2: Let $\mathcal{C}(R_0)$ denote the capacity region of the Type II Gaussian Z-interference channel with a digital realy link of limited rate R_0 from the interfered receiver to the interference-free receiver as shown in Fig. 1(b). Let

$$C^{\theta}_{sum}(R_0) = \max_{(C_1,C_2)\in\mathcal{C}(R_0)} \theta C_1 + (1-\theta)C_2. \tag{43}$$

Consider the weak interference regime, where $\mathsf{INR}_2 \leq \mathsf{SNR}_2$. For any $R_0 \geq 0$, a capacity upper bound for the Gaussian Z-relay-interference channel is

$$C^{\theta}_{sum}(R_0) \leq C^{\theta}_{sum}(0) + (1-\theta)\gamma\left(\frac{\mathsf{INR}_2}{1+\mathsf{SNR}_2}\right). \tag{44}$$

In particular, as $R_0 \to \infty$, the asymptotic sum capacity of the Type II Gaussian Z-interference channel with a relay link is

$$\gamma\left(\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\right) + \gamma\left(1+\mathsf{INR}_2+\mathsf{SNR}_2\right). \tag{45}$$

Finally, the sum capacity of the Type II Gaussian Z-relay-interference channel is upper bounded by the sum capacity of the Gaussian Z-interference channel without a relay link plus half a bit.

Proof: Clearly, the weighted sum capacity gain is a nondecreasing function of R_0. Thus, we focus on the case of $R_0 = \infty$, i.e. when Y_2 has complete knowledge of Y_1. The proof starts with Fano's inequality:

$$\begin{aligned}&n(\theta R_1 + (1-\theta)R_2) \\ &\overset{(a)}{\leq} \theta I(X_1^n; Y_1^n) + (1-\theta)I(X_2^n; Y_1^n, Y_2^n) + n\epsilon_n \\ &= \theta I(X_1^n; Y_1^n) + (1-\theta)I(X_2^n; Y_2^n) + \\ &\qquad (1-\theta)I(X_2^n; Y_1^n|Y_2^n) + n\epsilon_n \\ &\overset{(b)}{\leq} nC^{\theta}_{sum}(0) + (1-\theta)I(X_2^n; Y_1^n|Y_2^n) + n\epsilon_n \\ &\overset{(c)}{\leq} nC^{\theta}_{sum}(0) + (1-\theta)I(X_2^n; Y_1^n|X_1^n, Y_2^n) + n\epsilon_n\end{aligned} \tag{46}$$

where

(a) follows from Fano's inequality;
(b) follows from the definition of $C^{\theta}_{sum}(0)$;
(c) follows from the fact that X_1^n is independent of X_2^n given Y_2^n, in which case

$$\begin{aligned}I(X_2^n; Y_1^n|Y_2^n) &= h(X_2^n|Y_2^n) - h(X_2^n|Y_1^n, Y_2^n) \\ &\leq h(X_2^n|Y_2^n) - h(X_2^n|X_1^n, Y_1^n, Y_2^n) \\ &= h(X_2^n|X_1^n, Y_2^n) - h(X_2^n|X_1^n, Y_1^n, Y_2^n) \\ &= I(X_2^n; Y_1^n|X_1^n, Y_2^n).\end{aligned} \tag{47}$$

To complete the proof, we only need to show that $I(X_2^n; Y_1^n|X_1^n, Y_2^n) \leq n\gamma\left(\frac{\mathsf{INR}_2}{1+\mathsf{SNR}_2}\right)$.

Let $\mathbf{\Sigma}$ denote the covariance matrix of X_2^n. Define $\sigma_1^2 = \frac{N}{|h_{21}|^2}$, $\sigma_2^2 = \frac{N}{|h_{22}|^2}$. Let $\mathbf{I}$ be the $n \times n$ identity matrix. Then,

$$\begin{aligned}&I(X_2^n; Y_1^n|X_1^n, Y_2^n) \\ &= h(Y_1^n|X_1^n, Y_2^n) - h(Y_1^n|X_1^n, X_2^n, Y_2^n) \\ &= h(Y_1^n, Y_2^n|X_1^n) - h(Y_2^n|X_1^n) - h(Z_1^n) \\ &= h\left(\begin{bmatrix} h_{11}\mathbf{I} \\ h_{22}\mathbf{I}\end{bmatrix} X_2^n + \begin{bmatrix} Z_1^n \\ Z_2^n \end{bmatrix}\right) - h\left(h_{22}X_2^n + Z_2^n\right) - h(Z_1^n) \\ &\overset{(d)}{\leq} \max_{\mathrm{tr}(\mathbf{\Sigma})\leq nP_2} \frac{1}{2}\log\frac{\begin{vmatrix} \mathbf{\Sigma}+\sigma_1^2\mathbf{I} & \mathbf{\Sigma} \\ \mathbf{\Sigma} & \mathbf{\Sigma}+\sigma_2^2\mathbf{I}\end{vmatrix}}{|\mathbf{\Sigma}+\sigma_2^2\mathbf{I}|\cdot|\sigma_1^2\mathbf{I}|} \\ &= \max_{\mathrm{tr}(\mathbf{\Sigma})\leq nP_2} \frac{1}{2}\log\frac{|\mathbf{\Sigma}+\sigma_2^2\mathbf{I}|\cdot|\mathbf{\Sigma}+\sigma_1^2\mathbf{I}-\mathbf{\Sigma}(\mathbf{\Sigma}+\sigma_2^2\mathbf{I})^{-1}\mathbf{\Sigma}|}{|\mathbf{\Sigma}+\sigma_2^2\mathbf{I}|\cdot|\sigma_1^2\mathbf{I}|} \\ &= \max_{\mathrm{tr}(\mathbf{\Sigma})\leq nP_2} \frac{1}{2}\log\frac{|(\mathbf{\Sigma}^{-1}+(\sigma_2^2)^{-1}\mathbf{I})^{-1}+\sigma_1^2\mathbf{I}|}{|\sigma_1^2\mathbf{I}|} \\ &= \max_{\mathrm{tr}(\mathbf{\Sigma})\leq nP_2} \frac{1}{2}\log\frac{|\sigma_2^2\mathbf{I}-\sigma_2^2\mathbf{I}(\mathbf{\Sigma}+\sigma_2^2\mathbf{I})^{-1}\sigma_2^2\mathbf{I}+\sigma_1^2\mathbf{I}|}{|\sigma_1^2\mathbf{I}|} \\ &\overset{(e)}{=} n\gamma\left(\frac{\mathsf{INR}_2}{1+\mathsf{SNR}_2}\right)\end{aligned} \tag{48}$$

where

(d) can be derived using the extremal entropy inequality of Liu and Viswanath [13], which shows that a

Gaussian X_2^n maximizes the entropy difference term immediately above;

(e) follows from the fact that the maximizing $\mathbf{\Sigma}$ is $P_2\mathbf{I}$, which can be shown by examining the KKT condition of the optimization problem;

and the intermediate steps make repeated use of the fact that $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |D| \cdot \left|A - BD^{-1}C\right|$ and the matrix inversion lemma $(A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}$. Combining (46), (48) and noting that $\epsilon_n \to 0$ as $n \to \infty$ give us the desired result (44).

Next, note that the sum capacity of the Type II Gaussian Z-relay-interference channel with a relay link of capacity R_0 can be expressed as $2C_{sum}^{\frac{1}{2}}(R_0)$. Further, since the sum capacity of the Gaussian Z-interference channel without the relay is

$$2C_{sum}^{\frac{1}{2}}(0) = \gamma\left(\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\right) + \gamma(\mathsf{SNR}_2), \tag{49}$$

by (44), the following is a sum capacity upper bound for the Gaussian Z-interference channel with a relay link of rate R_0

$$\begin{aligned} 2C_{sum}^{\frac{1}{2}}(R_0) &\leq \gamma\left(\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\right) + \gamma(\mathsf{SNR}_2) \\ &\quad + \gamma\left(\frac{\mathsf{INR}_2}{1+\mathsf{SNR}_2}\right) \\ &= \gamma\left(\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\right) + \gamma\left(1 + \mathsf{INR}_2 + \mathsf{SNR}_2\right) \end{aligned}$$

By Theorem 1 and the computation earlier (i.e. (41)-(42)), we see that the above upper bound is also asymptotically achievable when $R_0 \to \infty$. This proves that (45) is the asymptotic sum capacity.

Finally, the half-bit bound on sum capacity follows from the fact that $\gamma\left(\frac{\mathsf{INR}_2}{1+\mathsf{SNR}_2}\right) \leq \frac{1}{2}$ when $\mathsf{INR}_2 \leq \mathsf{SNR}_2$. ■

Note that the capacity region of the Gaussian Z-interference without the relay link is not yet completely known, except for the sum capacity point. Theorem 2 bounds the capacity increase due to the relay link, without explicitly finding either capacity regions.

The asymptotic sum capacity (45) is essentially the sum capacity of a degraded Gaussian interference channel where the inputs are X_1 and X_2, and outputs are Y_1 and (Y_1, Y_2) of a Gaussian Z-interference channel in the weak interference regime. The capacity region for the general degraded interference channel is still open.

III. Concluding Remarks

This paper studies a class of Gaussian Z-interference channels with receiver cooperation, where a rate-limited digital link is provided from the interfered receiver to the interference-free receiver. In the strong interference regime, decode-and-forward is shown to be capacity achieving. In the weak interference regime, an achievable rate region is derived based a quantize-and-forward strategy. In the moderately strong interference regime, a combination of decode-and-forward and quantize-and-forward can be used.

It is interesting to note that the direction of the relay link critically influences the capacity gain due to relaying in the weak interference regime. As shown in the companion paper [1], when the relay link goes from the interference-free receiver to the interfered receiver, relaying can significantly enlarge the achievable rate region. In this case, the interference-free receiver may decode then bin-and-forward a part of the interference to the interfered receiver for interference subtraction. Such a strategy is effective in the sense that it asymptotically achieves the cut-set bound in the high SNR/INR limit—every bit of relay link rate results in one bit increase in sum capacity.

In a direct contract, the results in this paper show that when the direction of the relay link is reversed (i.e., when it goes from the interfered receiver to the interference-free receiver), the sum capacity increase is upper bounded by half a bit in the weak interference regime, regardless of the relay link capacity. Thus, a relay link from the interfered receiver to the interference-free receiver is much less effective.

Acknowledgment

The authors would like to thank LG Electronics for supporting their research.

References

[1] L. Zhou and W. Yu, "Gaussian Z-interference channel with a relay link: Achievable rate region and asymptotic sum capacity," in *Proc. Int. Symp. Inf. Theory and Its App. (ISITA)*, Auckland, New Zealand, Dec. 2008.

[2] O. Sahin and E. Erkip, "Achievable rates for the Gaussian interference relay channel," in *Proc. Global Telecommun. Conf. (Globecom)*, Nov. 2007, pp. 1627–1631.

[3] ——, "On achievable rates for interference relay channel with interference cancelation," in *Conf. Record Forty-First Asilomar Conf. Signals, Systems and Computers*, Nov. 2007, pp. 805–809.

[4] I. Marić, R. Dabora, and A. Goldsmith, "On the capacity of the interference channel with a relay," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Jul. 2008.

[5] R. Dabora, I. Marić, and A. Goldsmith, "Relay strategies for interference-forwarding," in *Proc. IEEE Inf. Theory Workshop (ITW)*, Porto, Portugal, May 2008, pp. 46–50.

[6] O. Simeone, O. Somekh, H. V. Poor, and S. Shamai, "Local base station cooperation via finite-capacity links for the uplink of simple cellular networks," *IEEE Trans. Inform. Theory*, vol. 55, no. 1, pp. 190–204, Jan. 2009.

[7] T. M. Cover and A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inform. Theory*, vol. 25, no. 5, pp. 572–584, Sep. 1979.

[8] T. S. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inform. Theory*, vol. 27, no. 1, pp. 49–60, Jan. 1981.

[9] A. D. Wyner and J. Ziv, "The rate-distortion function for source coding with side information at the decoder," *IEEE Trans. Inform. Theory*, vol. 22, no. 1, pp. 1–10, Jan. 1976.

[10] T. M. Cover and M. Chiang, "Duality between channel capacity and rate distortion with two-sided state information," *IEEE Trans. Inform. Theory*, vol. 48, no. 6, pp. 1629–1638, Jun. 2002.

[11] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 1st ed. Wiley, 1991.

[12] H. Sato, "The capacity of the Gaussian interference channel under strong interference," *IEEE Trans. Inform. Theory*, vol. 27, no. 6, pp. 786–788, Nov. 1981.

[13] T. Liu and P. Viswanath, "An extremal inequality motivated by multi-terminal information-theoretic problems," *IEEE Trans. Inform. Theory*, vol. 53, no. 5, pp. 1839–1851, May 2007.