Joint Scheduling and Dynamic Power Spectrum Optimization for Wireless Multicell Networks

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***Abstract*— This paper proposes a joint proportionally fair scheduling and dynamic power spectrum adaptation algorithm for wireless multicell networks. The proposed system allows multiple base-stations in a multicell network to be coordinated by exchanging interference pricing messages among each other. The messages summarize the effect of intercell interference, and they are functions of transmit power spectra, signal-to-noise ratios, direct and interfering channel gains, and the proportional fairness variables for each user. The use of interference pricing allows the transmit power spectra and user schedule within each base-station to be optimized jointly, while taking into consideration both the intercell interference and the fairness among the users in multiple cells. This paper proposes two power spectrum optimization methods, one based on the Karush-Kuhn-Tucker (KKT) condition of the optimization problem, and another based on the Newton's method. The proposed methods can achieve a throughput improvement of 40%-55% for users at the cell edge as compared to a conventional per-cell optimized system, while maintaining proportional fairness.**

I. Introduction

The capacity of wireless cellular networks is fundamentally limited by intercell interference. Traditional cellular networks manage interference with specific frequency reuse patterns so that nearby cell-edge users belonging to neighboring cells do not share the same frequency. However, as bandwidth becomes increasingly scarce and as modern cellular networks move increasingly toward the maximal frequency reuse of 1, where all the cells share the same frequency, the management of intercell interference via dynamic spectrum optimization is expected take a central role in future networks. The aim of this paper is to study adaptive scheduling, power control, and bandwidth allocation methods for interference mitigation.

Traditional cellular networks are designed on a per-cell basis. Transmitters in such a network typically operate at fixed power-spectrum density (PSD) levels, while receivers typically assume the worst-case interference.

This paper envisions an advanced wireless cellular network in which base-stations cooperate with each other in the joint scheduling of users and in the joint optimization of transmit power spectra over the frequencies for all users across the cells for interference mitigation. Base-station coordination can be realized by the exchange of messages among the base-stations. The main contribution of this paper is a set of numerical power spectrum optimization methods based on a specific type of interference pricing messages. These messages reveal the effect of interference among neighboring cells. They also help maintain fairness among all users across the cells, and allow frequency allocation and power spectrum adaptation to be done in a distributed fashion.

A. System Model

Consider a wireless multicell network as shown in Fig. 1, where users within each cell are separated from each other using orthogonal frequency division multiple access (OFDMA) over a fixed bandwidth. The frequency assignments for users within each cell are non-overlapping. Thus, users experience intercell interference only and no intracell interference.

The system is assumed to employ an initial channel estimation and synchronization phase, in which the multipath fading channel information is obtained between every pair of transmitter and receiver in the network, including both uplink and downlink direct channels within each cell as well as the interfering channels between the base-stations and the remote terminals in neighboring cells. For example, downlink channel estimation may be performed using a scheme in which signature sequences are synchronously transmitted by all base-stations. The mobile users may then estimate all the downlink channels at the same time by matching to the different sequences. In time-division duplex (TDD) systems, the uplink channel may be inferred from the downlink channel.

The joint scheduling and power spectrum adaptation problem can be formulated as that of deciding for each user, which frequency tones they should transmit in, and at what transmit power level. Equivalently, the base-stations can be thought of making two decisions at each frequency tone:

- Scheduling: Which user should be served?
- Power spectrum allocation: What is the appropriate power spectral density level at this frequency tone?

To obtain a network-wide global optimal solution, these two questions must be answered jointly and across the base-stations. Further, the same optimization problem must be solved for uplink and downlink separately, and repeatedly as channels vary over time.

The service objective of a wireless cellular network is to provide the highest overall throughput to all users while

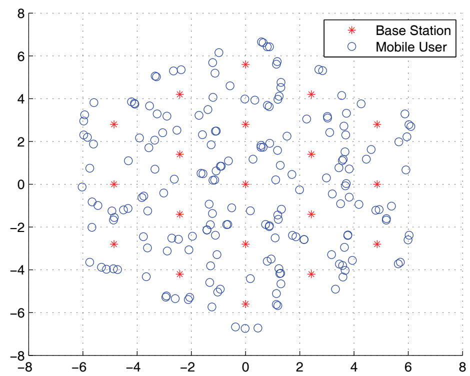

Fig. 1. A multicell network with 19 cells and with users distributed close to the cell edge.

maintaining fairness. Fairness is important in an interference-limited environment as one user's gain often comes at the expense of other users. This paper assumes a *proportionally fair* objective among all users in all cells. Specifically, the optimization objective is to solve

$$\max \sum_{lk} T_{lk} \log(\bar{R}_{lk}) \tag{1}$$

where $\bar{R}_{lk}$ is the long term average rate of the kth user in the lth cell, and T_{lk}'s are a set of weights corresponding to the quality-of-service target for the kth user in the lth cell. The logarithmic function is an example of a utility function, which provides a balance between the achievable rates among users.

B. Related Work

Conventional cellular networks are designed as single-cell systems—multicell coordination is limited to the use of soft handoff. In a single-cell design, scheduling and power spectrum adaptation can be considered separately. For example, in proportionally fair scheduling as originally proposed in [1], transmit power is assumed fixed. The scheduling algorithm then aims to find the best user to transmit at each timeslot to take advantage of multiuser diversity. Power spectrum adaptation is done in addition and independently of scheduling.

Power spectrum adaptation becomes much more challenging in a multicell setting. In this context, a lot of work [2], [3], [4], [5], [6] has been done in the digital subscriber lines (DSL) setting, where each "cell" has only one user. In this case, scheduling is not an issue; the main challenge is to find computationally efficient methods for optimizing power allocation across the frequencies. Particularly relevant to this paper are the works [5], [6] which use the idea of interference pricing for spectrum balancing. Interference pricing is a key concept which is also used in the design of power spectrum adaptation algorithms proposed in this paper.

The concept of interference pricing first appears in the literature for wireless ad-hoc networks with multiple transmitter-receiver pairs sharing the same physical medium [7], [8], [9], [10], [11], where again the focus is on power spectrum adaptation and not user scheduling. In the cellular network context, [12], [13] deal with a single-cell problem and propose a solution to the joint scheduling and power allocation problem via an integer relaxation method. For the multicell networks, [14] considers an idealized network and proposes a multicell scheduling algorithm.

The proposed system is related to [15] where power spectrum adaptation methods in the DSL domain (e.g. [3], [4], [5]) are applied to the wireless setting. The main contribution of [15] is that it proposes a method of iterative optimization of user scheduling and power adaptation for maximizing the weighted rate sum over all users in all cells. However, the weights are fixed in [15]. This paper proposes automatic and continuous adjustment of weights based on the proportional fairness criterion. In addition, this paper proposes novel power spectrum adaptation methods with faster convergence.

The proposed system is also closely related to [16], which considers a proportionally fair joint scheduler and intercell power allocation scheme. The main difference of this paper as compared to [16] is that [16] takes a gradient approach for power allocation, while the system proposed in this paper takes into account the second derivative information in addition, and therefore has faster convergence.

II. Joint Scheduling and Power Spectrum Optimization

A. Problem Formulation

Consider a multicell environment with L cells and K users per cell employing an OFDMA scheme with N tones over a fixed bandwidth. Both the base-station and the remote users are equipped with a single antenna only. For simplicity, we assume channel reciprocity and let h^n_{lmk} denote the channel response between the lth base-station and the kth remote user in mth cell in nth tone for both uplink and downlink. The system allows only a single user to transmit or to receive at any given tone in a cell. The uplink and downlink user schedules are determined by the assignment functions $f_U(l,n)$ and $f_D(l,n)$, which assign a (possibly different) user k to the lth cell in the nth tone for the uplink and downlink, respectively. The uplink and downlink transmit PSDs are denoted as $P^n_{U,l}$ and $P^n_{D,l}$, respectively. The uplink and downlink are separated via TDD.

For the downlink, the proportionally fair joint scheduling and transmit power-spectrum adaptation problem is that of choosing the scheduling function $k = f_D(l,n)$ and the transmit power spectrum $P^n_{D,l}$ over the frequencies and across all the cells to maximize (1), i.e.

$$\begin{aligned}
\max \quad & \sum_{l,k} T_{D,lk} \log\left(\bar{R}_{D,lk}\right) \\
\text{s.t.} \quad & R_{D,lk} = \sum_{n \in \mathcal{D}_{lk}} \log\left(1 + \frac{P^n_{D,l}|h^n_{llk}|^2}{\Gamma(\sigma^2 + \sum_{j \neq l} P^n_{D,j}|h^n_{jlk}|^2)}\right) \\
& \bar{P}_{D,l} \leq P^{max}_{D,l} \qquad \forall l \\
& 0 \leq P^n_{D,l} \leq S^{max}_D \qquad \forall l, n
\end{aligned} \tag{2}$$

where the summation in the expression for $R_{D,lk}$ is over all frequency tones assigned to the kth user in the lth cell, i.e. $n \in \mathcal{D}_{lk}$ where $\mathcal{D}_{lk} = \{n | k = f_D(l, n)\}$. Here, $R_{D,lk}$ is the instantaneous downlink rate and $\bar{R}_{D,lk}$ is the time averaged rate for the kth user in lth cell. Further, $\bar{P}_{D,lk}$ denotes the time averaged downlink total transmit power (summed across the frequencies), which needs to satisfy a maximum power constraint $P_{D,l}^{max}$; Γ is the SNR gap corresponding to the choice of modulation and coding schemes; σ^2 is the background noise; S_D^{max} is the transmit PSD constraint at the base-stations. We note that the uplink problem can be stated similarly.

The optimization problem (2) is a mixed discrete (user scheduling) and continuous (power allocation) optimization problem. This paper provides a local optimum solution for (2) using an approach (as in [15]) that iterates between the scheduling and power allocation steps. In the scheduling step, power allocation is assumed to be fixed. In the power allocation step, scheduling is assumed to be fixed.

B. Proportionally Fair Scheduling

Proportionally fair scheduling [1] is widely used for wireless systems. The main idea is to use a scheduling policy which is a function of the current average rates of the users. For single-cell systems, a proportionally fair scheduler selects the user

$$k^* = \text{argmax} \frac{R_k}{\bar{R}_k} \tag{3}$$

in each time epoch, where R_k is the user's requested rate if it were scheduled, and $\bar{R}_k$ is an exponentially weighted average rate for the kth user. This scheduling policy maximizes log utility, because $\frac{1}{\bar{R}_k}$ can be thought of as the marginal increase in the utility function $\log(\bar{R}_k)$ for the kth user.

This same idea can be applied to the multiuser setting, as commonly done in the network utility maximization literature (e.g. [12], [16]). In the multiuser case, the instantaneous rates for all users in all cells form a capacity region—tradeoffs between the individual rates within the region are possible.

The idea of proportional fair scheduling can also be applied to the multicell setting in the downlink. The key observation that allows this to be done is that in the downlink, the interference produced by each base-station is a function of the transmit power spectral density only and is independent of the user assignment at the base-station. Thus, if the power allocation $P_{D,l}^n$ is fixed, user scheduling can be done independently on a per-cell basis without affecting the interference level elsewhere in the network.

For a given cell l, in order to maximize the marginal increase in the proportional fairness utility $\sum_k T_{D,lk} \log(\bar{R}_{D,lk})$, the user scheduling algorithm should maximize the weighted rate sum with weights set as the derivative of the utility function:

$$\max \sum_k \left(\frac{T_{D,lk}}{\bar{R}_{D,lk}} \right) R_{D,lk}. \tag{4}$$

Since $R_{D,lk}$ is the sum of bit rates across the frequency tones, the above maximization decouples in a tone-by-tone basis. Now, because only one user can be active in a given tone, the user scheduling algorithm should assign user k in each tone n as

$$f_D(l,n) = \text{argmax}_k \left\{ \frac{T_{D,lk}}{\bar{R}_{D,lk}} \log \left(1 + \frac{P_{D,l}^n |h_{llk}^n|^2}{\Gamma(\sigma^2 + \sum_{j \neq l} P_{D,j}^n |h_{jlk}^n|^2)} \right) \right\}. \tag{5}$$

In other words, the scheduler simply chooses k in each tone such that the weighted instantaneous rate is maximized. The weights are computed as $\left(\frac{T_{D,lk}}{\bar{R}_{D,lk}}\right)$ with

$$\bar{R}_{D,lk} = \alpha \bar{R}_{D,lk} + (1-\alpha) R_{D,lk} \tag{6}$$

where $0 < \alpha < 1$ is a forgetting factor, and the instantaneous rate $R_{D,lk}$ is computed from the fixed power spectrum allocation as in (2).

Note that the observation that the interference level in the network is independent of the user schedule is true for the downlink only, and is not true for the uplink. Thus, applying proportionally fair scheduling to the uplink requires coordination across the cells. To avoid excessive coordination, this paper proposes a heuristic approach that uses the same scheduling policy for both uplink and downlink, i.e.

$$f_U(l,n) = f_D(l,n). \tag{7}$$

This clearly is a suboptimal choice, but it can be roughly justified for TDD systems where uplink and downlink channels are reciprocals of each other using a fact known as uplink-downlink duality, i.e. under the same sum power constraint, the rate regions of the uplink and downlink are the same. Although the sum power constraint does not exactly correspond to the problem setup here, system-level simulation indicates that this approach is reasonable.

C. Power Spectrum Adaptation

The power spectrum adaptation step assumes a fixed user schedule and finds the optimal transmit power spectrum in both uplink and downlink. The objective is again proportional fairness, thus the maximization of the marginal increase in $\sum_{l,k} T_{D,lk} \log(\bar{R}_{D,lk})$ is equivalent to

$$\max \sum_{l,k} \left(\frac{T_{D,lk}}{\bar{R}_{D,lk}} \right) R_{D,lk} \tag{8}$$

for the downlink, and likewise for the uplink. The above step transforms the problem into a weighted rate sum maximization problem with weights set as

$$w_{D,lk} = \frac{T_{D,lk}}{\bar{R}_{D,lk}} \tag{9}$$

These weights automatically adapt to the channel condition and the user scheduling; they provide a natural way of balancing the competing rate requirements from different users.

The transformation of the log-utility maximization to a weighted rate-sum maximization is crucial, as it allows the optimization problem to be decoupled on a tone-by-tone basis using a dual optimization approach [2], [3]. More specifically, for the downlink, by dualizing with respect to the total power constraint, the weighted rate-sum maximization is

$$\begin{aligned}\max \quad & \sum_{l,k} w_{D,lk} \sum_{n:\ k=f_D(l,n)} \log \\ & \left(1+\frac{P^n_{D,l}|h^n_{llk}|^2}{\Gamma(\sigma^2+\sum_{j\neq l} P^n_{D,j}|h^n_{jlk}|^2)}\right) - \sum_l \lambda_{D,l}\bar{P}_{D,l} \\ \text{s.t.} \quad & 0 \le P^n_{D,l} \le S^{max}_D \qquad \forall l,n. \end{aligned} \tag{10}$$

which decouples into N independent optimization problems, one per each tone $n = 1, \cdots, N$:

$$\begin{aligned}\max \quad & \sum_l w_{D,lk} \log\left(1+\frac{P^n_{D,l}|h^n_{llk}|^2}{\Gamma(\sigma^2+\sum_{j\neq l} P^n_{D,j}|h^n_{jlk}|^2)}\right) \\ & -\lambda_{D,l}P^n_{D,l} \\ \text{s.t.} \quad & 0 \le P^n_{D,l} \le S^{max}_D \qquad \forall l. \end{aligned} \tag{11}$$

where $k = f_D(l,n)$. This per-tone problem has L variables, instead of the NL variables as the case for (10). Thus, it is much more manageable.

Note that the purpose of $\lambda_{D,l}$ is to ensure that the power constraint is satisfied. An appropriate $\lambda_{D,L}$ can be found using a subgradient approach. For the remaining of this section, we focus on the numerical solution to (11) for fixed $\lambda_{D,l}$.

1) KKT Method: The objective in (11) is a well known nonconvex function for which finding the global optimum solution is believed to be difficult. This paper focuses on iterative approaches to achieve at least a local optimum solution. Our first idea is to look at its Karush-Kuhn-Tucker (KKT) condition, i.e. take the derivative of the objective function with respect to $P^n_{D,l}$ and set it to zero:

$$\frac{w_{D,lk}|h^n_{llk}|^2}{P^n_{D,l}|h^n_{llk}|^2+\Gamma(\sigma^2+\sum_{j\neq l} P^n_{D,j}|h^n_{jlk}|^2)} = \sum_{j\neq l} t^n_{D,jl} + \lambda_{D,l}, \tag{12}$$

with $k = f_D(l,n)$, for $l = 1, \cdots, L$, where

$$t^n_{D,jl} = w_{D,jk'} \frac{\Gamma|h^n_{ljk'}|^2}{P^n_{D,j}|h^n_{jjk'}|^2}\left(\frac{(\mathrm{SINR}^n_{D,j})^2}{1+\mathrm{SINR}^n_{D,j}}\right), \tag{13}$$

and

$$\mathrm{SINR}^n_{D,j} = \frac{P^n_{D,j}|h^n_{jjk'}|^2}{\Gamma(\sigma^2+\sum_{i\neq j} P^n_{D,i}|h^n_{ijk'}|^2)}, \tag{14}$$

with $k' = f_D(j,n)$. The KKT condition (12) is essentially a water-filling condition if the terms $t^n_{D,jl}$ are held fixed. In this case, (12) gives the following power update equation, which we call the "KKT method": (see also [5])

$$P^n_{D,l,new} = \left[\frac{w_{D,lk}}{\sum_{j\neq l} t^n_{D,jl}+\lambda_{D,l}} - \frac{\Gamma(\sigma^2+\sum_{j\neq l} P^n_{D,j}|h^n_{jlk}|^2)}{|h^n_{llk}|^2}\right]_0^{S^{max}_D} \tag{15}$$

where $k = f_D(l,n)$, and the notation $[x]^b_a$ denotes x upper bounded above by b and lower bounded below by a. The second term in the right-hand side of (15) is the effective combined downlink noise and interference in the nth tone of the lth base-station, which can be measured at the remote terminal locally. Thus, to compute (15), the base-station only has to know $t^n_{D,jl}$. In this paper, we propose to pass $t^n_{D,jl}$ as messages from neighboring base-stations. In this case, $P^n_{D,l,new}$ can be effectively computed in an iterative process. Note that the computation of $t^n_{D,jl}$ requires not only the proportional fairness weights, the transmit power and the SINR, but also the ratio of the direct and the interfering channel gains, which has to be estimated in the initialization phase.

The terms $t^n_{D,jl}$ have a pricing interpretation, which comes from the fact that $t^n_{D,jl}$ is the derivative of the jth base-station's data rate with respect to the lth base-station's power, weighted by the proportional fairness variable, i.e. $-w_{D,jk'}\partial R_{D,jk'}/\partial P_{D,l}$, where $k' = f_D(j,n)$. It summarizes the interfering effect of $P^n_{D,l}$ on the jth neighbor.

For practical implementation, the update according to (15) may be too aggressive, and it may lead to non-convergence. We propose a damped iteration where the next iteration of $P^n_{D,l}$ is set as follows in a dB scale:

$$\begin{aligned} & 10\log_{10}(P^n_{D,l}[\kappa+1]) = \\ & \quad \gamma 10\log_{10}(P^n_{D,l,new}) + (1-\gamma)10\log_{10}(P^n_{D,l}[\kappa]) \end{aligned} \tag{16}$$

where the index κ denotes the iteration number, and $0 < \gamma < 1$. In practice, $\gamma = 0.5$ is found to work well.

The implementation of the algorithm depends critically on the availability of the pricing messages $t^n_{D,jl}$. As messages are often updated asynchronously, computing the sum may incur delay. Observe that since only the sum of $t^n_{D,jl}$ enters the computation, it is sensible to approximate the sum by $\max_{j\neq l}\{t^n_{D,jl}\}$. To compensate for the fact that maximum is strictly less than the sum, we propose to adjust the maximum by a constant factor c, in which case the update becomes

$$P^n_{D,l,new} = \left[\frac{w_{D,lk}}{c\cdot\max_{j\neq l}\{t^n_{D,jl}\}+\lambda_{D,l}} - \frac{\Gamma(\sigma^2+\sum_{j\neq l} P^n_{D,j}|h^n_{jlk}|^2)}{|h^n_{llk}|^2}\right]_0^{S^{max}_D} \tag{17}$$

which we call the "max-price KKT" method. In practice, $c = 2$ is found to work well.

To summarize, to solve (11), we propose to start with the current power allocations $\{P^n_{D,1}, P^n_{D,2}, \cdots P^n_{D,L}\}$, and update the power according to (16) and (15) or (17). The process iterates until convergence or until a maximum number of iterations is reached.

2) Newton's Method: We now propose a second method for solving (11) which has a faster convergence speed than the KKT method. The idea is to do a distributed Newton's search directly on the objective function of (11). Let

$$r^n_{D,lk} = \log\left(1+\frac{P^n_{D,l}|h^n_{llk}|^2}{\Gamma(\sigma^2+\sum_{j\neq l} P^n_{D,j}|h^n_{jlk}|^2)}\right), \tag{18}$$

where $k = f_D(l,n)$. Then,

$$\frac{\partial r^n_{D,lk}}{\partial P^n_{D,l}} = \frac{1}{P^n_{D,l}}\left(1+\frac{1}{\text{SINR}^n_{D,l}}\right), \quad (19)$$

$$\frac{\partial r^n_{D,jk}}{\partial P^n_{D,l}} = \frac{-\Gamma|h^n_{ljk'}|^2}{P^n_{D,j}|h^n_{jjk'}|^2}\cdot\frac{(\text{SINR}^n_{D,j})^2}{1+\text{SINR}^n_{D,j}}, \quad (20)$$

and

$$\frac{\partial^2 r^n_{D,lk}}{\partial (P^n_{D,l})^2} = \frac{-1}{\left(P^n_{D,l}\right)^2}\left(1+\frac{1}{\text{SINR}^n_{D,l}}\right)^2 \quad (21)$$

$$\frac{\partial^2 r^n_{D,jk}}{\partial (P^n_{D,l})^2} = \frac{(\Gamma|h^n_{ljk'}|^2)^2}{(P^n_{D,j}|h^n_{jjk'}|^2)^2}\frac{(\text{SINR}^n_{D,j})^3(2+\text{SINR}^n_{D,j})}{(1+\text{SINR}^n_{D,j})^2} \quad (22)$$

where again $k = f_D(l,n)$, $k' = f_D(j,n)$, and $j \neq l$.

The idea is to improve the objective function (11)

$$g(P^n_{D,1},\cdots,P^n_{D,L}) = \sum_l w_{D,lk} r^n_{D,lk} - \lambda_{D,l}P^n_{D,l} \quad (23)$$

by incrementing the transmit power $(P^n_{D,1},\cdots,P^n_{D,L})$ in a Newton's direction. The Newton's direction is

$$[\Delta P^n_{D,1},\cdots,\Delta P^n_{D,L}] = -(\nabla^2 g)^{-1}\nabla g. \quad (24)$$

In practice, inverting the Hessian matrix $\nabla^2 g$ is computationally expensive. One trick [17] is to ignore the off-diagonal terms of the Hessian, and to invert the diagonal terms only, i.e.

$$\Delta P^n_{D,l} = -\frac{(\nabla g)_l}{(\nabla^2 g)_{ll}}. \quad (25)$$

However, the above method works only if the objective function g is concave, in which case $(\nabla^2 g)_{ll}$ is negative, and $\Delta P^n_{D,l}$ always increases in the direction of the gradient $(\nabla g)_l$. As the objective function of (11) is not concave, the $\Delta P^n_{D,l}$ above does not necessarily give an increment direction (see e.g. [18]). For the proposed system, we modify the search direction as follows:

$$\Delta P^n_{D,l} = \frac{(\nabla g)_l}{|(\nabla^2 g)_{ll}|}. \quad (26)$$

This heuristics works very well in practice.

Now, the lth element of the gradient vector is:

$$(\nabla g)_l = \frac{w_{D,lk}}{P^n_{D,l}}\left(1+\frac{1}{\text{SINR}^n_{D,l}}\right) - \sum_{j\neq l} t^n_{D,jl} - \lambda_{D,l}. \quad (27)$$

Likewise, the lth diagonal term of the Hessian matrix is:

$$(\nabla^2 g)_{ll} = \frac{-w_{D,lk}}{\left(P^n_{D,l}\right)^2}\left(1+\frac{1}{\text{SINR}^n_{D,l}}\right)^2 + \sum_{j\neq l} w^n_{D,jk}$$
$$\left(\frac{\Gamma|h^n_{ljk'}|^2}{P^n_{D,j}|h^n_{jjk'}|^2}\right)^2\cdot\frac{(\text{SINR}^n_{D,j})^3(2+\text{SINR}^n_{D,j})}{(1+\text{SINR}^n_{D,j})^2} \quad (28)$$

Substituting (27)-(28) into (26) gives the "Newton's method".

Note that in order to implement the above Newton's method in a distributed fashion, the base-stations need to pass not only the pricing messages $t^n_{D,jl}$ in (27), but also the additional terms in (28). To facilitate distributed implementation with messages $t^n_{D,jl}$ only, we propose a further modification of the Hessian that includes the first term of (28) only. Finally, as in KKT method, we replace $\sum_{j\neq l} t^n_{D,jl}$ by $c \cdot \max_{j\neq l}\{t^n_{D,jl}\}$. These modifications lead to the following update equation:

$$\Delta P^n_{D,l} = \frac{\frac{w_{D,lk}}{P^n_{D,l}}\left(1+\frac{1}{\text{SINR}^n_{D,l}}\right) - c\cdot\max_{j\neq l}\{t^n_{D,jl}\} - \lambda_{D,l}}{\frac{w_{D,lk}}{\left(P^n_{D,l}\right)^2}\left(1+\frac{1}{\text{SINR}^n_{D,l}}\right)^2} \quad (29)$$

which gives the "max-price Newton" method. Again, $c = 2$ is found to work well.

To summarize, in Newton's method, each base-station iteratively updates its power allocation according to

$$P^n_{D,l}(\kappa+1) = \left[P^n_{D,l}(\kappa) + \Delta P^n_{D,l}\right]_0^{S_D^{max}}. \quad (30)$$

where $\Delta P^n_{D,l}$ is computed either by (26), (27) and (28), or by (29) until convergence.

D. Overall Iterative Algorithm

The power spectrum adaptation and the user scheduling steps are iterated until convergence. Each step is nondecreasing in the optimization objective. Thus, the iterations converge.

Base-to-base Distance	2.8km		1.4km	
	UL	DL	UL	DL
Fixed PSD	125	129	137	142
KKT method	185	181	228	227
KKT max-price	179	177	215	217
Newton's method	187	183	229	228
Newton max-price	179	178	216	219
Improvement	43%	38%	58%	54%

TABLE I
SUM RATE (IN MBPS) OVER 19 CELLS WITH 40 USERS PER CELL AT THE CELL EDGE WITH PROPORTIONAL FAIRNESS.

III. SIMULATION

The performance of the proposed algorithm is evaluated on a wireless cellular network with 19 cells and 40 users per cell over 10MHz bandwidth. The cell radius is chosen to be 2.8km (corresponding to a typical WiMax or LTE setting) and 1.4km (corresponding to an overlapped deployment). Frequency selective channels with both log-normal shadowing and Rayleigh fading are simulated. Both the base-stations and the mobile users have a maximum PSD constraint of -27dBm/Hz. For simplicity, no maximum power constraint is imposed. Perfect channel estimation is assumed. For evaluation purposes, the channels are assumed fixed throughout.

Dynamic power spectrum optimization is expected to bring the largest benefit to cell-edge users. To illustrate the benefit for cell-edge users, the mobile users in this simulation are distributed on purpose close to the cell edge at distances between $0.8r$ to $0.9r$, where r is the cell radius.

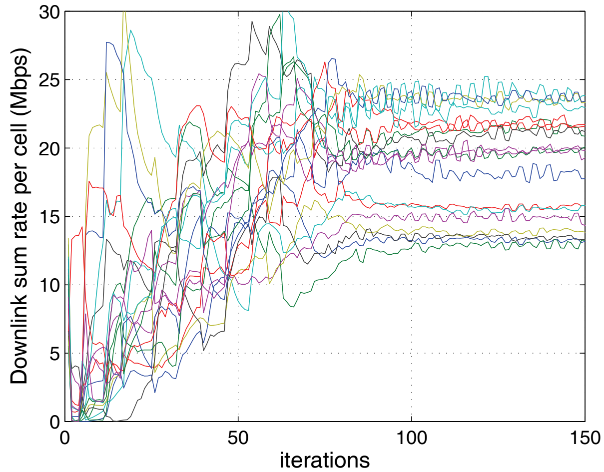

Fig. 2. Convergence of downlink sum rates in each of the 19 cells using KKT method both with max-price simplification.

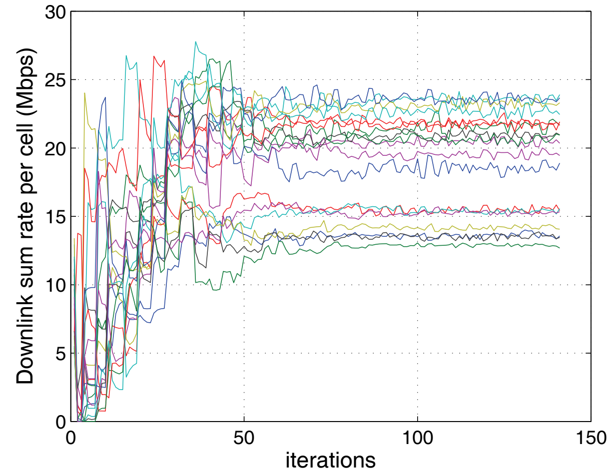

Fig. 3. Convergence of downlink sum rates in each of the 19 cells using Netwon's method both with max-price simplification.

Table I shows the achieved sum rates across all 19 cells with 40 cell-edge users per cell with and without the dynamic power spectrum adaptation. Without dynamic power spectrum adaptation, each transmitter simply transmits at the maximum fixed PSD level to maximize its own transmission rate. Table I shows that all four dynamic power spectrum optimization algorithms proposed in this paper produce significant rate gains for both uplink and downlink as compared to the fixed PSD case. The rate improvement is in the 40%-55% range on average. Note that these figures pertain specifically to cell-edge users. When users are uniformly placed in the cell, the improvement in the average rate would be about 15%-20%.

Table I also shows that the performance gain is more pronounced when base-stations are closer to each other, as expected. In addition, the max-price simplification produces near-optimum performance, losing about 5% in sum rate.

Figs. 2-3 illustrate the convergence behaviors of the max-price KKT and Newton's methods. The per-cell sum rates for each of the 19 cells are plotted against the iteration number. Each iteration step here consists of either a power spectrum adaptation or a user scheduling step. Up to 10 sub-iterations are performed per each power spectrum adaptation step. The proportional fairness weights are also updated at the same time. We observe that the Newton's method converges faster, within about 50 iterations as compared to about 80 iterations for the KKT method. Note that the proportional fairness criterion ensures a fair rate allocation across all the cells. The minimum per-cell sum rate is well above 10Mbps.

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