Competition and Cooperation in Multiuser Communication Environments

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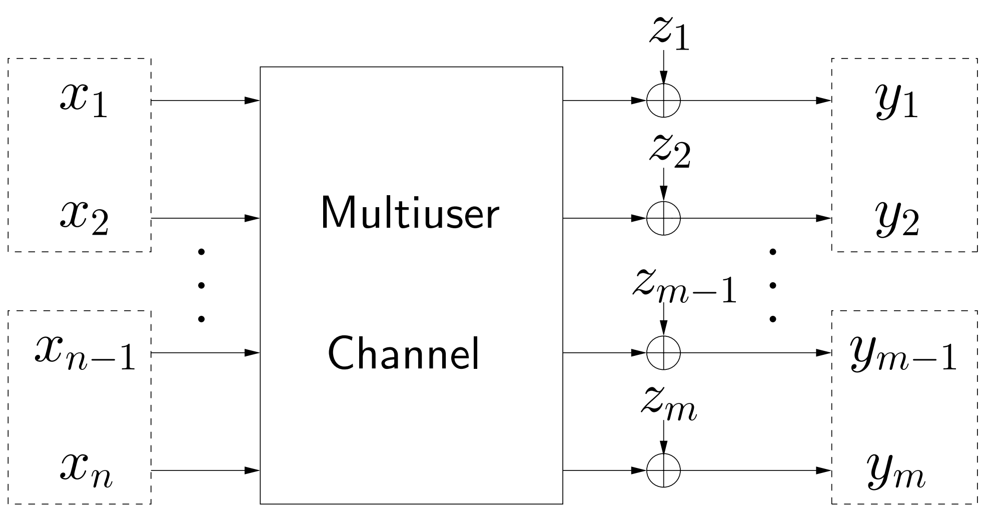
Introduction

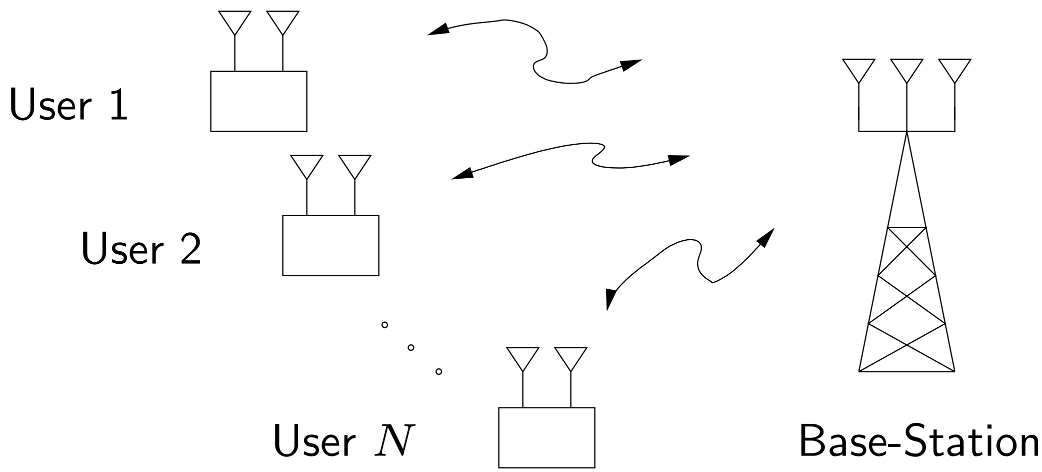
- A multiuser communication environment is a competitive environment.



  - What is the role of competition?
  - What is the value of cooperation?

Multi-Antenna Wireless Communication

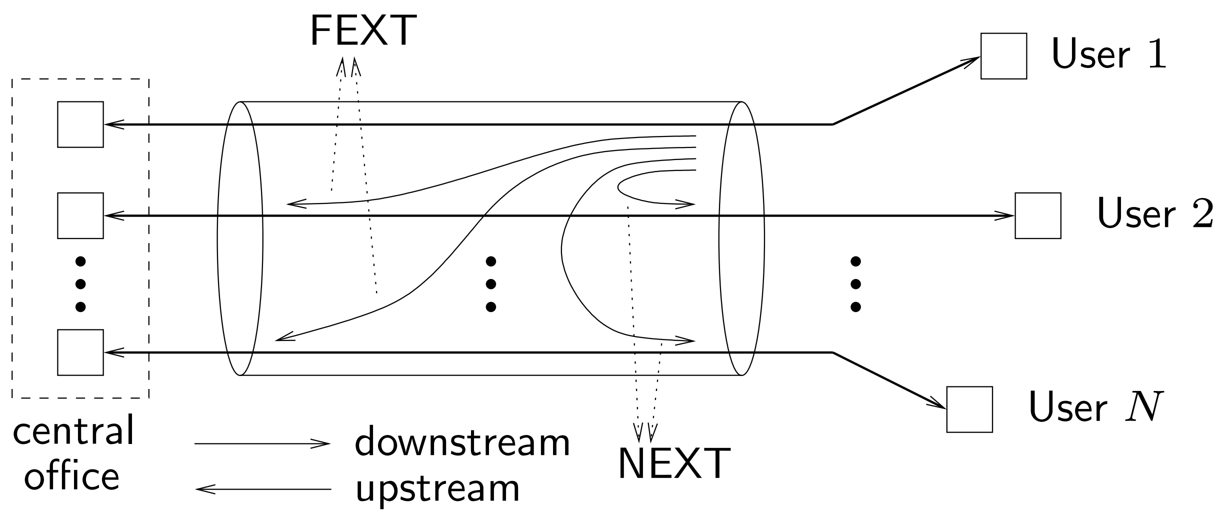
- Examples: broadcast wireless access (802.11), wireless local loop (WLL)



  - Cooperation among multiple antennas within the same user
  - Competition among the users

Digital Subscriber Lines (DSL), Ethernet

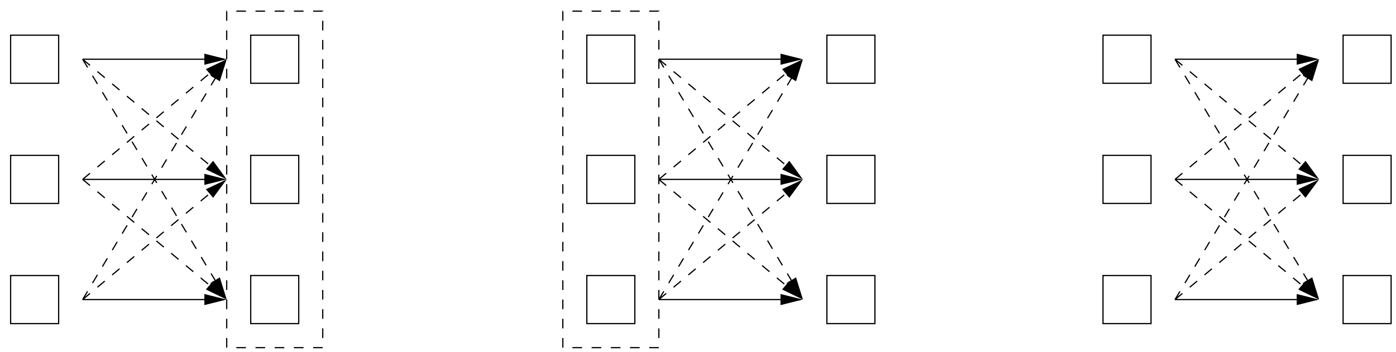
- DSL and Ethernet environments are interference-limited.



  - Explore the benefit of cooperation.
  - Manage the competition.

Goal

- To characterize channel capacity, optimum spectrum, and coding for

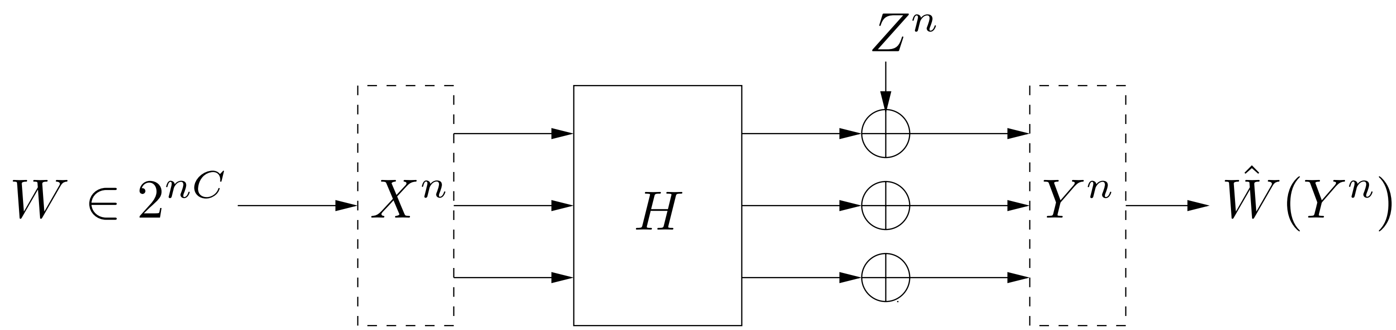


Multiple Access   Broadcast   Interference

  - assuming Gaussian noise.
- To illustrate the value of cooperation in these scenarios.

Gaussian Vector Channel

- Capacity: $C = \max I(\mathbf{X}; \mathbf{Y})$.

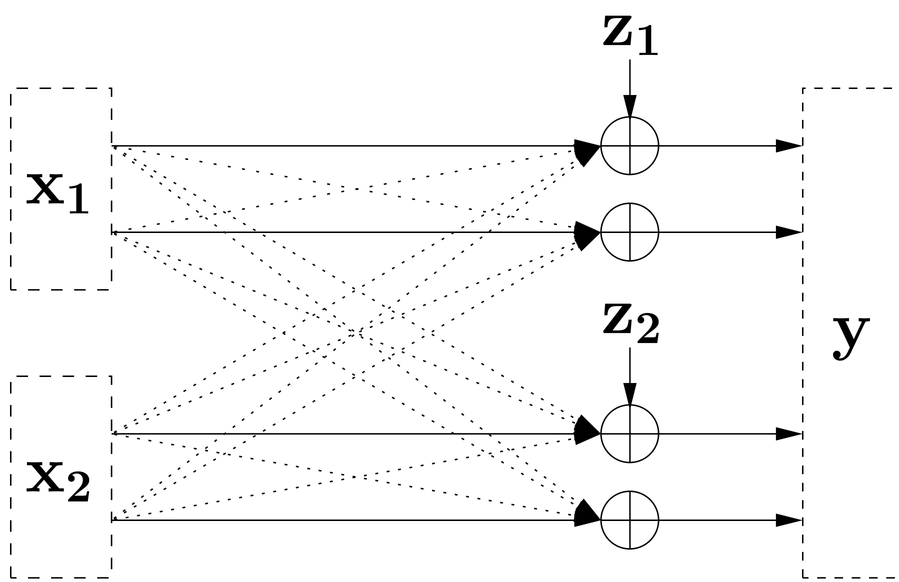


- Optimum Spectrum: Water-filling

$$\begin{array}{ll} \text{maximize} & \dfrac{1}{2}\log\dfrac{|HS_{xx}H^T + S_{zz}|}{|S_{zz}|} \\ \text{subject to} & \mathrm{tr}(S_{xx}) \leq P, \\ & S_{xx} \geq 0. \end{array}$$

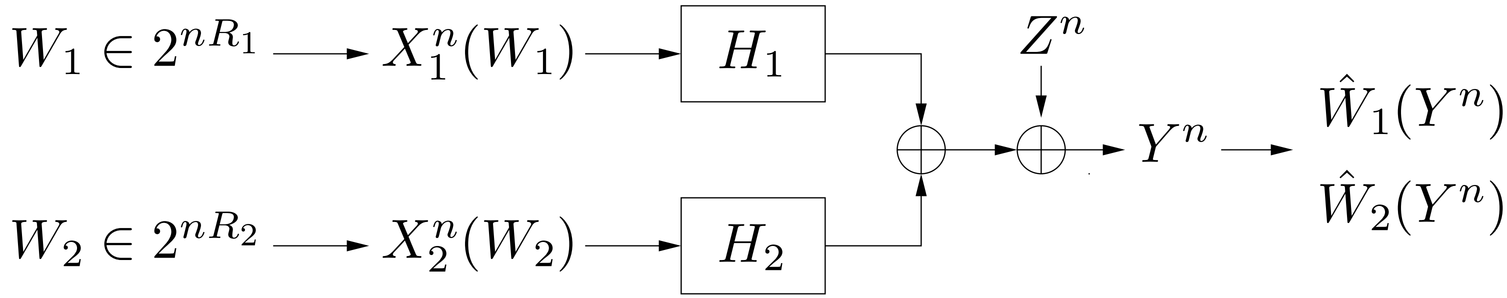
Multiple Access Channel

- No transmitter coordination. Only receiver coordination.



  - Capacity? Optimum Spectrum? Coding?

Capacity Region for Multiple Access Channel



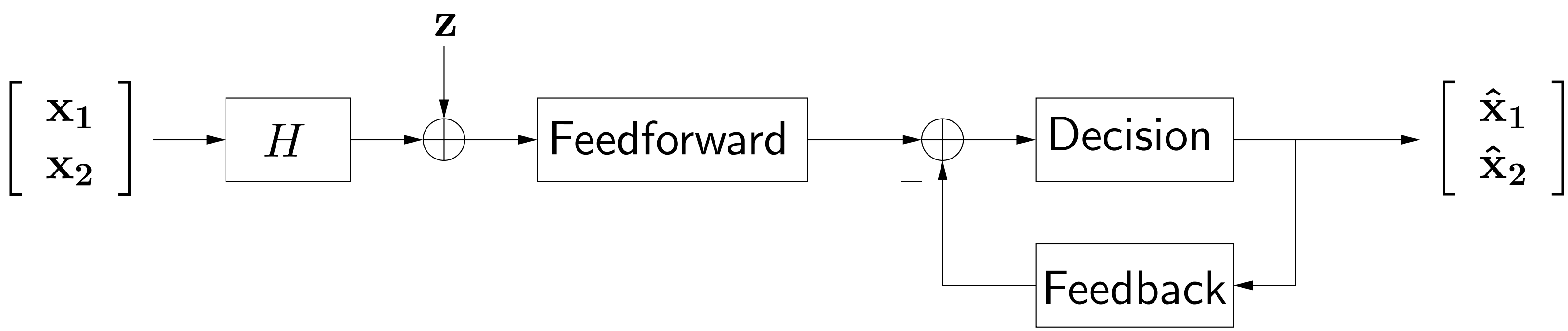
- Capacity region:

$$R_1 \leq I(\mathbf{X}_1; \mathbf{Y} | \mathbf{X}_2);$$
$$R_2 \leq I(\mathbf{X}_2; \mathbf{Y} | \mathbf{X}_1);$$
$$R_1 + R_2 \leq I(\mathbf{X}_1, \mathbf{X}_2; \mathbf{Y}).$$

  – Ahlswede ('71), Liao ('72), Cover-Wyner ('73)

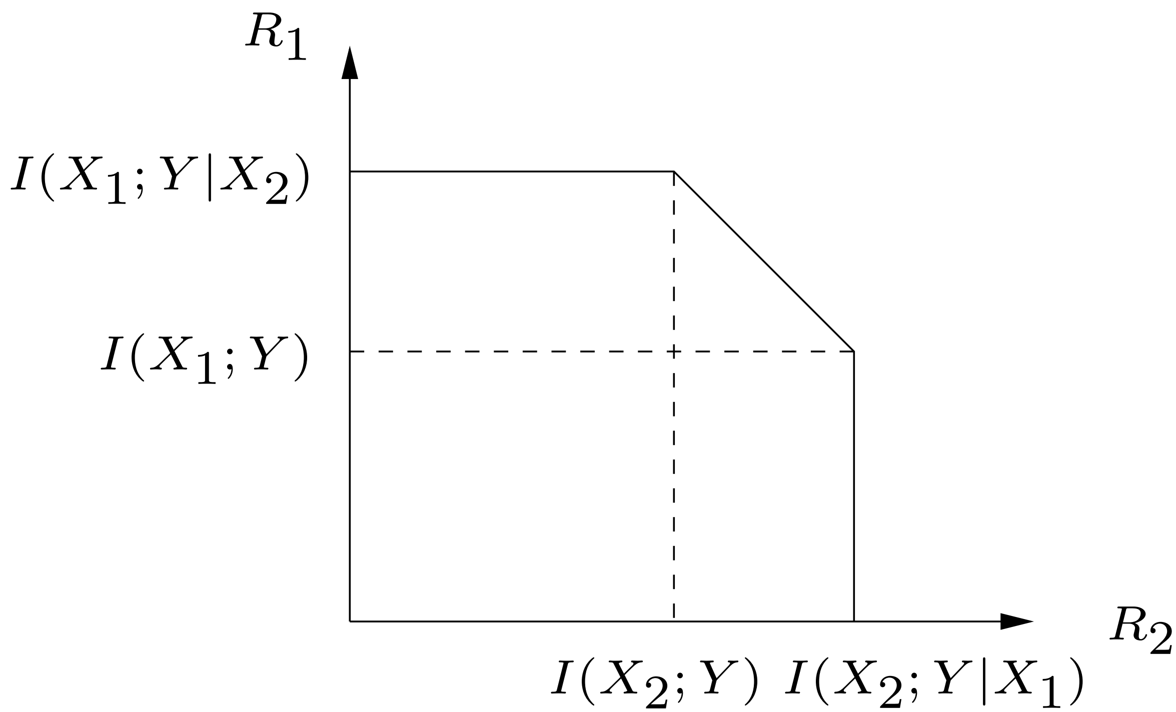
Coding for Multiple Access Channel

- Superposition coding and successive decoding achieves $I(\mathbf{X}_1, \mathbf{X}_2; \mathbf{Y})$:



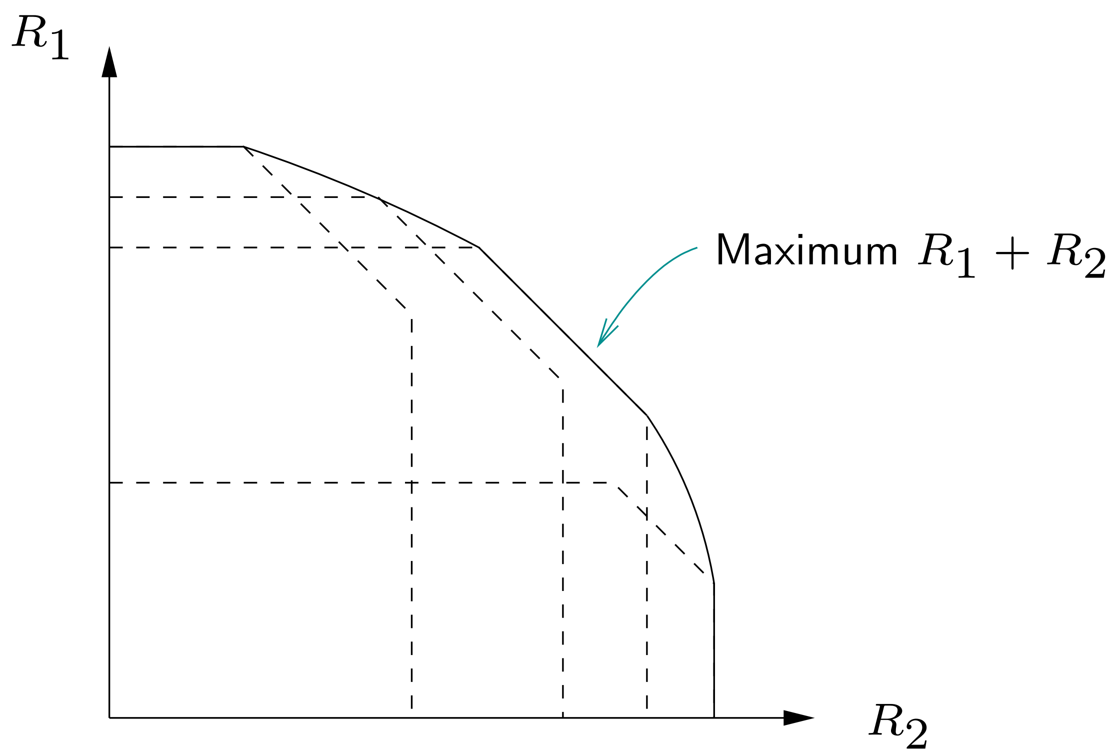
- Implementation: Generalized Decision-Feedback Equalizer (GDFE). Cioffi, Forney ('97), Varanasi, Guess ('97)

Capacity Pentagon



- Fix an input distribution $p(x_1)p(x_2)$, the capacity region is a pentagon.

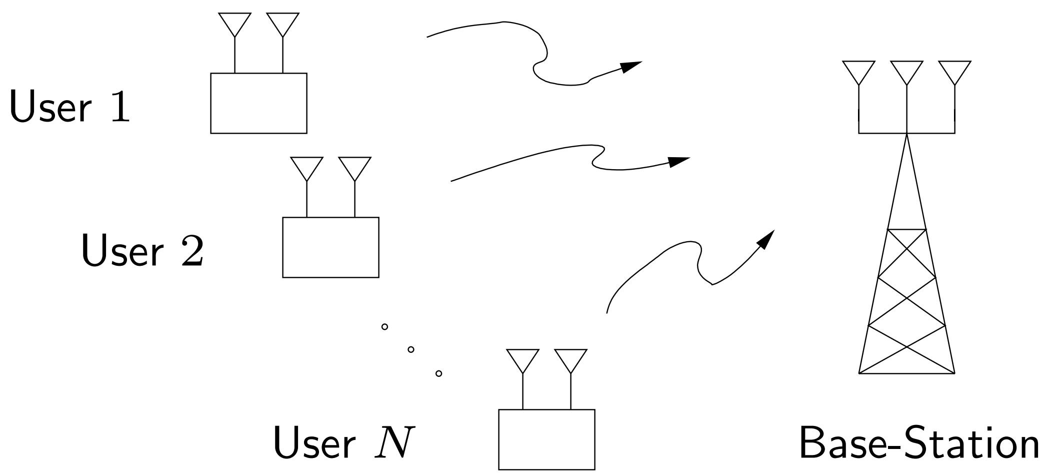
Vector Multiple Access Capacity Region



$$\max(R_1 + R_2) \Longleftrightarrow \max I(\mathbf{X_1}, \mathbf{X_2}; \mathbf{Y}) \text{ over all } p(x_1)p(x_2).$$

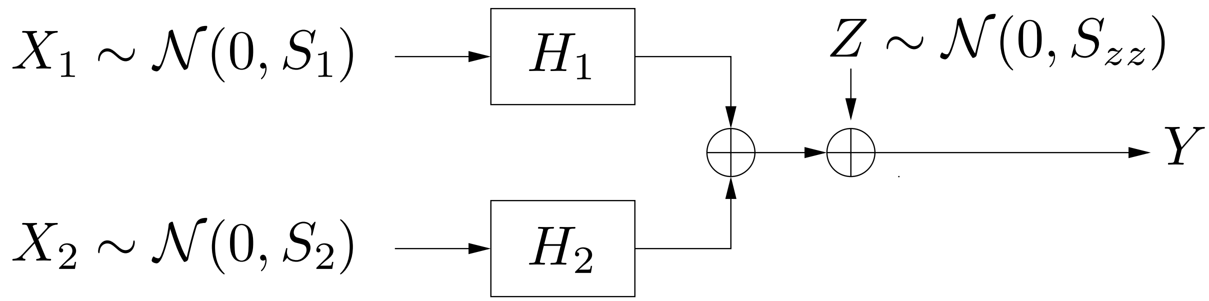
Uplink Power Control in Wireless Systems

- Successive decoding achieves capacity in multiple access channels.



  - If channel state is known at the transmitter...
  - What is the optimal power allocation?

Sum Capacity Maximization

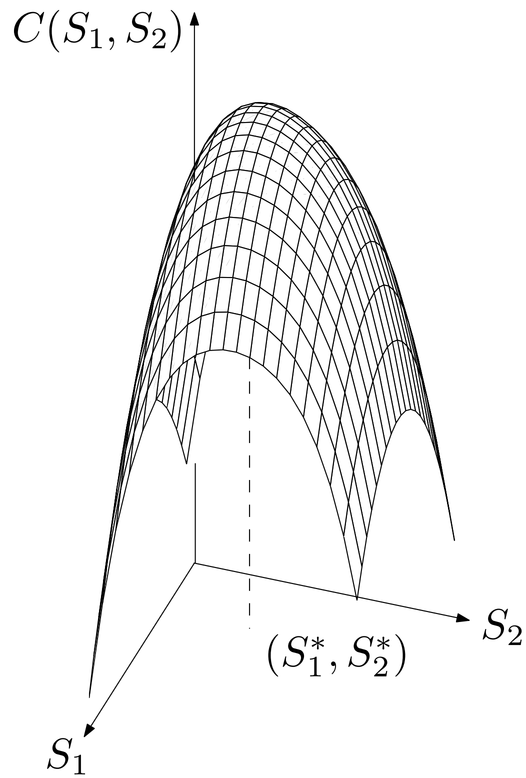


$$
\begin{array}{lcr}
\text{maximize} & \dfrac{1}{2}\log\dfrac{|H_1 S_1 H_1^T + H_2 S_2 H_2^T + S_{zz}|}{|S_{zz}|} & \\
\text{subject to} & \mathrm{tr}(S_i) \le P_i, & i = 1, 2 \\
 & S_i \ge 0, & i = 1, 2
\end{array}
$$

Maximizing a **concave** objective with **convex** constraints.

Competitive Optimum

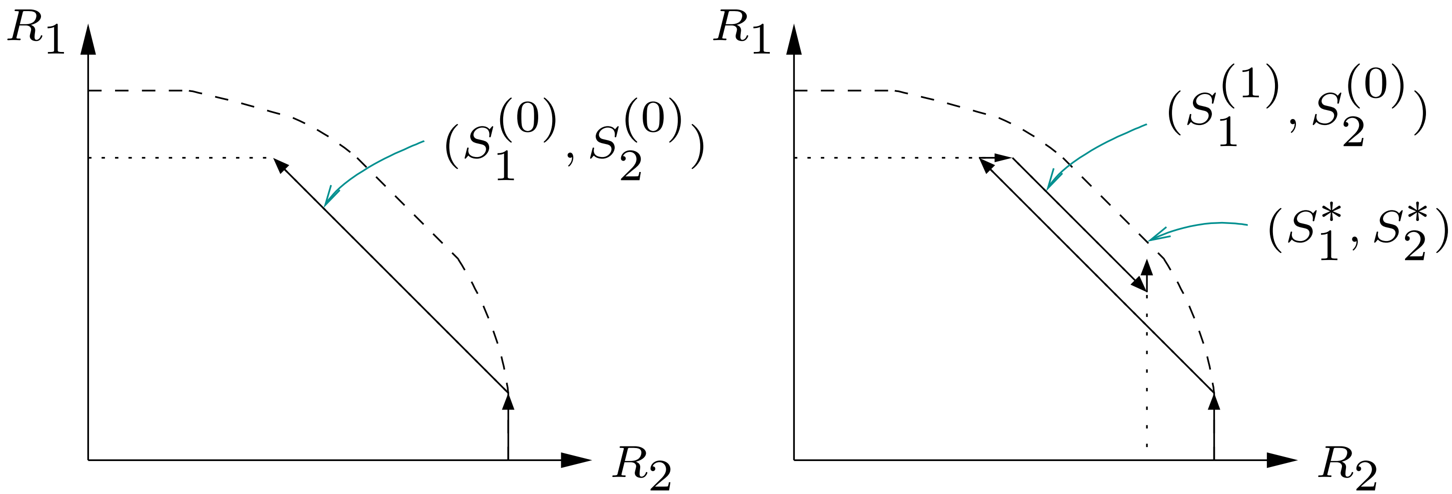
- Optimum S_1^* is a water-filling covariance against S_2^*.
- Optimum S_2^* is a water-filling covariance against S_1^*.
- (S_1^*, S_2^*) can be reached by each user iteratively water-filling against each other.



Multiple Access Channel Sum Capacity = Competitive Optimum

Iterative Water-filling

Theorem 1. *The iterative water-filling process, where each user water-fills against the combined interference and noise, converges to the sum capacity of a Gaussian vector multiple access channel.*



$$S_1^{(0)} \longrightarrow S_2^{(0)} \longrightarrow S_1^{(1)} \longrightarrow S_2^{(1)} \longrightarrow \cdots \longrightarrow (S_1^*, S_2^*)$$

Related Works

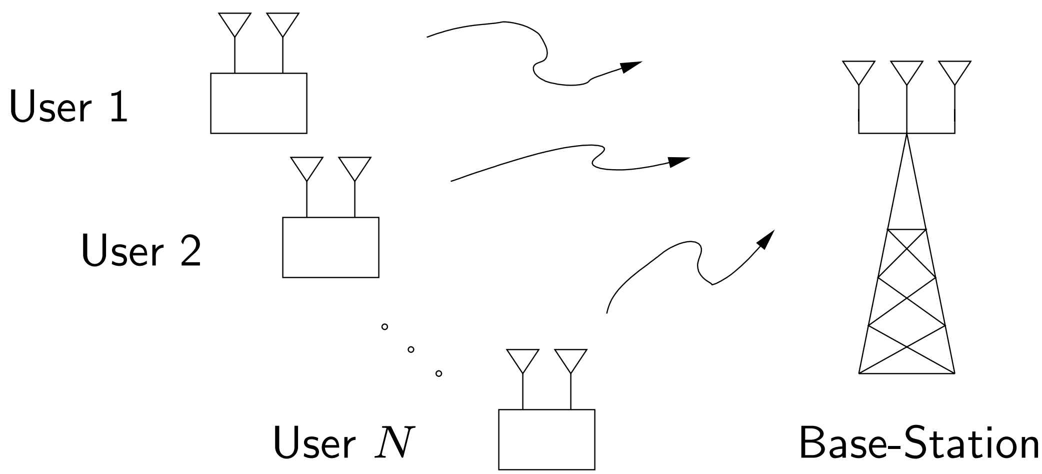
- Multiple access channel with ISI: Cheng and Verdu ('93).

- Multiple access fading channel:
  - Single-antenna: Knopp and Humblet ('95), Hanly and Tse, ('98).
  - Multi-antenna (asymptotic): Viswanath, Tse, Anantharam ('00)
  - Multi-path fading channels: Medard ('00)
  - CDMA channels: Viswanath and Anantharam ('99), Yates, Rose ('00)

- Iterative water-filling is a generalization for vector multi-access channels

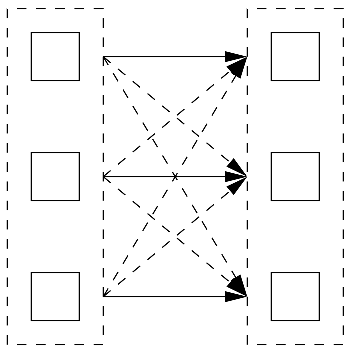
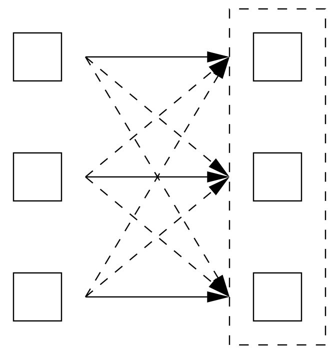
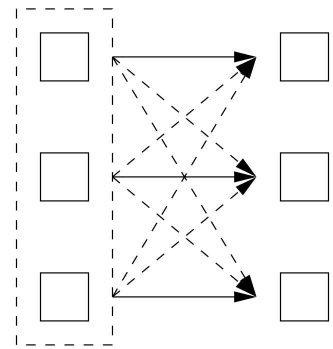
Multi-user Diversity in Wireless Systems

- Solves the power control problem for multi-antenna fading channels:



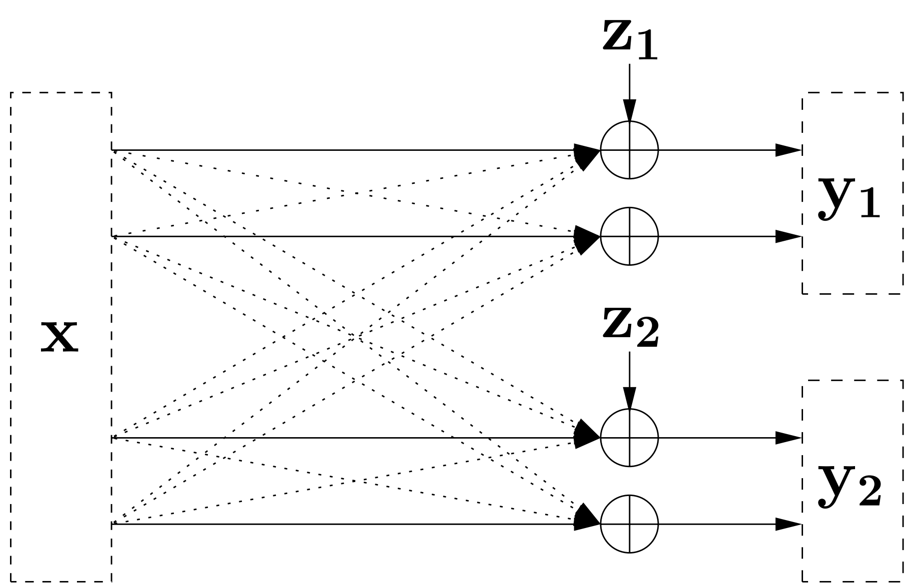
  - Single receive antenna: one user should transmit at the same time.
  - Multiple receive antennas: multiple users transmit at the same time.

Results So Far

Vector Channel	Multiple Access	Broadcast
		
$C = \max I(\mathbf{X};\mathbf{Y})$	$C = \max I(\mathbf{X_1},\mathbf{X_2};\mathbf{Y})$	??
Water-filling	Iterative Water-filling	??
Vector Coding	GDFE	??

Broadcast Channel

- Coordination at transmitter. No receiver coordination.



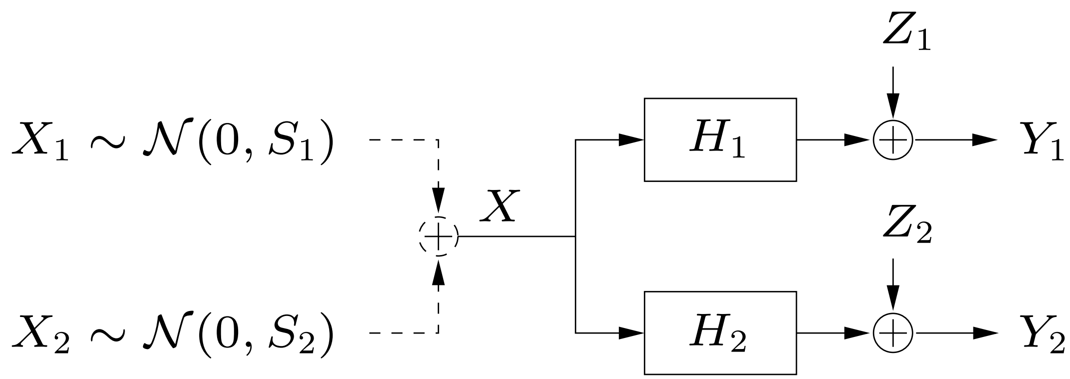
  - Capacity? Optimum Spectrum? Coding?

Broadcast Channel Capacity

- Introduced by Cover ('72)
  - Superposition coding: Cover ('72).
  - Degraded broadcast channel: Bergman ('74), Gallager ('74)
  - Coding using binning: Marton ('79), El Gamal, van der Meulen ('81)
  - Sum and product channels: El Gamal ('80)
  - Gaussian vector channel, 2×2 case: Caire, Shamai ('00)

- General capacity region is a well-known open problem.
  - We focus on a non-degraded Gaussian vector broadcast channel.
  - Simultaneous and independent work was done by Vishwanath, Jindal, Goldsmith, and Viswanath, Tse.

Gaussian Vector Broadcast Channel

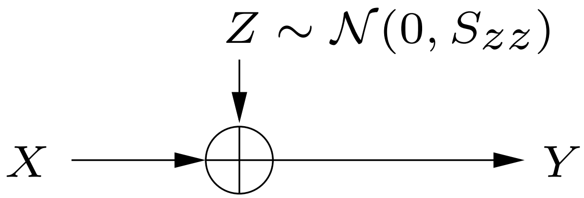


- Superposition coding gives:

$$
\begin{aligned}
R_1 &= I(\mathbf{X_1}; \mathbf{Y_1}) = \frac{1}{2} \log \frac{|H_1 S_1 H_1^T + H_1 S_2 H_1^T + S_{z_1 z_1}|}{|H_1 S_2 H_1^T + S_{z_1 z_1}|} \\
R_2 &= I(\mathbf{X_2}; \mathbf{Y_2}) = \frac{1}{2} \log \frac{|H_2 S_2 H_2^T + H_2 S_1 H_2^T + S_{z_2 z_2}|}{|H_2 S_1 H_2^T + S_{z_2 z_2}|}
\end{aligned}
$$

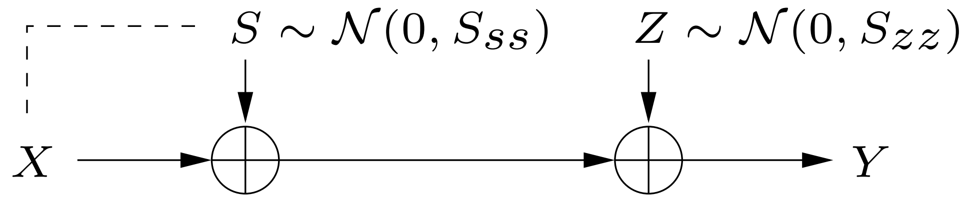
Writing on Dirty Paper

Gaussian Channel



$$C = \frac{1}{2} \log \frac{|S_{xx} + S_{zz}|}{|S_{zz}|}$$

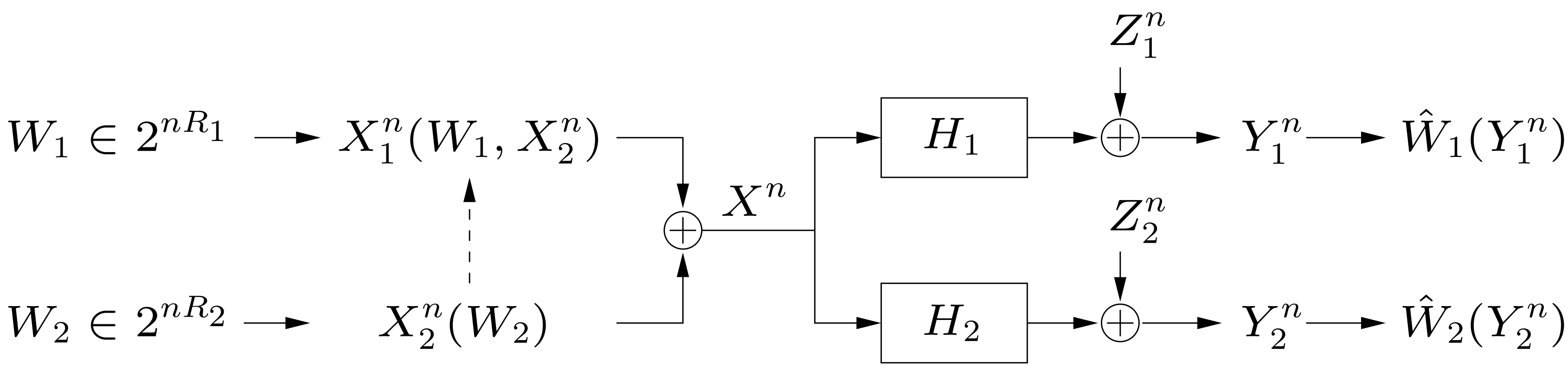
... with Transmitter Side Information



$$C = \frac{1}{2} \log \frac{|S_{xx} + S_{zz}|}{|S_{zz}|}$$

- Capacities are the same if S is known *non-causally* at the transmitter.
  - Based on Gel'fand and Pinsker ('80), Heegard and El Gamal ('83).
  - Gaussian scalar channel: Costa ('81). Vector channel: Yu, $et\ al$ ('01).
  - Generalizations: Cohen, Lapidoth ('01), Erez, Zamir, Shamai ('01).

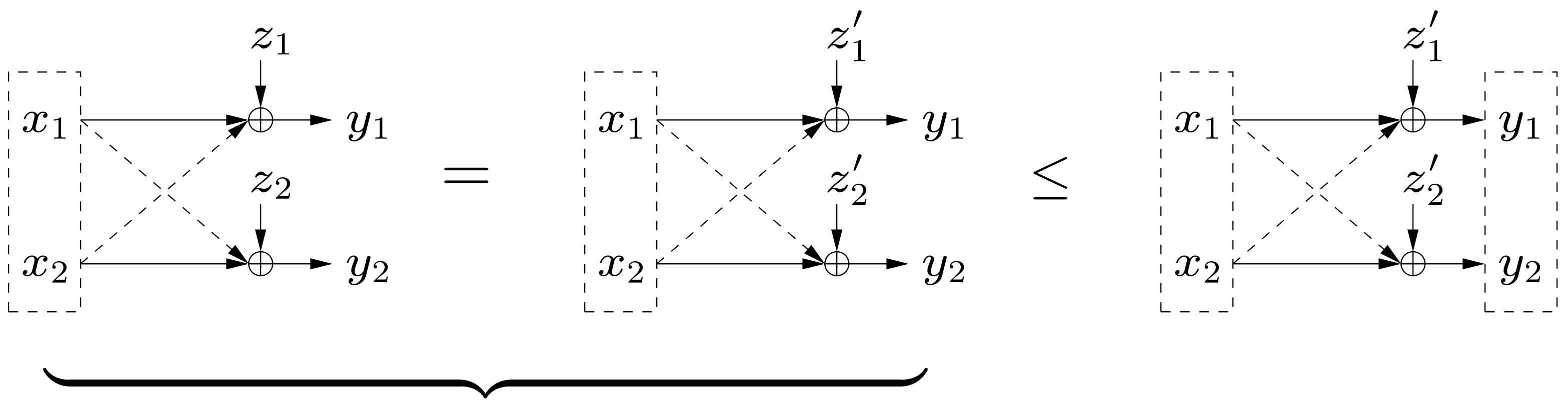
New Achievable Region



$$
\begin{aligned}
R_1 &= I(\mathbf{X_1}; \mathbf{Y_1} | \mathbf{X_2}) &= \frac{1}{2} \log \frac{|H_1 S_1 H_1^T + S_{z_1 z_1}|}{|S_{z_1 z_1}|} \\
R_2 &= I(\mathbf{X_2}; \mathbf{Y_2}) &= \frac{1}{2} \log \frac{|H_2 S_2 H_2^T + H_2 S_1 H_2^T + S_{z_2 z_2}|}{|H_2 S_1 H_2^T + S_{z_2 z_2}|}
\end{aligned}
$$

Converse

- Broadcast capacity does not depend on noise correlation: Sato ('78).



if $\begin{cases} p(z_1) = p(z_1') \\ p(z_2) = p(z_2') \end{cases}$, not necessarily $p(z_1, z_2) = p(z_1', z_2')$.

- Thus, sum-capacity $C \leq \min\limits_{S_{zz}} \max\limits_{S_{xx}} I(\mathbf{X}; \mathbf{Y})$.

Least Favorable Noise

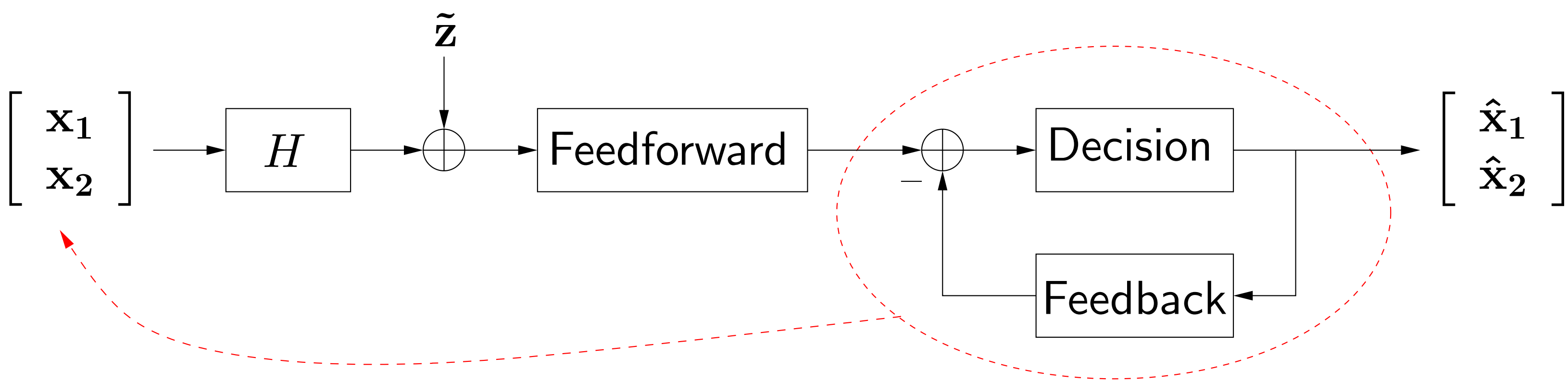
- Fix Gaussian input S_{xx}:

$$\begin{aligned} \text{minimize} \quad & \frac{1}{2}\log\frac{|HS_{xx}H^T + S_{zz}|}{|S_{zz}|} \\ \text{subject to} \quad & S_{zz} = \begin{bmatrix} S_{z_1z_1} & \star \\ \star & S_{z_2z_2} \end{bmatrix} \\ & S_{zz} \geq 0 \end{aligned}$$

- Minimizing a **convex** function over **convex** constraints.

- Optimality condition: $S_{zz}^{-1} - (HS_{xx}H^T + S_{zz})^{-1} = \begin{bmatrix} \Psi_1 & 0 \\ 0 & \Psi_2 \end{bmatrix}$.
  - if $S_{zz} > 0$ at minimum.

GDFE Revisited



- Least Favorable Noise $\Longleftrightarrow$ Feedforward filter is diagonal!
- Decision-feedback may be moved to the transmitter by precoding.

$$R = \min_{S_{zz}} I(\mathbf{X};\mathbf{Y}) \text{ \textit{(i.e. with least favorable noise) is achievable.}}$$

Gaussian Broadcast Channel Sum Capacity

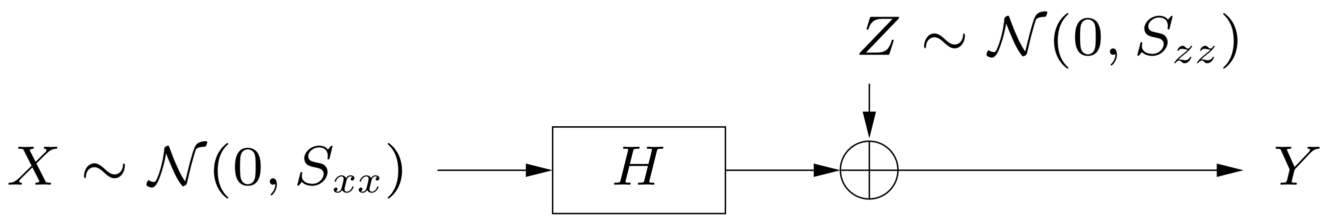
- Achievability: $C \geq \max_{S_{xx}} \min_{S_{zz}} I(\mathbf{X};\mathbf{Y})$.
- Converse (Sato): $C \leq \min_{S_{zz}} \max_{S_{xx}} I(\mathbf{X};\mathbf{Y})$.
- (Diggavi, Cover '98): $\min_{S_{zz}} \max_{S_{xx}} I(\mathbf{X};\mathbf{Y}) = \max_{S_{xx}} \min_{S_{zz}} I(\mathbf{X};\mathbf{Y})$.

Theorem 2. *Gaussian vector broadcast channel sum capacity is:*

$$C = \max_{S_{xx}} \min_{S_{zz}} \frac{1}{2} \log \frac{|HS_{xx}H^T + S_{zz}|}{|S_{zz}|}$$

whenever $S_{zz} > 0$ *at the saddle-point.*

Gaussian Mutual Information Game

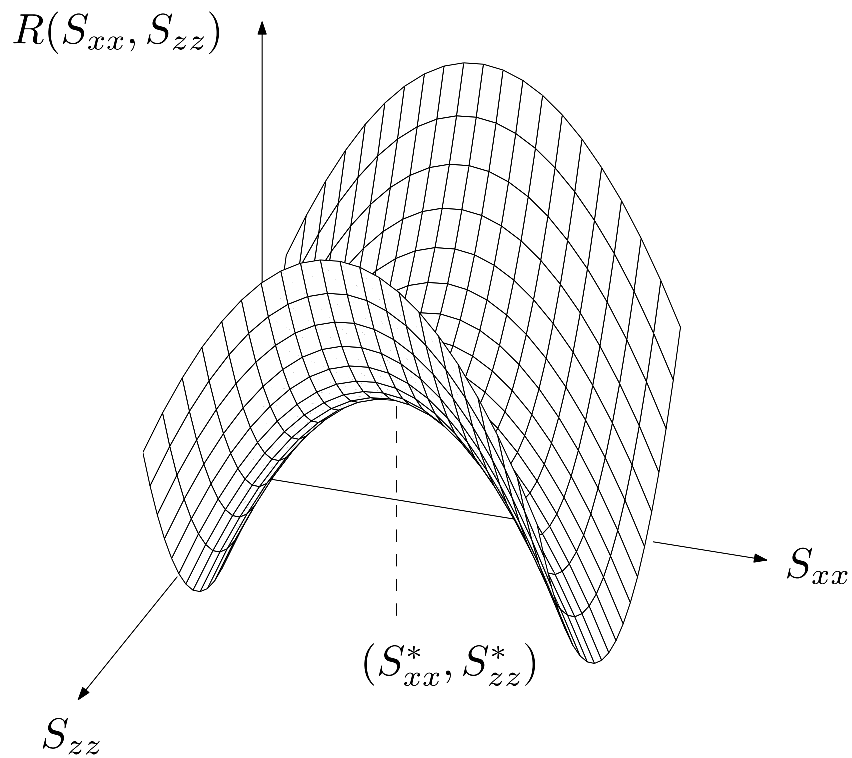


	Strategy	Objective
Signal Player	$\{S_{xx} : \mathrm{trace}(S_{xx}) \leq P\}$	$\max I(\mathbf{X};\mathbf{Y})$
Fictitious Noise Player	$\left\{ S_{zz} : S_{zz} = \begin{bmatrix} S_{z_1z_1} & \star \\ \star & S_{z_2z_2} \end{bmatrix} \geq 0 \right\}$	$\min I(\mathbf{X};\mathbf{Y})$

Competitive equilibrium exists.

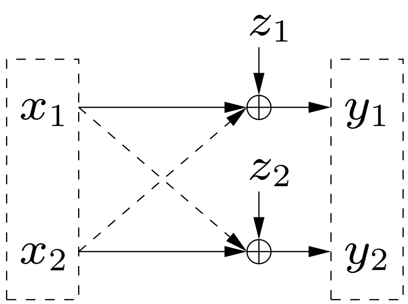
Saddle-Point is the Broadcast Capacity

- The optimum S_{xx}^* is a water-filling covariance against S_{zz}^*.
- The optimum S_{zz}^* is a least-favorable noise for S_{xx}^*.



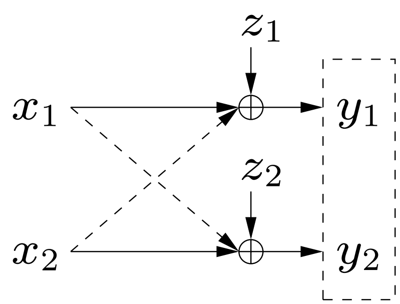
Broadcast Channel Sum Capacity = Competitive Equilibrium

The Value of Cooperation



$$\max_{S_{xx}} I(\mathbf{X};\mathbf{X}+\mathbf{Z})$$

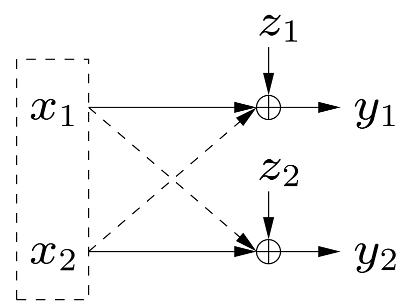
$$\text{s.t.} \quad \mathrm{trace}(S_{xx}) \le P$$



$$\max_{S_{xx}} I(\mathbf{X};\mathbf{X}+\mathbf{Z})$$

$$\text{s.t.} \quad S_{xx} = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix}$$

$$\mathrm{trace}(S_i) \le P_i,$$

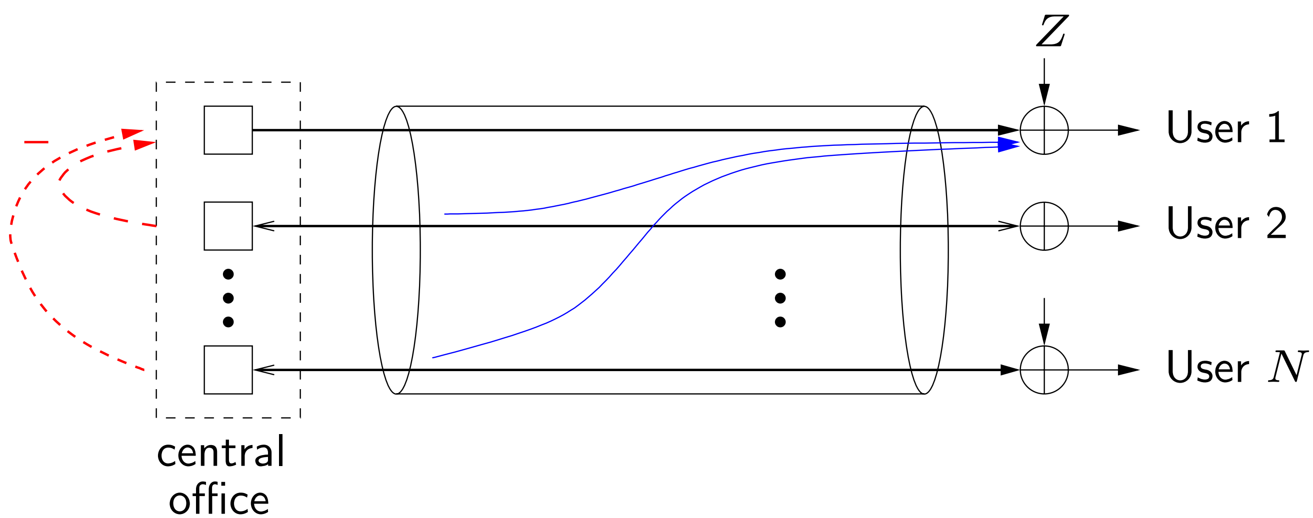


$$\min_{S_{zz}} \max_{S_{xx}} I(\mathbf{X};\mathbf{X}+\mathbf{Z})$$

$$\text{s.t.} \quad S_{zz} = \begin{bmatrix} S_{z_1 z_1} & \star \\ \star & S_{z_2 z_2} \end{bmatrix}$$

$$\mathrm{trace}(S_{xx}) \le P$$

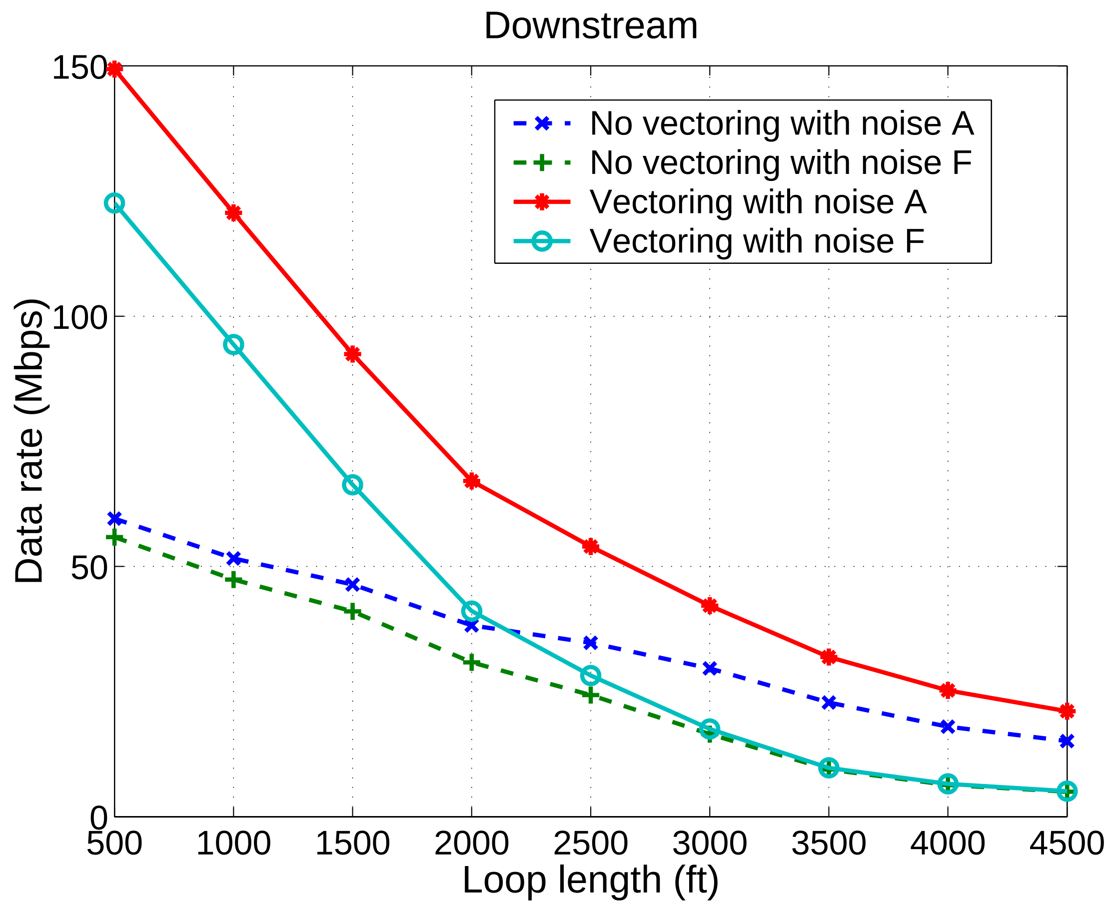
Application: Multi-line Transmission in DSL



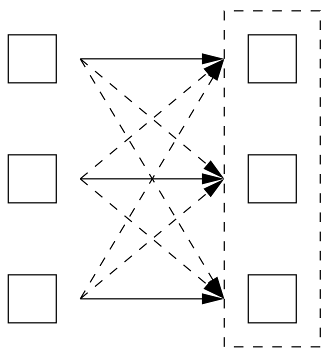
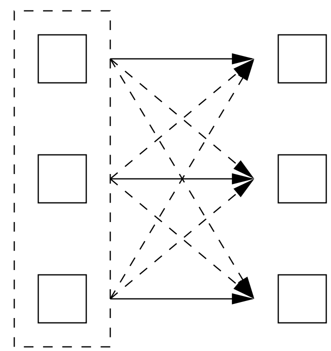
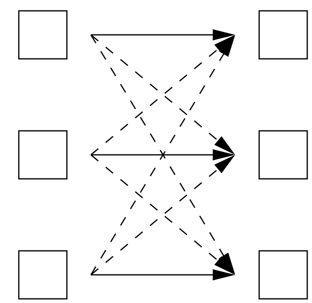
- With coordination, crosstalk can be "pre-subtracted".
  - Practical pre-subtraction: Tomlinson precoding.
  - Optimal pre-subtraction: "Dirty-paper" precoding.

Performance: Vector DSL/Ethernet

- 20 VDSL lines
- Large improvement
  - for short loops
- Courtesy:
  - George Ginis

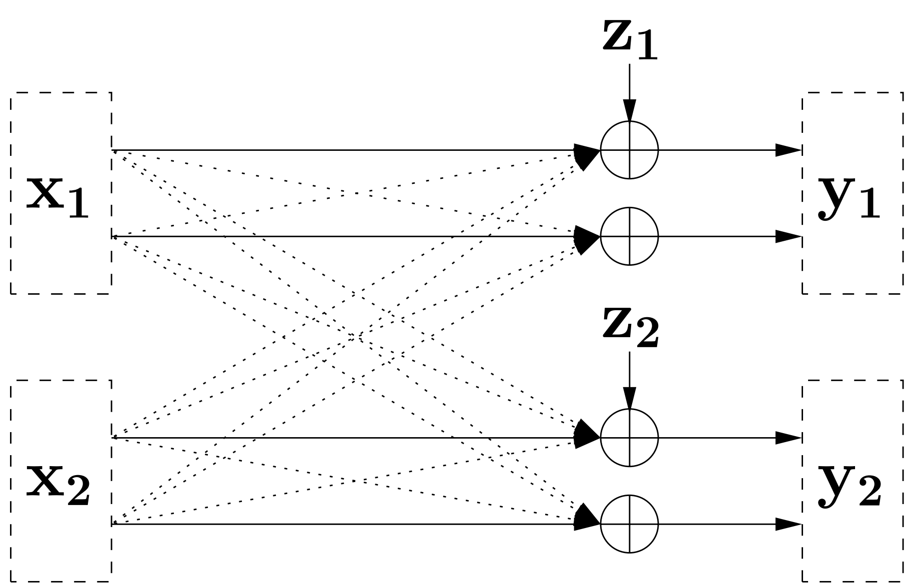


Results So Far

Multiple Access	Broadcast	Interference
		
$C = \max I(\mathbf{X_1}, \mathbf{X_2}; \mathbf{Y})$	$C = \max \min I(\mathbf{X}; \mathbf{Y})$	??
Iterative Water-filling	Minimax	??
GDFE	GDFE Precoder	??

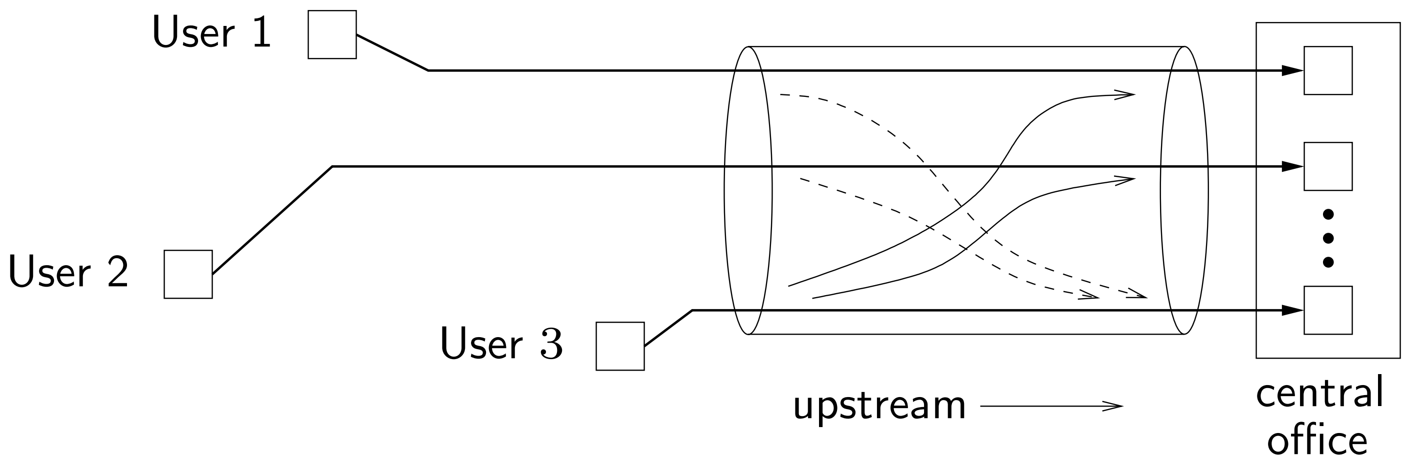
Interference Channel

- No transmitter coordination. No receiver coordination.



  - Capacity is a difficult open problem.

DSL Interference Environment



- Near-far problem: The closer user emits too much interference.
  - Power back-off is necessary.
  - Current system imposes a maximum power-spectral-density limit.

Power Control Problem

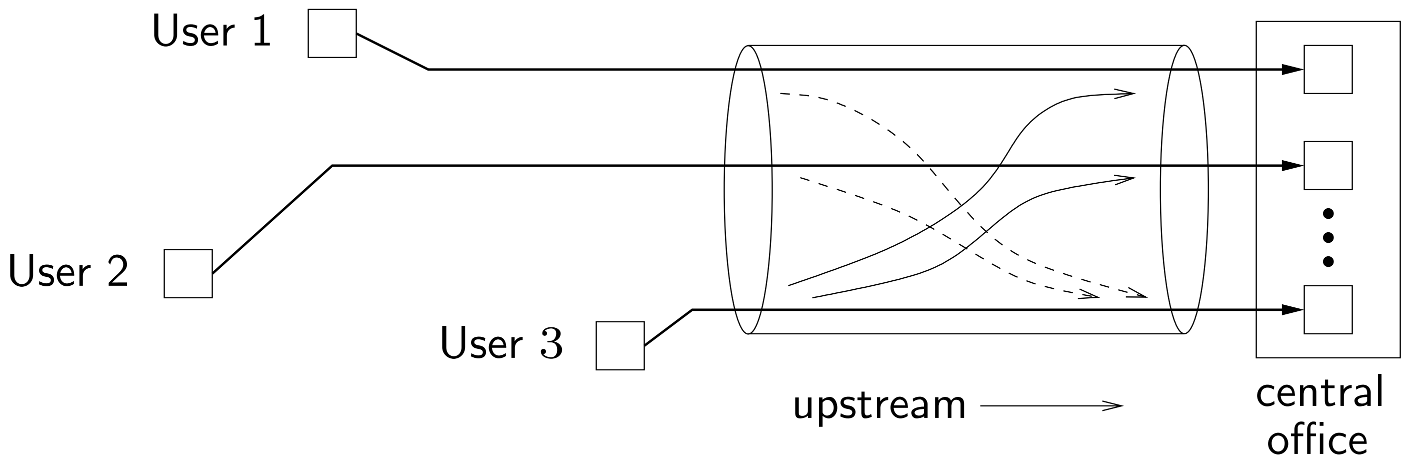
- Find an optimum $(P_1(f), P_2(f))$ to maximize:

$$
\begin{aligned}
R_1 &= \int_0^W \log\left(1 + \frac{|H_{11}(f)|^2 P_1(f)}{N_1(f) + |H_{21}(f)|^2 P_2(f)}\right) df, \\
R_2 &= \int_0^W \log\left(1 + \frac{|H_{22}(f)|^2 P_2(f)}{N_2(f) + |H_{12}(f)|^2 P_1(f)}\right) df.
\end{aligned}
$$

$$
\text{s.t.} \quad \int_0^W P_1(f)df \le \mathcal{P}_1, \qquad \int_0^W P_2(f)df \le \mathcal{P}_2
$$

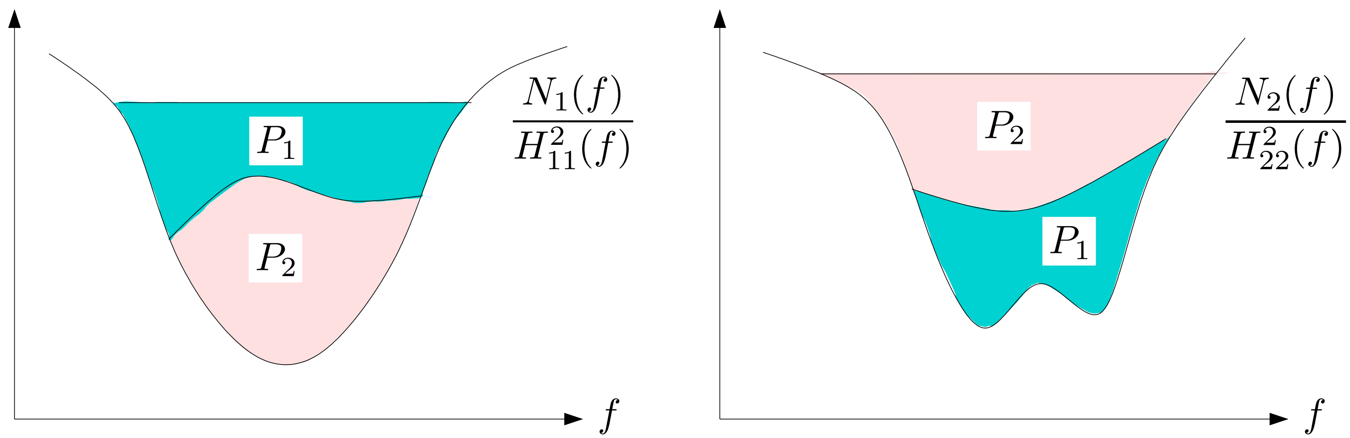
- Finding the global optimum is computationally difficult.

Competitive Environment



- Each user maximizes its *own* data rate regarding other users as noise.
  - Non-cooperative game.

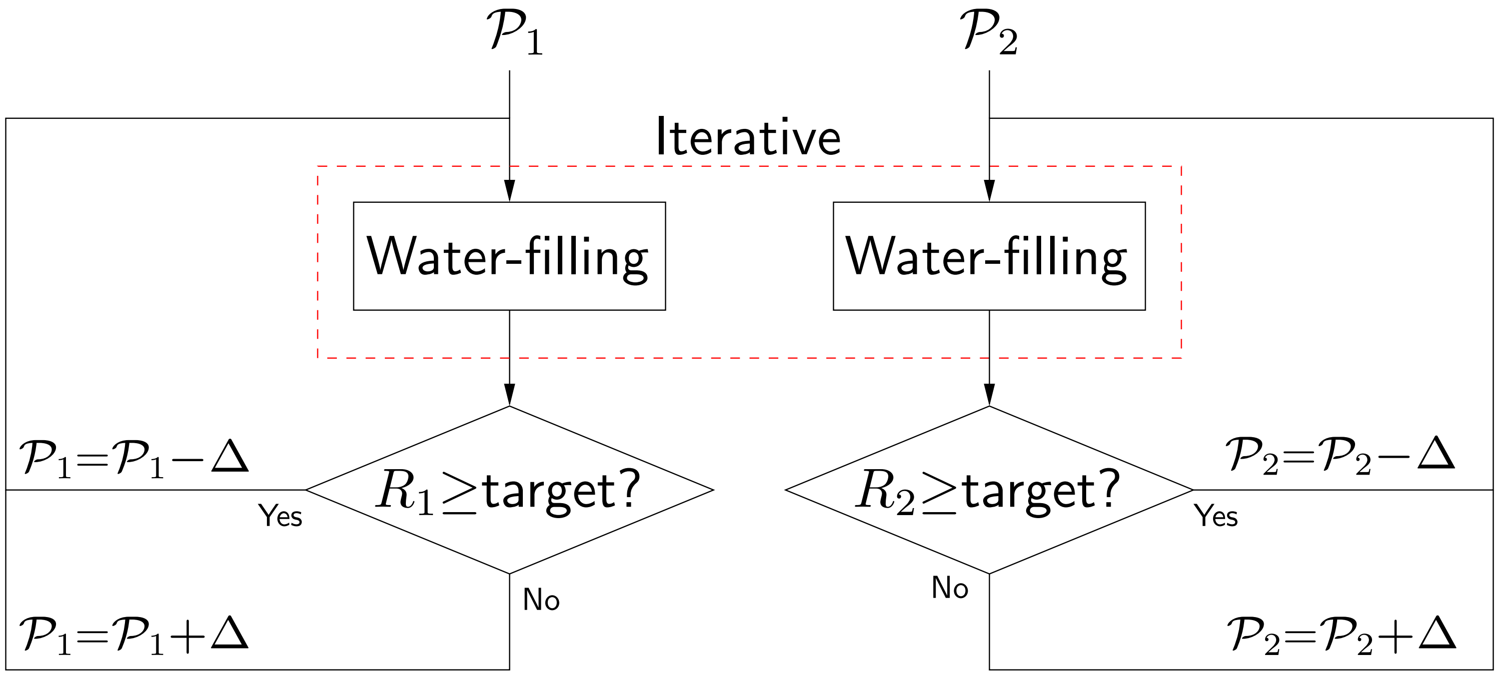
Iterative Water-filling



$$P_1^{(0)}(f) \to P_2^{(0)}(f) \to P_1^{(1)}(f) \to P_2^{(1)}(f) \to \cdots$$

Theorem 3. *Under a mild condition, the two-user Gaussian interference game has a competitive equilibrium. The equilibrium is unique, and it can be reached by iterative water-filling.*

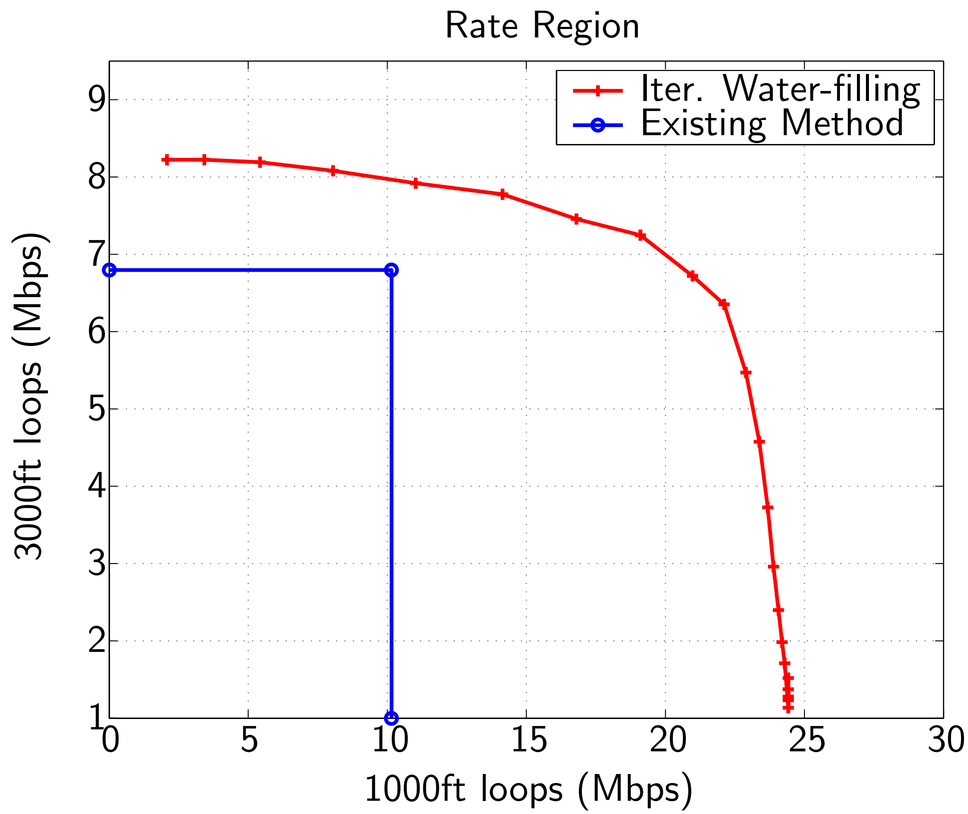
Distributed Power Control for DSL



Control $(P_1(f), P_2(f))$ *by setting* $(\mathcal{P}_1, \mathcal{P}_2)$*.*

Performance

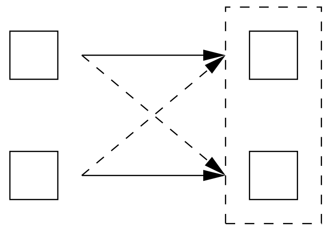
- 4 VDSL lines at 3000ft
  - rates: 6.7Mbps
- 4 VDSL lines at 1000ft
  - 10Mbps $\Rightarrow$ 21Mbps.



Competitive optimal points are much better than existing methods.

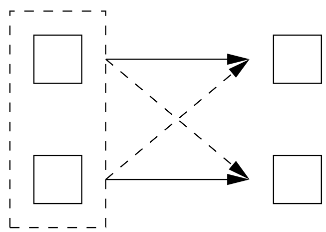
Conclusion

Multiple Access



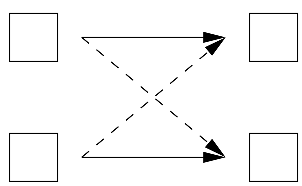
Iterative water-filling achieves sum capacity.

Broadcast



Sum Capacity is a saddle-point of a mutual information game.

Interference



Competitive optimum is a desirable operating point.

Future Work

- Network Information Theory
- Multi-antenna/Multi-line Signal Processing
- Multiuser system design: physical layer vs network layer
- Applications to broadband access networks:
  - Wireless Local Area Networks
  - High-speed Ethernet
  - Digital Subscriber Lines
  - Computer Interconnects