Optimal Multi-user Spectrum Management for Digital Subscriber Lines

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***Abstract*— Crosstalk is a major issue in modern DSL systems such as ADSL and VDSL. Static spectrum management, the traditional way of ensuring spectral compatibility, employs spectral masks which can be overly conservative and lead to poor performance.**

In this paper we present a centralized algorithm for optimal spectrum management (OSM) in DSL. The algorithm uses a dual decomposition to solve the spectrum management problem in an efficient and computationally tractable way. The algorithm shows significant performance gains over existing DSM techniques, e.g. in a downstream ADSL scenario the centralized OSM algorithm can outperform a distributed DSM algorithm such as *iterative waterfilling* by up to 135%

I. Introduction

Crosstalk is a major issue in modern DSL systems such as ADSL and VDSL. Typically 10-20 dB larger than the background noise, crosstalk is *the* dominant source of performance degradation.

Whilst it is possible to do *crosstalk cancellation*[1][2], in many scenarios this may not be feasible due to complexity issues or as a result of unbundling. In this case the effects of crosstalk must be mitigated through *spectral management*. With spectral management the transmit spectra of the modems within a network are limited in some way to minimize the negative effects of crosstalk.

Static spectrum management (SSM) is the traditional approach. In SSM spectral masks are employed which are identical for all modems. To ensure widespread deployment, these masks are based on worst case scenarios[3]. As a result they can be overly restrictive and lead to poor performance.

Dynamic spectrum management (DSM), a new paradigm, overcomes this problem by designing the spectra of each modem to match the specific topology of the network[4]. These spectra are adapted based on the direct and crosstalk channels seen by the different modems. They are customized to suit each modem in each particular situation.

A DSM algorithm known as *iterative waterfilling* (IW) was recently proposed[5] and demonstrates the spectacular performance gains which are possible. An unanswered question at this point is: How much better can we do?

This work was carried out in the frame of IUAP P5/22, *Dynamical Systems and Control: Computation, Identification and Modelling* and P5/11, *Mobile multimedia communication systems and networks*; the Concerted Research Action GOA-MEFISTO-666, *Mathematical Engineering for Information and Communication Systems Technology*; IWT SOLIDT Project, *Solutions for xDSL Interoperability, Deployment and New Technologies*; FWO Project G.0196.02, *Design of efficient communication techniques for wireless time-dispersive multi-user MIMO systems* and was partially sponsored by Alcatel-Bell.

Wei Yu is supported by Bell Canada University Laboratories, Communications and Information Technology Ontario (CITO), Natural Science and Engineering Research Council (NSERC) of Canada, and the Canada Research Chairs Program.

In this paper we address this question. We focus on the problem of spectrum management where a centralized spectrum management center (SMC) is responsible for setting the spectra of the modems within the network. We present an algorithm for *optimal spectrum management* (OSM) in the DSL interference channel. This algorithm can achieve the best possible trade-offs between the rates of the modems within the network, allowing operation at any point on the rate region boundary.

The algorithm is suitable for direct application when a SMC is available. In the absence of a SMC this algorithm is also useful as it provides an upper bound on the performance of all other DSM algorithms, both centralized and distributed. The spectra generated by the algorithm also give insight into the design of distributed DSM algorithms.

One may ask, if centralized control is available (via a SMC) why not do full-blown crosstalk cancellation which leads to greater performance than with DSM alone. The fundamental difference between DSM and crosstalk cancellation is complexity. DSM involves only setting the PSD levels of currently-available modems. This can be done without any change to the modem hardware. Crosstalk cancellation uses signal level coordination, requiring an entirely new design of the *DSL access multiplexer* (DSLAM) and *customer premises* (CP) modems. DSM can potentially be applied right now, where-as it may be several years before systems with crosstalk cancellation become economically viable.

Optimal spectrum management has been investigated previously. Unfortunately the resulting optimisation is non-convex which leads to an exponential complexity in the number of tones K in the system. In ADSL $K = 256$ whilst in VDSL $K = 4096$. This results in a computationally intractable problem.

The fundamental problem is that the total power constraints on the modems couple the optimisation across frequency. As such the optimisation must be done jointly across all tones which leads to an exponential complexity in K. We overcome this problem through use of the dual decomposition method. This technique allows us to replace the constrained optimisation problem with an unconstrained maximization of a Lagrangian. The Lagrangian incorporates the constraints implicitly into the cost function, removing the need for the constraints to be explicitly enforced. As a result the optimisation can be decoupled across frequency and an optimal solution can be found in a per-tone fashion. This leads to a linear rather than exponential complexity in K and a computationally tractable problem.

In [6] an attempt was made to formulate an optimal spectrum management algorithm for VDSL based on simulated annealing. Due to the complexity of the problem the PSDs of each modem were forced to be flat within each transmission band. Only the level of the PSD in each band could be varied which led to a sub-optimal solution. Even with this restriction the algorithm had a large complexity $\mathcal{O}(e^K)$. Furthermore since the algorithm is based on simulated annealing it is not possible to guarantee that the global optimum has been obtained. Sub-optimal DSM algorithms, both distributed [5], [7], [8] and centralized [9] have also been proposed.

II. System Model

Assuming that *discrete multi-tone* (DMT) modulation is employed we can model transmission independently on each tone

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k \tag{1}$$

The vector $\mathbf{x}_k \triangleq \left[x_k^1, \cdots, x_k^N\right]$ contains transmitted signals on tone k. There are N lines in the binder and x_k^n is the signal transmitted onto line n at tone k. $\mathbf{y}_k$ and $\mathbf{z}_k$ have similar structures. $\mathbf{y}_k$ is the vector of received signals on tone k. $\mathbf{z}_k$ is the vector of additive noise on tone k and contains thermal noise, alien crosstalk, RFI etc. Recall that $1 \le k \le K$ where K is the number of tones within the system. We denote the noise PSD on line n as $\sigma_k^n \triangleq \mathcal{E}\left\{|z_k^n|^2\right\}$. $\mathbf{H}_k$ is the $N \times N$ channel transfer matrix on tone k. $h_k^{n,m} \triangleq [\mathbf{H}_k]_{n,m}$ is the channel from TX m to RX n on tone k. The diagonal elements of $\mathbf{H}_k$ contain the direct-channels whilst the off-diagonal elements contain the crosstalk channels. We denote the transmit PSD $s_k^n \triangleq \mathcal{E}\left\{|x_k^n|^2\right\}$. For convenience we denote the vector containing the PSD of user n on all tones as $\mathbf{s}_n \triangleq [s_1^n, \ldots, s_K^n]$.

We assume that each modem can only support a maximum bitloading of $b_{\max}$. $b_{\max}$ lies in the range 8-15 in current standards[10]. We also assume that modems can only support integer bitloading which is typically the case in practice. The algorithm could also be modified in a straight-forward fashion to include fractional bitloadings. Under these assumptions the achievable bitloading of user n on tone k is

$$b_k^n \triangleq \left\lfloor \log_2 \left(1 + \frac{1}{\Gamma} \frac{|h_k^{n,n}|^2 \, s_k^n}{\sum_{m \neq n} |h_k^{n,m}|^2 \, s_k^m + \sigma_k^n} \right) \right\rfloor \tag{2}$$

where $\lfloor x \rfloor$ is $\min(x, b_{\max})$ rounded down to the nearest integer. Γ is the SNR-gap to capacity and is a function of the desired BER, coding gain and noise margin. The data-rate on line n is thus

$$R_n = \sum_k b_k^n$$

III. Spectrum Management

A. The Spectrum Management Problem

We restrict our attention to the two user case for ease of explanation. Extensions to more than two users are straight-forward. The spectrum management problem for the two user case is defined as

$$\max_{\mathbf{s}_1, \mathbf{s}_2} R_2 \quad \text{s.t.} \quad R_1 \ge R_1^{\text{target}} \tag{3}$$

B. Constraints

The optimisation (3) is typically subject to a *total power constraint* on each modem

$$\sum_k s_k^n \le P_n, \; n = 1, 2 \tag{4}$$

This arises from limitations on each modem's analog front-end. *Spectral mask constraints* may also apply

$$s_k^n \le s_k^{n,\max}, \; \forall k, \; n = 1, 2 \tag{5}$$

C. Mapping from Bitloading to Powerloading

Since each modem only supports integer bitloading, we can reduce our search space to the PSDs corresponding to exact bitloadings. This reduces complexity considerably without affecting optimality. To find the PSDs corresponding to a particular bitloading we proceed as follows. Define

$$\mathbf{A} \triangleq \begin{bmatrix} 0 & \alpha_k^{1,2} \\ \alpha_k^{2,1} & 0 \end{bmatrix}$$

where $\alpha_k^{n,m} \triangleq \Gamma \, |h_k^{n,m}|^2 \, |h_k^{n,n}|^{-2}$. Also define $\boldsymbol{\sigma}_k \triangleq \Gamma[\sigma_k^1, \, \sigma_k^2]^T$ and $\Lambda_k \triangleq \operatorname{diag}\{2^{b_k^1} - 1, \, 2^{b_k^2} - 1\}$. The PSD pair required to support a particular bitloading pair b_k^1, b_k^2 is then

$$\begin{bmatrix} s_k^1 \\ s_k^2 \end{bmatrix} = \left(\mathbf{I}_N - \Lambda_k \mathbf{A}_k\right)^{-1} \Lambda_k \boldsymbol{\sigma}_k \tag{6}$$

In the following we use $s_k^n(b_k^1, b_k^2)$ to denote the PSD of user n corresponding to the bitloadings b_k^1, b_k^2 as calcu lated by (6).

At this point we could propose a simplistic algorithm to find the optimal PSDs based on an exhaustive search. For each possible bitloading pair on each tone calculate the corresponding PSD pair. Taking all possible combinations of bitloadings across all tones results in $(b_{\max} + 1)^{2K}$ possible PSD pairs. Determine the feasibility of each PSD pair based on any power constraints as described in Sec. III-B, and on the target rate constraint for user 1. Choose the PSD pair which maximizes the data-rate of user 2.

Unfortunately whilst this algorithm is simple to implement, its complexity is $\mathcal{O}((b_{\max} + 1)^{2K})$. With $K = 256$ in ADSL and $K = 4096$ in VDSL, this results in a computationally intractable problem.

D. Dual Decomposition

As we saw in the previous section, an exhaustive search for the optimal PSDs leads to a computationally intractable problem. The reason behind this is as follows. The total power constraint on each line causes the power allocation problem to become coupled across frequency. As such we must jointly search the PSDs across all tones. This results in an exponential complexity in K and an intractable problem.

To overcome this we replace the power constrained optimisation (3), with an unconstrained optimisation of a Lagrangian (9). In the Lagrangian the total power constraints are enforced through the use of the Lagrangian multipliers λ_1 and λ_2 which form part of the cost function. When λ_1 and λ_2 are chosen correctly, maximizing the Lagrangian will implicitly enforce the power constraints. The power constraints need not be explicitly enforced and the problem can be decoupled across frequency.

When the problem is decoupled we can solve the optimisation by maximizing the Lagrangian independently on each tone. This leads to a complexity which is linear rather than exponential in K and the problem becomes computationally tractable. This is the main innovation in this paper.

We begin in Sec. III-E by replacing the original optimisation problem (3), with a weighted rate-sum maximization (7). With a correctly chosen weight w, maximizing (7) implicitly enforces the target rate constraint on user 1. The weight w is in itself a form of Lagrangian multiplier.

In Sec. III-F we append the Lagrangian multipliers to the weighted rate-sum to form the Lagrangian. We will see that maximizing this Lagrangian is equivalent to solving the original optimisation problem (3). We will also see that this Lagrangian can be decoupled and maximized independently on each tone.

Using a Lagrangian to solve a constrained optimisation in an unconstrained way is a commonly used approach in convex optimisation theory and is known as the dual decomposition method. The dual decomposition has been applied in other communication problems with convex cost functions such as joint routing and resource allocation[11] and power allocation in the vector multiple access channel[12]. In this work we show that the dual decomposition method can also be applied to non-convex optimizations.

E. Rate Regions

The rate region is a plot of all possible operating points (rate pairs) that can be supported by a multi-user channel. Operating points on the boundary of the region are said to be optimal. It is our goal to find the PSDs corresponding to these points.

Theorem 1: In *convex* rate regions there exists some w such that

$$\max_{\mathbf{s}_1,\mathbf{s}_2} wR_1 + (1-w)R_2 \tag{7}$$

and (3) are equivalent. That is spectrum management is equivalent to a weighted rate-sum maximization.

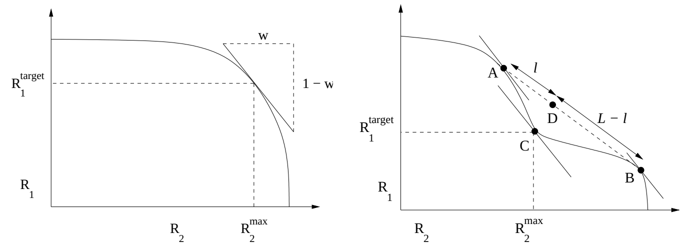
Proof: This proof will be made by illustration. Examining the convex rate region in Fig. 1 (a) we see that there is only one point which maximizes the weighted rate-sum for a given w. For this particular w we achieve a rate R_1^{target} on line 1. Since the rate region is convex, any higher R_2 necessarily leads to a smaller R_1. Thus $R_2^{\max}$ is the highest rate for line 2 which will allow the target rate for line 1 to be achieved.

Due to this one to one mapping between w and R_1^{target}, optimality in terms of (7) is a sufficient condition for optimality in terms of the original spectrum management problem (3).

Consider the converse. Examine the non-convex rate region in Fig. 1 (b). At point C the spectrum management problem (3) is solved for a particular R_1^{target}. However this point has no corresponding w for which it is optimal in terms of a weighted rate-sum. Points A and B are superior in this sense, although B does not satisfy $R_1 \geq R_1^{\text{target}}$ and A is inferior in terms of R_2. So in non-convex rate regions not all points can be expressed as weighted rate-sum optimizations. ■

In the wireline medium there is some correlation between the channels on neighbouring tones. If we sample the channel finely enough then neighbouring tones will see almost the same channels (both direct and crosstalk).

Imagine that the tone spacing is fine enough such that $h_k^{n,m} \simeq h_{k+l}^{n,m},\ 0 \leq l \leq L-1$. Consider two points in the rate region, $A = (R_1^a, R_2^a)$ and $B = (R_1^b, R_2^b)$ and their



(a) Convex (b) Non-convex

Fig. 1. Rate Region Examples

corresponding PSDs $(s_k^{1,a}, s_k^{2,a})$ and $(s_k^{1,b}, s_k^{2,b})$. It is possible to operate at a point $D = (\frac{l}{L}R_1^a + \frac{L-l}{L}R_1^b, \frac{l}{L}R_2^a + \frac{L-l}{L}R_2^b)$ for any $0 \leq l \leq L-1$. This is done by setting the PSDs to $(s_k^{1,a}, s_k^{2,a})$ on tones $k \in \{pL+1, \ldots pL+l\}$ for all integer values of p, and to $(s_k^{1,b}, s_k^{2,b})$ on all other tones.

For example, to operate at a point 2/3 between A and B (on the side closer to A), we require $l = 2$ and $L = 3$. Thus we set the PSDs to $(s_k^{1,a}, s_k^{2,a})$ on tones $k \in \{1, 2, 4, 5, 7, 8, \ldots, K\}$ and to $(s_k^{1,b}, s_k^{2,b})$ on tones $k \in \{3, 6, 9, \ldots, K\}$. For this to work the tone spacing must be small enough such that the channel is approximately flat over $L = 3$ neighbouring tones. That is, we must have $h_k^{n,m} \simeq h_{k+1}^{n,m} \simeq h_{k+2}^{n,m},\ \forall k \in \{1, 4, \ldots, K\}$.

For large L (small tone spacing), practically any operating point between A and B can be achieved. Thus for any two points in the rate region, any point between them is also within the rate region. This is the definition of a convex set. As such the rate region is convex in DMT systems with small tone spacings. In ADSL and VDSL the tone spacing is 4.3125 kHz. In both measured and empirical wireline channels we have found this tone spacing to be small enough such that the rate regions are convex.

F. The Lagrangian

We can incorporate the total power constraints (4) into the optimization problem by defining the Lagrangian

$$L \triangleq wR_1 + (1-w)R_2 + \lambda_1(P_1 - \sum_k s_k^1) + \lambda_2(P_2 - \sum_k s_k^2) \tag{8}$$

Here λ_n is the Lagrangian multiplier for user n and is chosen such that either the power constraint on user n is tight $\sum_k s_k^n = P_n$ or $\lambda_n = 0$. The constrained optimization (7) can now be solved via the unconstrained optimization

$$\max_{\mathbf{s}_1,\mathbf{s}_2} L(w, \lambda_1, \lambda_2, s_k^1, s_k^2) \tag{9}$$

Define the Lagrangian on tone k

$$L_k \triangleq wb_k^1 + (1-w)b_k^2 - \lambda_1 s_k^1(b_k^1, b_k^2) - \lambda_2 s_k^2(b_k^1, b_k^2)$$

Clearly the Lagrangian (8) can be decomposed into a sum across tones of L_k and a term which is independent of s_k^1 and s_k^2

$$L = \sum_k L_k + \lambda_1 P_1 + \lambda_2 P_2$$

As a result we can split the optimization into K per-tone optimizations which are coupled only through w, λ_1 and λ_2.

IV. Optimal Spectrum management

The *optimal spectrum management* (OSM) algorithm is listed as Alg. 1. Spectral mask constraints can be incorporated into the optimisation by setting L_k to $-\infty$ if $s_k^1 > s_k^{1,\max}$ or $s_k^2 > s_k^{2,\max}$.

The algorithm operates as follows. We need to search through both λ_1 and λ_2 to find values which place sufficient importance on the total power constraint terms within the Lagrangian (8). We must also search through w to find the value which achieves the right trade-off between the rates of the two users, thereby maximizing the rate of user 2 whilst still achieving the target rate of user 1. The algorithm contains three loops, an outer loop which searches for w, an intermediate loop which searches for λ_1 and an inner loop which searches for λ_2. Bisection is used in each of these searches.

When searching for λ_n, we first find a value of λ_n which ensures that the power constraint of user n is satisfied. This value is stored in $\lambda_n^{\max}$. Note that a larger λ_n places more emphasis on the power constraint of user n in the Lagrangian. As a result, using a larger λ_n will result in a lower total power for user n.

Once $\lambda_n^{\max}$ is found the algorithm proceeds to bisection. Note that after the algorithm has completed, for each user either $\sum_k s_k^n = P_n$ or the corresponding Lagrangian multiplier is driven to zero ($\lambda_n = 0$). Thus the Lagrangian and the original objective become equivalent. More rigorously,

Theorem 2: For convex rate regions Alg. 1 converges. At convergence Alg. 1 yields the optimal PSDs for the spectrum management problem (3), that is

$$\begin{aligned} \mathbf{s}_1, \mathbf{s}_2 &= \arg\max_{\mathbf{s}_1,\mathbf{s}_2} R_2 \\ \text{s.t.} \quad & R_1 \geq R_1^{\text{target}} \\ & \sum_k s_k^n \leq P_n, \ \forall n \end{aligned} \tag{10}$$

Proof: See [13] ∎

Note that by solving the optimization independently on each tone we require only $K(b_{\max}+1)^2$ evaluations of L_k each time the function *optimize_s* is called, so the complexity becomes linear rather than exponential in K. In contrast solving the problem jointly across all tones would have required $(b_{\max}+1)^{2K}$ evaluations of $s_k^1(b_k^1, b_k^2)$ and $s_k^2(b_k^1, b_k^2)$ which is computationally intractable.

In this paper we have only presented the algorithm and optimality proof for 2 user channels. Extensions to more than 2 users are straight-forward and follow naturally from the algorithm and proof presented here.

Algorithm 1 Optimal Spectrum Management

Main Function

$w_{\min} = 0$, $w_{\max} = 1$
while $|R_1 - R_1^{\text{target}}| > \epsilon$
  $w = (w_{\max} + w_{\min})/2$
  $\mathbf{s}_1, \mathbf{s}_2$ = optimize_$\lambda_1(w)$
  if $R_1(\mathbf{s}_1, \mathbf{s}_2) > R_1^{\text{target}}$, then $w_{\max} = w$, else $w_{\min} = w$
end

Function $\mathbf{s}_1, \mathbf{s}_2$ = ***optimize_***$\lambda_1(w)$
$\lambda_1^{\max} = 1$, $\lambda_1^{\min} = 0$
while $\sum_k s_k^1 > P_1$
  $\lambda_1^{\max} = 2\lambda_1^{\max}$
  $\mathbf{s}_1, \mathbf{s}_2$ = optimize_$\lambda_2(w, \lambda_1^{\max})$
end
repeat
  $\lambda_1 = (\lambda_1^{\max} + \lambda_1^{\min})/2$
  $\mathbf{s}_1, \mathbf{s}_2$ = optimize_$\lambda_2(w, \lambda_1)$
  if $\sum_k s_k^1 > P_1$, then $\lambda_1^{\min} = \lambda_1$, else $\lambda_1^{\max} = \lambda_1$
until convergence

Function $\mathbf{s}_1, \mathbf{s}_2$ = ***optimize_***$\lambda_2(w,\lambda_1)$
$\lambda_2^{\max} = 1$, $\lambda_2^{\min} = 0$
while $\sum_k s_k^2 > P_2$
  $\lambda_2^{\max} = 2\lambda_2^{\max}$
  $\mathbf{s}_1, \mathbf{s}_2$ = optimize_$s(w, \lambda_1, \lambda_2^{\max})$
end
repeat
  $\lambda_2 = (\lambda_2^{\max} + \lambda_2^{\min})/2$
  $\mathbf{s}_1, \mathbf{s}_2$ = optimize_$s(w, \lambda_1, \lambda_2)$
  if $\sum_k s_k^2 > P_2$, then $\lambda_2^{\min} = \lambda_2$, else $\lambda_2^{\max} = \lambda_2$
until convergence

Function $\mathbf{s}_1, \mathbf{s}_2$= ***optimize_***$s(w, \lambda_1, \lambda_2)$
for $k = 1 \ldots K$
  $b_k^1, b_k^2 = \arg\max_{b_k^1, b_k^2} L_k(b_k^1, b_k^2, w, \lambda_1, \lambda_2)$
  $s_k^1 = s_k^1(b_k^1, b_k^2)$, $s_k^2 = s_k^2(b_k^1, b_k^2)$
end

V. Performance

We now examine the performance of OSM when compared with other spectrum management techniques. We simulate downstream transmission in ADSL with a 5 km CO distributed line and a 3 km RT distributed line. The RT is located 4 km from the CO as depicted in Fig. 2. A maximum transmit power of 20.4 dBm is applied to each modem. The usual PSD constraint is not applied. Background noise includes crosstalk from 10 ISDN, 4 HDSL, and 10 SSM (legacy) ADSL disturbers. We use 0.5 mm (24-Gauge) lines and the target symbol error probability is 10^{-7} or less. The coding gain and noise margin are set to 3 dB and 6 dB respectively.

Fig. 3 shows the rate regions corresponding to various spectrum management algorithms. For comparison the rate regions with IW and flat PBO are shown. No PBO method for RT distributed ADSL modems has been defined in standardization and this is still an open issue[3].

Examining the PSDs derived with the OSM algorithm allows us to gain some intuition into how it operates. The PSDs corresponding to a 1 Mbps service on the CO distributed line are depicted in Fig. 4 and Fig. 5. The main goal is to protect the performance of the CO line. Crosstalk coupling increases with frequency. As a result we see that the PSD on the RT line decreases with frequency to protect the CO. In the high frequencies above 430 kHz the CO line cannot reliably communicate even in the absence of crosstalk due to its large direct channel attenuation. As a result it is not necessary for the RT to do PBO in frequencies above 430 kHz and we see a sudden increase in the PSD on the RT line.

As shown in Tab. I using OSM instead of IW allows us to increase the data-rate on the RT distributed line from 3.1 Mbps to 7.3 Mbps whilst still maintaining a 1 Mbps service on the CO distributed line.

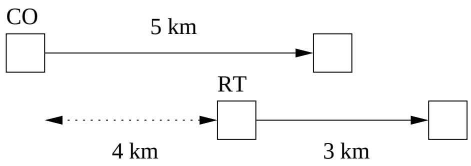


Fig. 2. Downstream ADSL Scenario

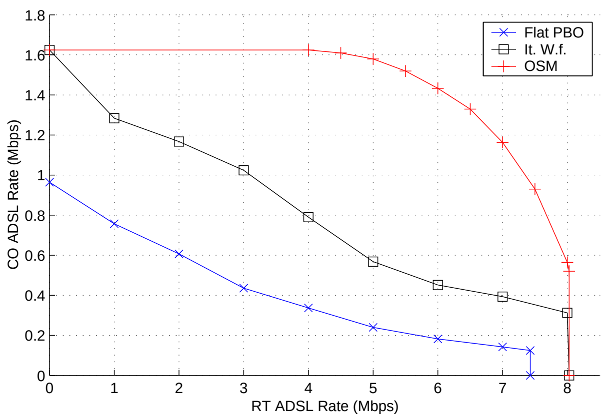


Fig. 3. Rate Regions

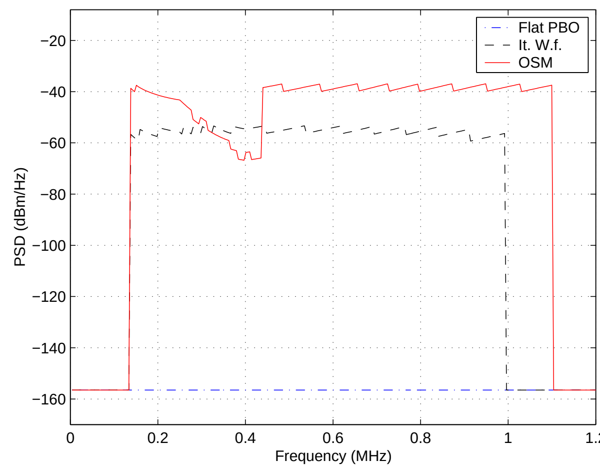


Fig. 5. PSDs on RT line (CO Line @ 1 Mbps)

Scheme	CO Rate	RT Rate
Flat PBO	1.0 Mbps	0.0 Mbps
IW	1.0 Mbps	3.1 Mbps
OSM	1.0 Mbps	7.3 Mbps

TABLE I
ACHIEVABLE RATES

VI. CONCLUSIONS

In this paper we presented an algorithm for optimal spectrum management (OSM) in DSL. This algorithm calculates the spectra required for the modems within a network to achieve maximal performance, thereby operating on the rate region boundary. The algorithm can operate under a combination of total power and/or spectral mask constraints.

Through the use of a dual decomposition the algorithm solves the spectrum management problem independently on each tone. The result is a computationally tractable and efficient algorithm.

Simulations show that the algorithm yields significant gains over existing spectrum management techniques, e.g. in a downstream ADSL scenario the OSM algorithm can outperform another DSM algorithm *iterative waterfilling* by up to 135%.

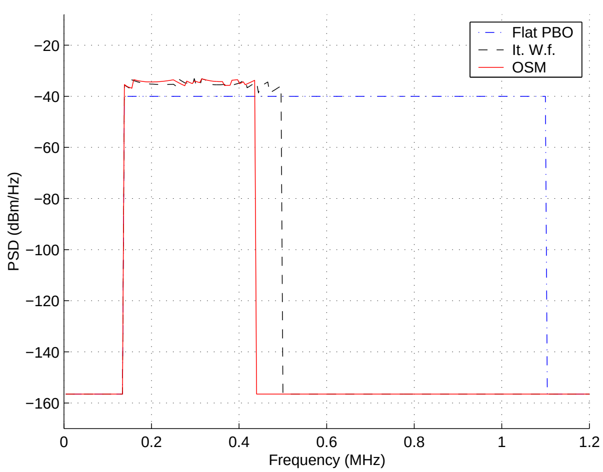


Fig. 4. PSDs on CO line (CO Line @ 1 Mbps)

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