Costas Receiver and Quadrature-Carrier Multiplexing

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Costas Receiver and Quadrature-Carrier Multiplexing

Reference:

Sections 3.4 and 3.5 of

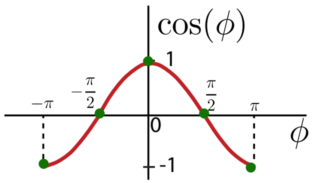
S. Haykin and M. Moher, Introduction to Analog & Digital Communications, 2nd ed., John Wiley & Sons, Inc., 2007. ISBN-13 978-0-471-43222-7.

$\boxed{\text{Costas Receiver}}$

Coherent Demodulation Output

$$v_0(t) = \frac{1}{2} A_c A_c' \cos(\phi) m(t)$$

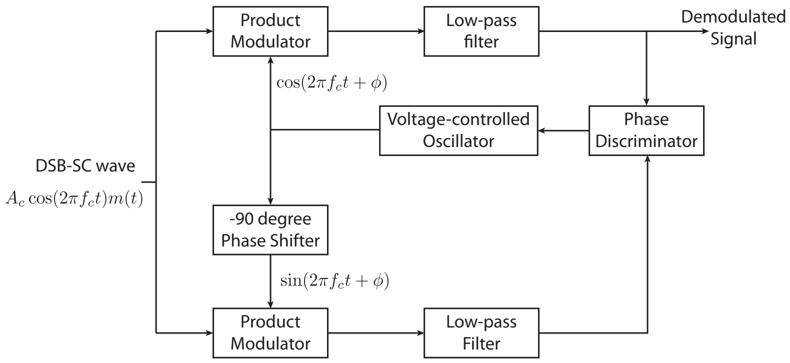
- To maximize $v_0(t)$, would like $\phi \approx 0$.



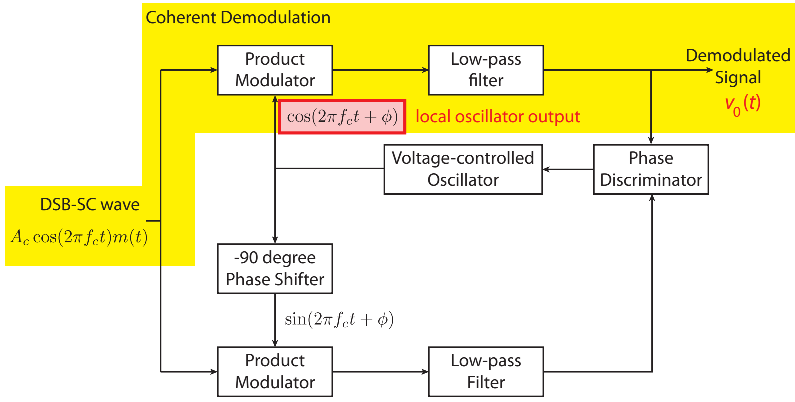
- For $v_0(t)$ to be proportional to $m(t)$, ϕ should be constant.

Thus, we wish to synchronize the local oscillator.
Let $A_c' = 1$ for simplicity.

Costas Receiver



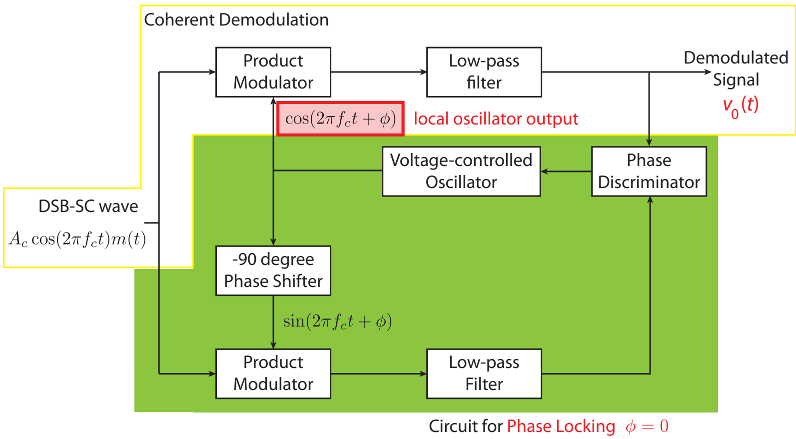
Costas Receiver: Coherent Demodulation



Goals: (1) Coherent demodulation of DSB-SC input signal.

(2) Tweak the local oscillator phase such that $\phi = 0$.

Costas Receiver: Phase Lock Circuit

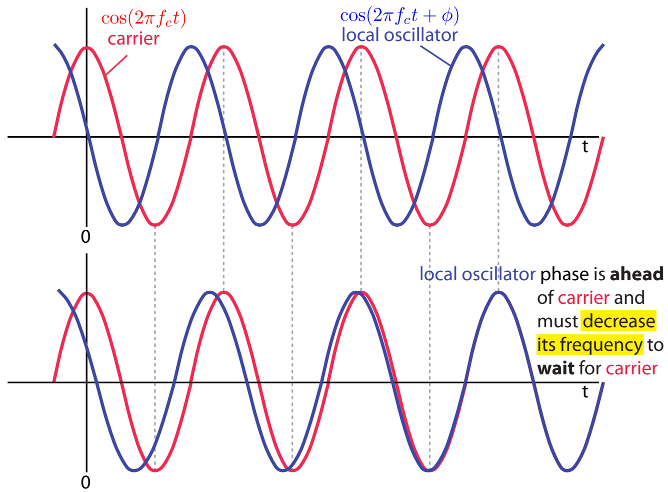


Goals: (1) Coherent demodulation of DSB-SC input signal.

(2) Tweak the local oscillator phase such that $\phi = 0$.

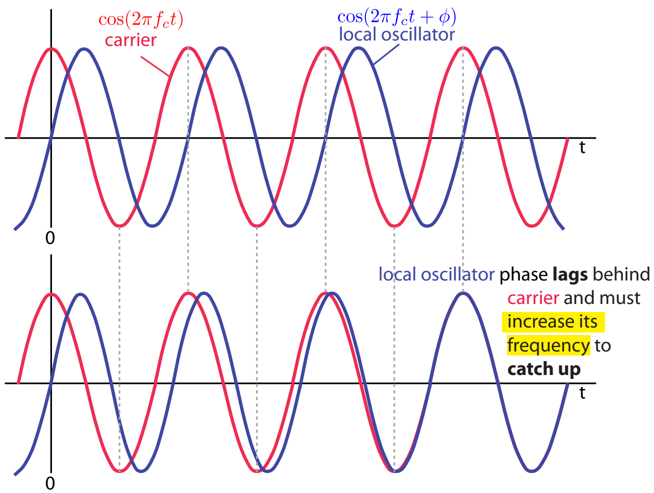
Costas Receiver: Phase Lock Circuit

$\phi > 0$: Freq of local oscillator needs to temporarily decrease

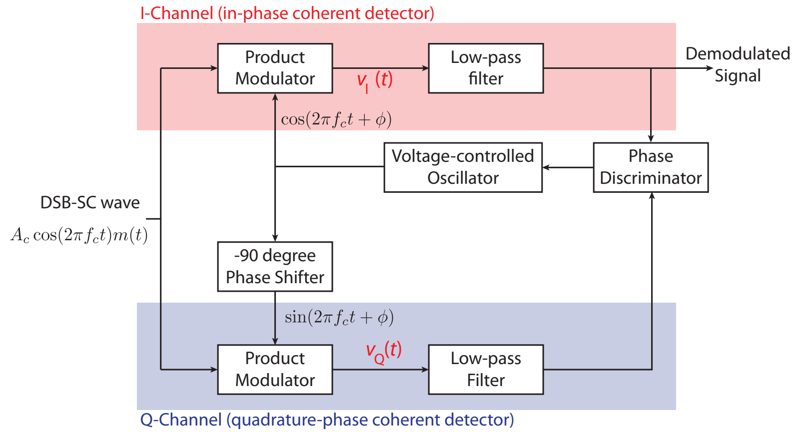


Costas Receiver: Phase Lock Circuit

$\phi < 0$: Freq of local oscillator needs to temporarily increase



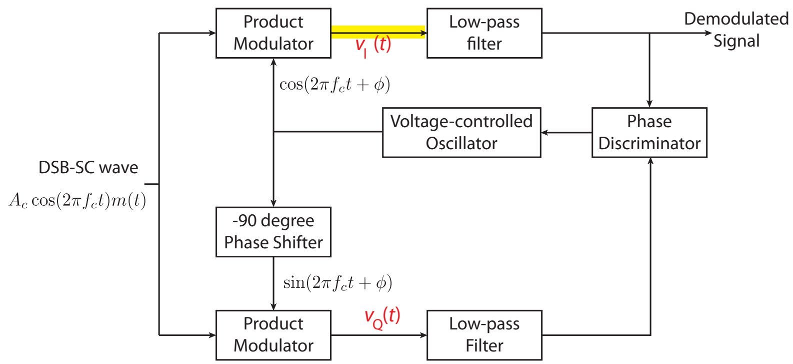
Costas Receiver: In-Phase and Quadrature-Phase



Costas Receiver: In-Phase Coherent Detector

$\cos(A)\cos(B) = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(B-A)$

$$A_c \cos(2\pi f_c t)\cos(2\pi f_c t + \phi) m(t)$$
$$= \frac{A_c}{2}\cos(4\pi f_c t + \phi) m(t) + \frac{A_c}{2}\cos(\phi) m(t)$$



Costas Receiver: In-Phase Coherent Detector

$$\begin{aligned} s(t) &= A_c \cos(2\pi f_c t) m(t) \\ v_I(t) &= s(t) \cdot \cos(2\pi f_c t + \phi) \end{aligned}$$

Recall

$$\cos(2\pi f_c t + \phi) = \frac{e^{j\phi}}{2} e^{j2\pi f_c t} + \frac{e^{-j\phi}}{2} e^{-j2\pi f_c t} \rightleftharpoons \frac{e^{j\phi}}{2}\delta(f - f_c) + \frac{e^{-j\phi}}{2}\delta(f + f_c)$$

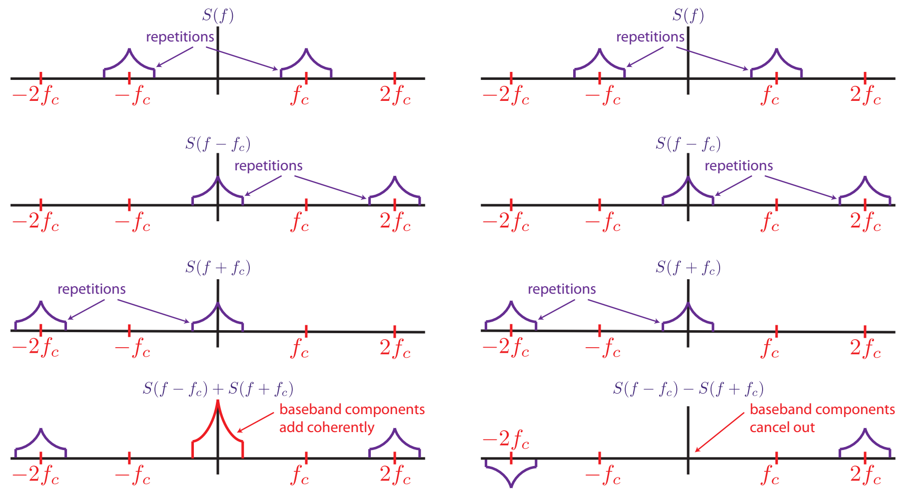
$$\begin{aligned} V_I(f) &= S(f) \star \left[\frac{e^{j\phi}}{2}\delta(f - f_c) + \frac{e^{-j\phi}}{2}\delta(f + f_c)\right] \\ &= \frac{e^{j\phi}}{2} S(f) \star \delta(f - f_c) + \frac{e^{-j\phi}}{2} S(f) \star \delta(f + f_c) \\ &= \frac{e^{j\phi}}{2} S(f - f_c) + \frac{e^{-j\phi}}{2} S(f + f_c) \end{aligned}$$

Costas Receiver: In-Phase Coherent Detector

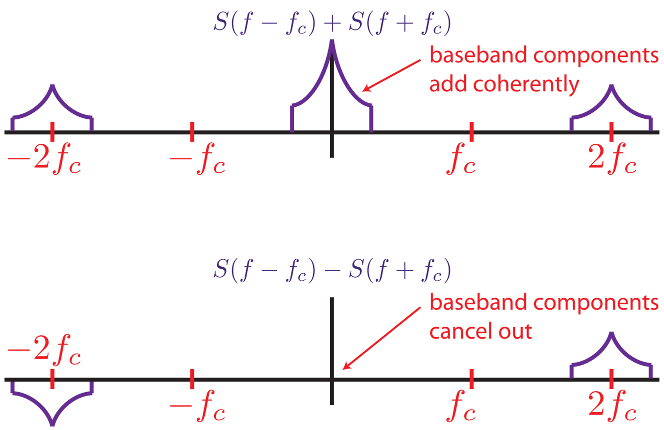
$$
\begin{aligned}
V_I(f) &= \frac{e^{j\phi}}{2} S(f-f_c) + \frac{e^{-j\phi}}{2} S(f+f_c) \\
&= \frac{\cos(\phi) + j\sin(\phi)}{2} S(f-f_c) + \frac{\cos(-\phi) + j\sin(-\phi)}{2} S(f+f_c) \\
&= \frac{\cos(\phi) + j\sin(\phi)}{2} S(f-f_c) + \frac{\cos(\phi) - j\sin(\phi)}{2} S(f+f_c) \\
&= \underbrace{\frac{\cos(\phi)}{2}}_{\approx 1/2} [S(f-f_c) + S(f+f_c)] + j \underbrace{\frac{\sin(\phi)}{2}}_{\text{small}} [S(f-f_c) - S(f+f_c)]
\end{aligned}
$$

for $\phi \ll 1$.

Costas Receiver: In-Phase Coherent Detector



Costas Receiver: In-Phase Coherent Detector



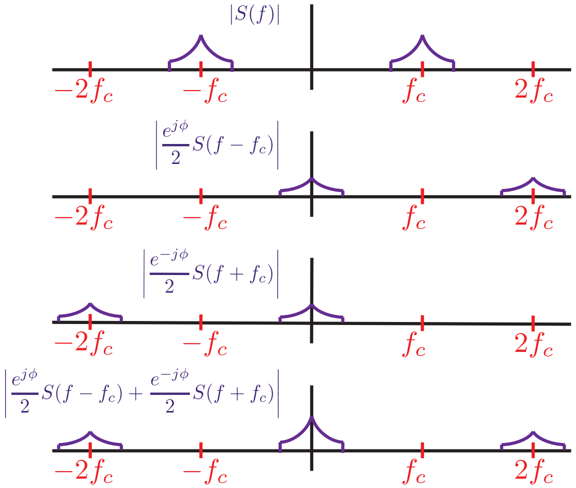
Costas Receiver: In-Phase Coherent Detector

$$
\begin{aligned}
V_I(f) &= \frac{e^{j\phi}}{2} S(f-f_c) + \frac{e^{-j\phi}}{2} S(f+f_c) \\
&= \frac{\cos(\phi) + j\sin(\phi)}{2} S(f-f_c) + \frac{\cos(-\phi) + j\sin(-\phi)}{2} S(f+f_c) \\
&= \frac{\cos(\phi) + j\sin(\phi)}{2} S(f-f_c) + \frac{\cos(\phi) - j\sin(\phi)}{2} S(f+f_c) \\
&= \underbrace{\frac{\cos(\phi)}{2}}_{\approx 1/2} \left[\underbrace{S(f-f_c) + S(f+f_c)}_{\text{significant baseband}} \right] + j \underbrace{\frac{\sin(\phi)}{2}}_{\text{small}} \left[\underbrace{S(f-f_c) - S(f+f_c)}_{\text{negligible baseband}} \right]
\end{aligned}
$$

for $\phi \ll 1$.

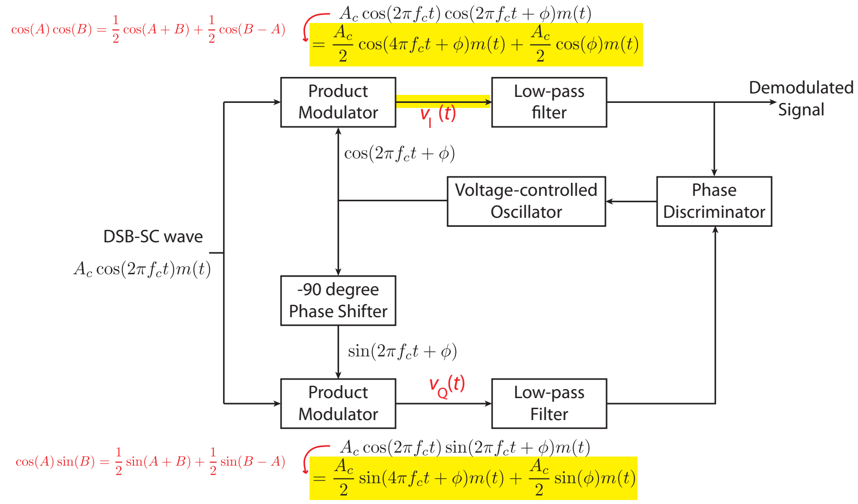
The closer ϕ is to zero, the more significant the baseband term of $v_I(t)$ and vice versa.

Costas Receiver: In-Phase Coherent Detector



For ϕ small.

Costas Receiver: Quadrature-Phase Detector



Costas Receiver: Quadrature-Phase Detector

$$\begin{aligned}
s(t) &= A_c\cos(2\pi f_c t)m(t)\\
v_Q(t) &= s(t)\cdot\sin(2\pi f_c t+\phi)
\end{aligned}$$

Recall

$$\sin(2\pi f_c t+\phi)=\frac{e^{j\phi}}{2j}e^{j2\pi f_c t}-\frac{e^{-j\phi}}{2j}e^{-j2\pi f_c t}\rightleftharpoons\frac{e^{j\phi}}{2j}\delta(f-f_c)-\frac{e^{-j\phi}}{2j}\delta(f+f_c)$$

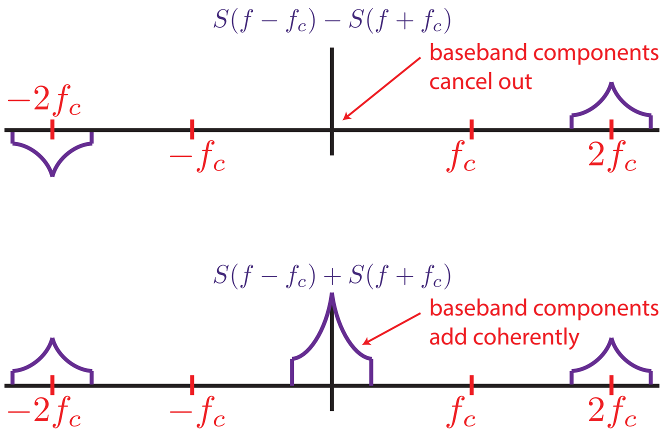
$$\begin{aligned}
V_Q(f) &= S(f)\star\left[\frac{e^{j\phi}}{2j}\delta(f-f_c)-\frac{e^{-j\phi}}{2j}\delta(f+f_c)\right]\\
&= \frac{e^{j\phi}}{2j}S(f)\star\delta(f-f_c)-\frac{e^{-j\phi}}{2j}S(f)\star\delta(f+f_c)\\
&= \frac{e^{j\phi}}{2j}S(f-f_c)-\frac{e^{-j\phi}}{2j}S(f+f_c)
\end{aligned}$$

Costas Receiver: Quadrature-Phase Detector

$$\begin{aligned}
V_Q(f) &= \frac{e^{j\phi}}{2j}S(f-f_c)-\frac{e^{-j\phi}}{2j}S(f+f_c)\\
&= \frac{\cos(\phi)+j\sin(\phi)}{2j}S(f-f_c)-\frac{\cos(-\phi)+j\sin(-\phi)}{2j}S(f+f_c)\\
&= \frac{\cos(\phi)+j\sin(\phi)}{2j}S(f-f_c)-\frac{\cos(\phi)-j\sin(\phi)}{2j}S(f+f_c)\\
&= \underbrace{\frac{\cos(\phi)}{2j}}_{\approx 1/2j}[S(f-f_c)-S(f+f_c)]+j\underbrace{\frac{\sin(\phi)}{2j}}_{\text{small}}[S(f-f_c)+S(f+f_c)]
\end{aligned}$$

for $\phi \ll 1$.

Costas Receiver: Quadrature-Phase Detector



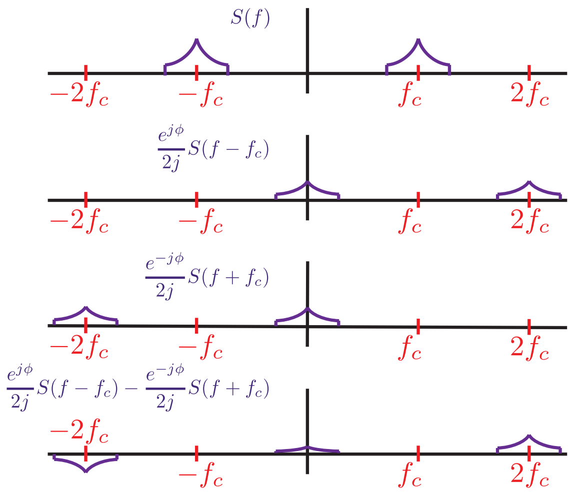
Costas Receiver: Quadrature-Phase Detector

$$
\begin{aligned}
V_Q(f) &= \frac{e^{j\phi}}{2j}S(f-f_c) - \frac{e^{-j\phi}}{2j}S(f+f_c) \\
&= \frac{\cos(\phi)+j\sin(\phi)}{2j}S(f-f_c) - \frac{\cos(-\phi)+j\sin(-\phi)}{2j}S(f+f_c) \\
&= \frac{\cos(\phi)+j\sin(\phi)}{2j}S(f-f_c) - \frac{\cos(\phi)-j\sin(\phi)}{2j}S(f+f_c) \\
&= \underbrace{\frac{\cos(\phi)}{2j}}_{\approx 1/2j}\Bigg[\underbrace{S(f-f_c)-S(f+f_c)}_{\text{negligible baseband}}\Bigg] + j\underbrace{\frac{\sin(\phi)}{2j}}_{\text{small}}\Bigg[\underbrace{S(f-f_c)+S(f+f_c)}_{\text{significant baseband}}\Bigg]
\end{aligned}
$$

for $\phi \ll 1$.

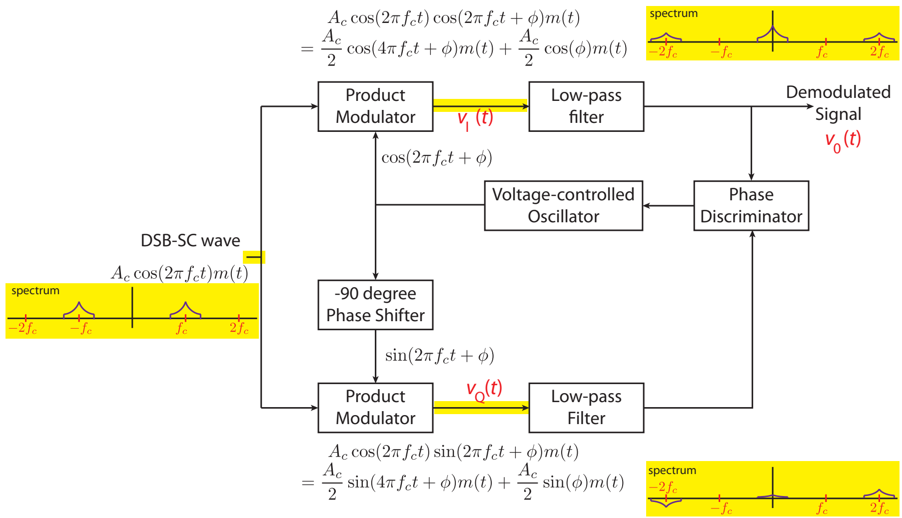
The closer ϕ is to zero, the more negligible the baseband term of $v_Q(t)$ and vice versa.

Costas Receiver: Quadrature-Phase Detector

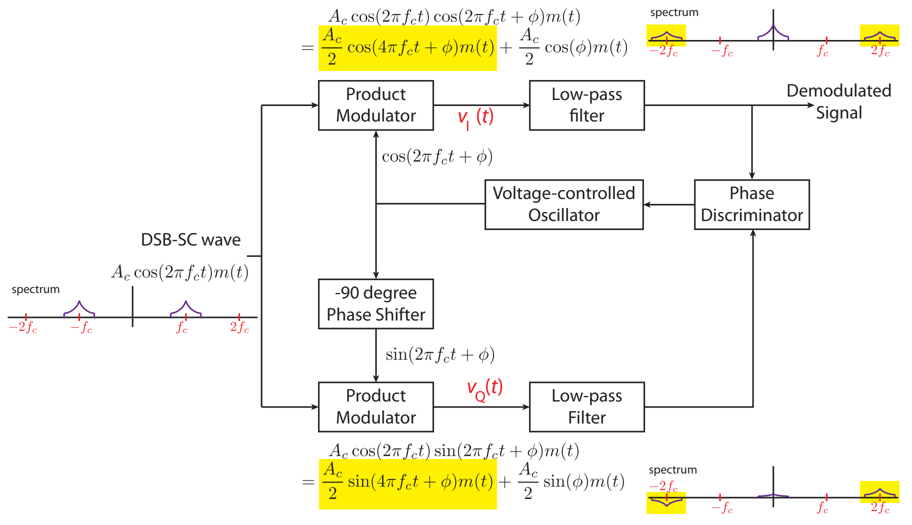


For ϕ small.

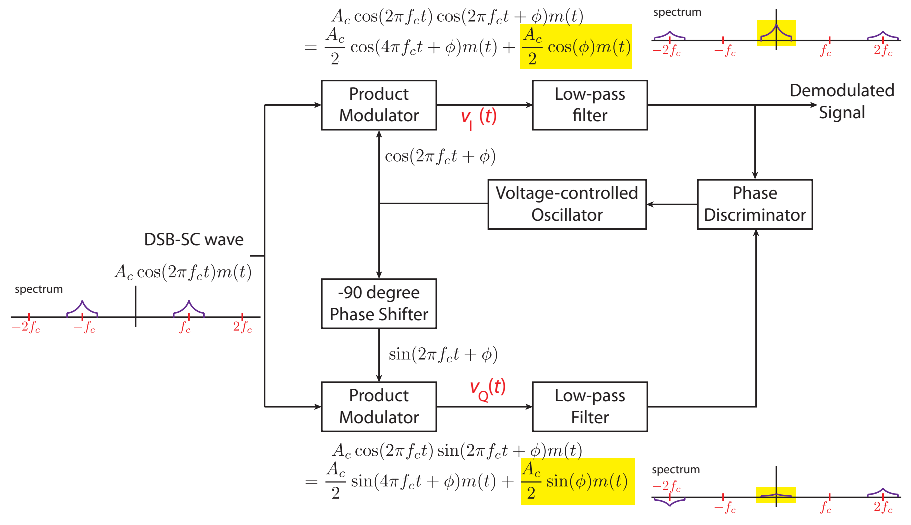
Costas Receiver: $v_I(t)$ and $v_Q(t)$



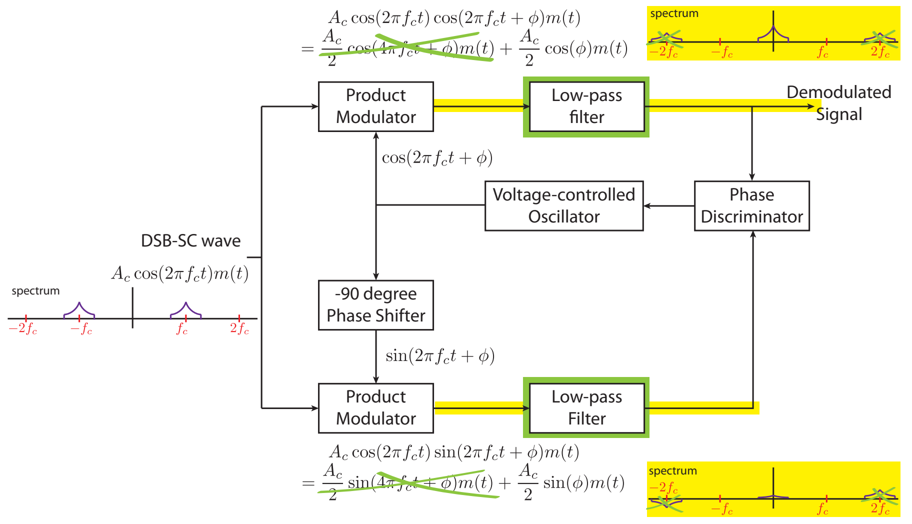
Costas Receiver: $v_I(t)$ and $v_Q(t)$



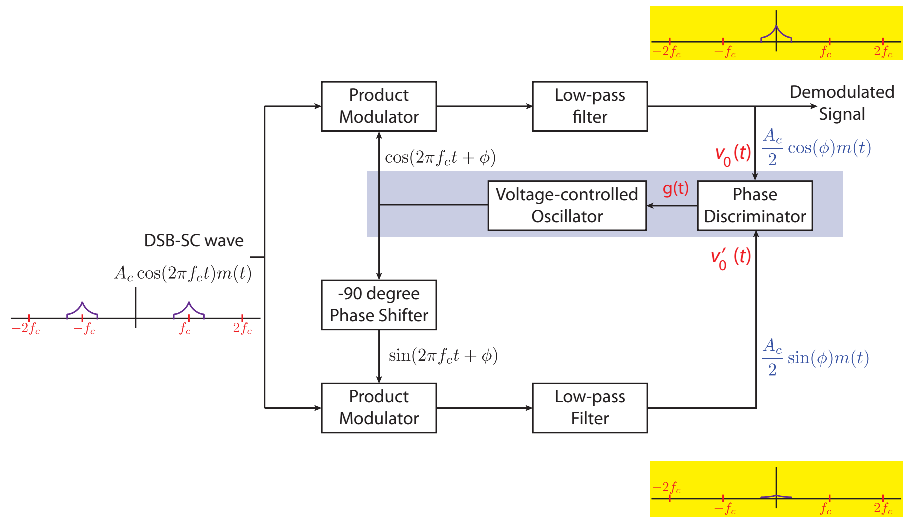
Costas Receiver: $v_I(t)$ and $v_Q(t)$



Costas Receiver: Low-Pass Filter



Costas Receiver: Local Oscillator Control



Costas Receiver: Phase Discriminator

Two components in sequence:

(1) multiplier

$$\begin{aligned} v_0(t) \cdot v_0'(t) &= \frac{A_c}{2}\cos(\phi)m(t) \cdot \frac{A_c}{2}\sin(\phi)m(t) \\ &= \frac{A_c^2}{4} \underbrace{\cos(\phi)}_{\approx 1} \underbrace{\sin(\phi)}_{\approx \phi} m^2(t) \\ &= \frac{A_c^2}{4}\phi m^2(t) \end{aligned}$$

for $\phi \ll 1$.

Costas Receiver: Phase Discriminator

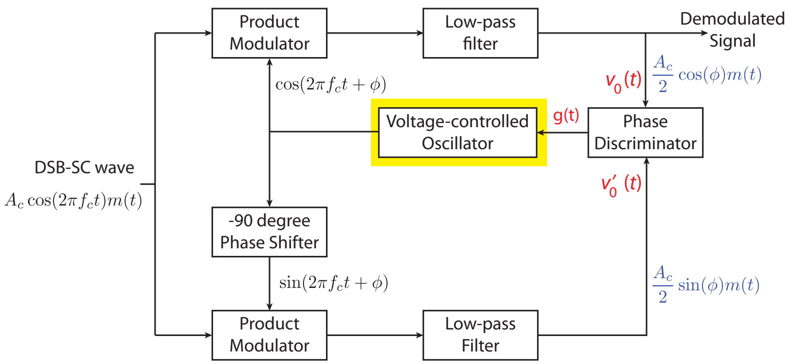
Two components in sequence:

(2) time-averaging unit

$$\begin{aligned} g(t) &= \frac{1}{2T}\int_{-T}^{T} \frac{A_c^2}{4}\phi m^2(t)dt \\ &= \frac{A_c^2}{4}\phi \underbrace{\frac{1}{2T}\int_{-T}^{T} m^2(t)dt}_{\text{power of } m(t)\text{; for } T \text{ large is constant}} \end{aligned}$$

Therefore, $g(t)$ is proportional to ϕ, and is the same sign as the phase error ϕ.

Costas Receiver: Voltage-controlled Oscillator



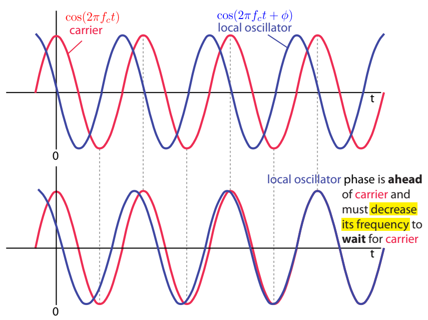
Costas Receiver: Voltage-controlled Oscillator

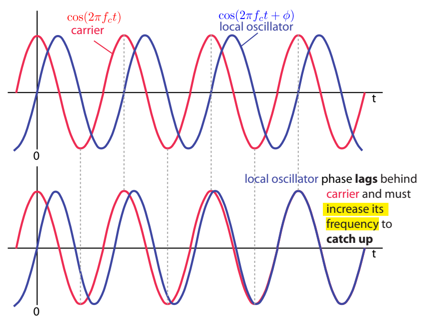
- If $g(t) > 0$ (or $\phi > 0$), then the local oscillator will decrease from f_c proportional to the value of $g(t)$ (or ϕ).
- If $g(t) < 0$ (or $\phi < 0$), then the local oscillator will increase from f_c proportional to the value of $g(t)$ (or ϕ).

Costas Receiver: Voltage-controlled Oscillator

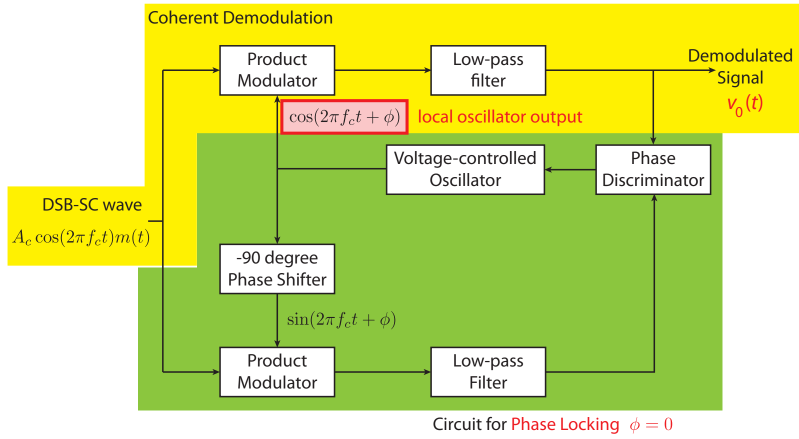
$\phi > 0$: Freq of local oscillator needs to temporarily decrease

$\phi < 0$: Freq of local oscillator needs to temporarily increase





Costas Receiver



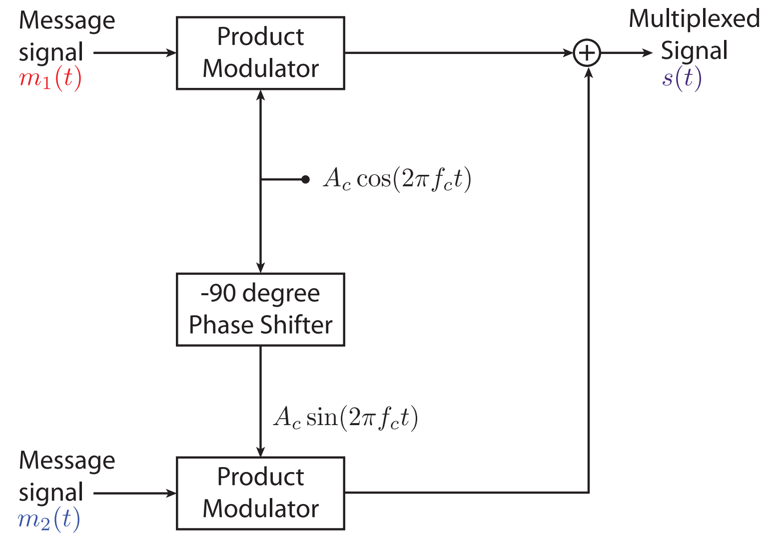
Quadrature Amplitude Multiplexing

Multiplexing and QAM

Multiplexing: to send multiple message simultaneously

Quadrature Amplitude Multiplexing (QAM): (a.k.a quadrature-carrier multiplexing) amplitude modulation scheme that enables two DSB-SC waves with independent message signals to occupy the same channel bandwidth (i.e., same frequency channel) yet still be separated at the receiver.

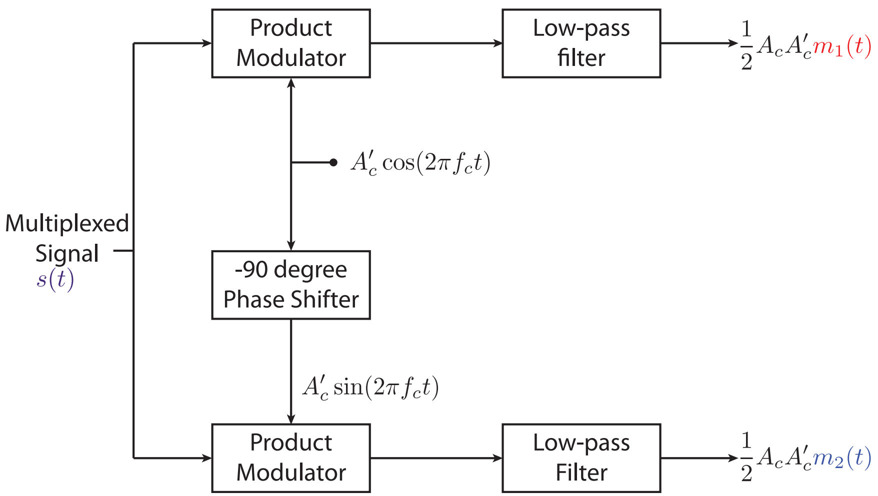
QAM: Transmitter



QAM: Transmitter

$$s(t) = A_c m_1(t) \cos(2\pi f_c t) + A_c m_2(t) \sin(2\pi f_c t)$$

QAM: Receiver



QAM: Receiver

- Costas receiver may be used to synchronize the local oscillator for demodulation.

■