

# Secret-Key Generation over Reciprocal Fading Channels

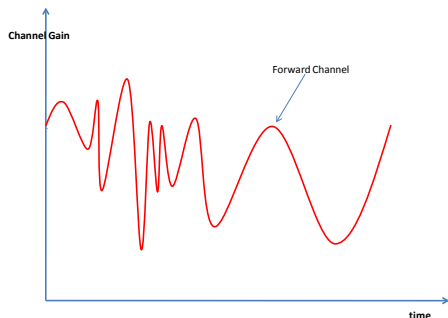
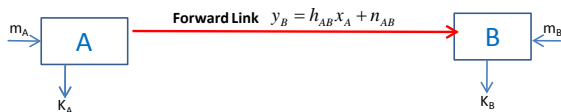
Ashish Khisti

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Nov. 14, 2012

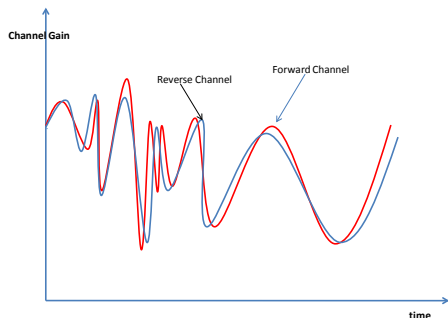
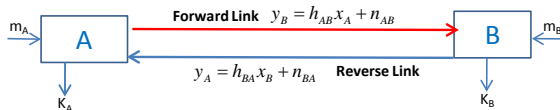
# Motivation

## Secret-Key Generation in Wireless Fading Channels



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## Secret-Key Generation in Wireless Fading Channels



**Fading:**

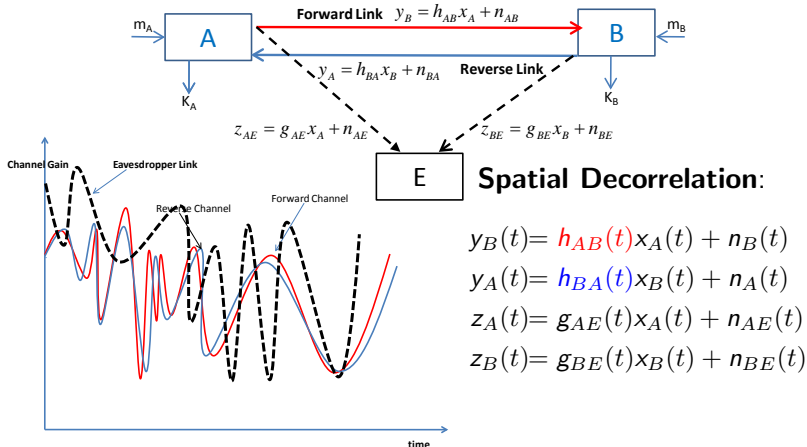
$$y_B(t) = h_{AB}(t)x_A(t) + n_B(t)$$

**Reciprocity:**

$$y_B(t) = h_{AB}(t)x_A(t) + n_B(t)$$

$$y_A(t) = h_{BA}(t)x_B(t) + n_A(t)$$

## Secret-Key Generation in Wireless Fading Channels



### Spatial Decorrelation:

$$y_B(t) = h_{AB}(t)x_A(t) + n_B(t)$$

$$y_A(t) = h_{BA}(t)x_B(t) + n_A(t)$$

$$z_A(t) = g_{AE}(t)x_A(t) + n_{AE}(t)$$

$$z_B(t) = g_{BE}(t)x_B(t) + n_{BE}(t)$$

## Secret-Key Generation in Wireless Systems

- A. Hassan, W. Stark, J. Hershey, and S. Chennakeshu ('96)
- **UWB Systems**: Wilson-Tse-Scholz ('07), M. Ko ('07), Madiseh-Neville-McGuire('12)
- **Experimental UWB**: Measurements for Key Generation Madiseh ('12)
- **Narrowband Systems**: Azimi Sadjadi- Kiayias-Mercado-Yener ('07), Mathur-Trappe-Mandayam -Ye-Reznick ('10), Patware and Kasera ('07)
- **OFDM reciprocity**: Haile ('09), Tsouri and Wulich ('09)
- **Quantization Techniques**: Ye-Reznick-Shah ('07), Hamida-Pierrot-Castelluccia ('09), Sun-Zhu-Jiang-Zhao ('11)
- **Adaptive Channel Probing**: Wei-Zheng-Mohapatra ('10)
- **Unauthenticated Channels, Impersonation Attacks, Spoofing**: Mathur et al. ('10), Xiao-Greenstein-Mandayam-Trappe ('07).
- **Mobility Assisted Key Generation**: Zhang-Kasera-Patwari ('10), Gungor-Chen-Koksal ('11)
- **Active Eavesdroppers**: Zafer-Agrawal-Srivatsa
- **Software Radio Implementations**: Jana et. al. ('09)
- **MIMO systems**: Wallace and Sharma ('10), Shimizu et al. Zeng-Wu-Mohapatra

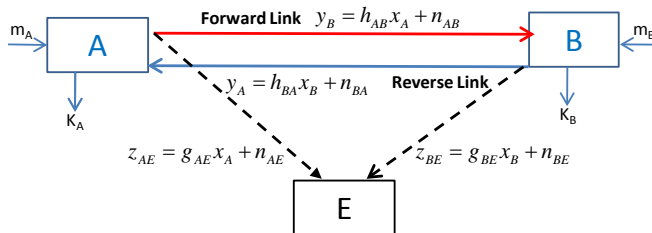
## Information Theoretic Secret-Key Generation:

- Information Theoretic Secrecy: Shannon '49
- [Secret-Key Generation from Correlated Randomness](#): Maurer ('93), Csiszar-Ahlsvede ('93)
- Strong Secrecy: Csiszar ('96), Maurer-Wolf ('00), Watanabe ('11)
- Secret-Key Generation over Unauthenticated Channels: Maurer and Wolf ('03)
- Multi-terminal Secret-Key Generation: Csiszar-Narayan ('04)
- Joint Source-Channel Coding: Khisti-Diggavi-Wornell ('12), Prabhakaran-Eswaran-Ramchandran ('12)
- Secret-Key Generation over Channels with State: Khisti-Diggavi-Wornell ('12), Khisti ('10), Zibaeenejad ('12)
- Secret-Key generation over Two-Way channels: Ahmadi and Safavi-Naini ('11)
- Network Coding for Secret-Key Agreement: Chan ('11)
- Authentication based on Secret-Key Generation: Willems and T. Ignatenko ('12)
- Minimum Rate for Secret-Key Generation: Tyagi ('12)

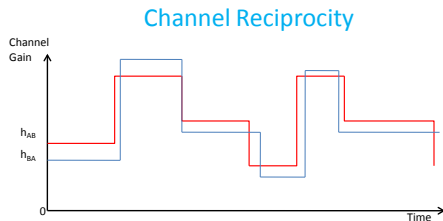
## Observation

- There exists a disconnect between the Information Theoretic Models and Practical Systems for Secret-Key Generation
- No Information Theoretic limits are known!
- No provably optimal signalling scheme is known.

# Problem Setup

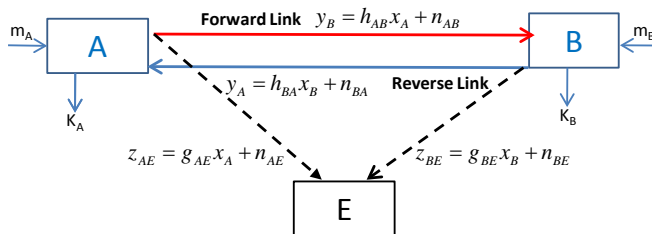


- **No CSI:**  $h_{AB}(i)$  and  $h_{BA}(i)$
- $g_A(i)$  &  $g_B(i)$  known to Eve
- **Block-Fading:**  
Coherence Period:  $T$ .
- **Approximate Reciprocity:**  
 $(h_{AB}, h_{BA}) \sim p_{h_{AB}, h_{BA}}(\cdot, \cdot)$
- **Independence:**  
 $(g_{AE}, g_{BE}) \perp (h_{AB}, h_{BA})$





# Problem Setup



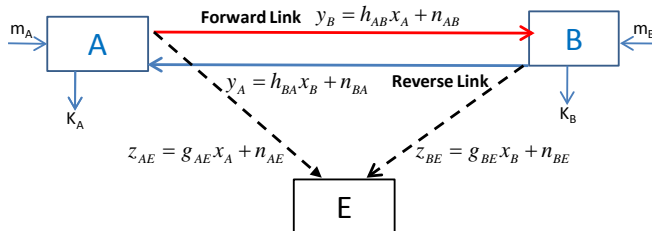
Two Way Channel:

$$y_B(i) = h_{AB}(i)x_A(i) + n_{AB}(i), \quad y_A(i) = h_{BA}(i)x_B(i) + n_{BA}(i)$$
$$z_{AE}(i) = g_{AE}(i)x_A(i) + n_{AE}(i), \quad z_{BE}(i) = g_{BE}(i)x_B(i) + n_{BE}(i)$$

Interactive Comm.:  $x_A(i) = f_A(m_A, y_A^{i-1})$ ,  $x_B(i) = f_B(m_B, y_B^{i-1})$

Average Power Constraint  $E[|x_A|^2] \leq P$ ,  $E[|x_B|^2] \leq P$ .

# Problem Setup



## Secret-Key Generation

- $k_A = \mathcal{K}_A(y_A^N, m_A)$ ,  $k_B = \mathcal{K}_B(y_B^N, m_B)$
- **Reliability:**  $\Pr(k_A \neq k_B) \leq \epsilon_N$
- **Secrecy:**  $I(k_A; z_A^N, z_B^N, g_A^N, g_B^N) \leq N\epsilon_N$
- Rate  $R = \frac{1}{N}H(k_A)$

## Secret-Key Capacity.

# Secret-Key Capacity — Upper Bound

Khisti'12

## Theorem

*An upper bound on the secret-key capacity is given by:*

$$R^+ \leq \frac{1}{T} I(\mathbf{h}_{AB}; \mathbf{h}_{BA}) + \max_{P(\mathbf{h}_{AB}) \in \mathcal{P}} \{I(y_B; x_A | \mathbf{h}_{AB}, z_A, \mathbf{g}_A)\} \\ + \max_{P(\mathbf{h}_{BA}) \in \mathcal{P}} I(y_A; x_B | \mathbf{h}_{BA}, z_B, \mathbf{g}_B)$$

*where:*  $p_{x_A | h_{AB}} \equiv \mathcal{CN}(0, P(\mathbf{h}_{AB}))$ ,  $p_{x_B | h_{BA}} \equiv \mathcal{CN}(0, P(\mathbf{h}_{BA}))$ .

## Theorem

An upper bound on the secret-key capacity is given by:

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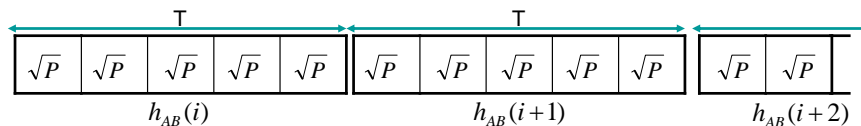
where:  $p_{x_A|h_{AB}} \equiv \mathcal{CN}(0, P(h_{AB}))$ ,  $p_{x_B|h_{BA}} \equiv \mathcal{CN}(0, P(h_{BA}))$ .

Interpretation of the Upper Bound:

- Channel Reciprocity:  $\frac{1}{T} I(h_{AB}; h_{BA})$
- Forward Channel:  $I(y_B; x_A | h_{AB}, z_A, g_A)$
- Reverse Channel:  $I(y_A; x_B | h_{BA}, z_B, g_B)$

# Training-Only Scheme

Probe  $K$  Coherence Blocks

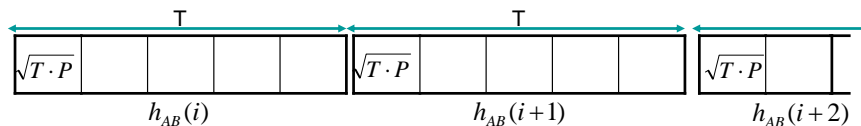


- $x_A(i, t) = \sqrt{P}$
- $\mathbf{y}_B(i) = \sqrt{P} \cdot h_{AB}(i) \cdot \mathbf{1} + \mathbf{n}(i)$
- $\hat{h}_{AB}(i)$ : MMSE estimate
- Estimate  $\hat{h}_{AB}^K$  on the forward link;  $\hat{h}_{BA}^K$  on the reverse link.

Secret-Key Rate:  $R^+ = \frac{1}{T} I(\hat{h}_{AB}; \hat{h}_{BA})$

# Training-Only Scheme

Probe  $K$  Coherence Blocks

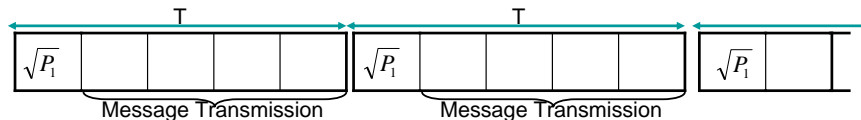


- $x_A(i, 1) = \sqrt{T \cdot P}$ ,  $x_A(i, t) = 0$ ,  $i = 1 \dots, K$ ,  $t = 2, \dots, T$ .
- $y_B(i) = \sqrt{T \cdot P} h_{AB}(i) + n(i)$ ,  $i = 1, 2, \dots, K$
- $\hat{h}_{AB}(i)$ : MMSE estimate
- Estimate  $\hat{h}_{AB}^K$  on the forward link;  $\hat{h}_{BA}^K$  on the reverse link.

Secret-Key Rate:  $R^+ = \frac{1}{T} I(\hat{h}_{AB}; \hat{h}_{BA})$

# Message Transmission

Lai-Liang-Poor '12



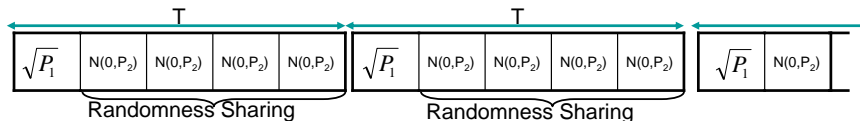
- **Training:**  $x_A(i, 1) = \sqrt{P_1}$ ,  $R_T = \frac{1}{T} I(\hat{h}_{AB}; \hat{h}_{BA})$
- **Secure Msg. Transmission:**  $\{x_A(i, 2), \dots, x_A(i, T)\}_{i=1,2,\dots,K}$   
 $R_M = \frac{T-1}{T} E \left[ \log(1 + P_2(\hat{h}_{AB})|\hat{h}_{AB}|^2) - \log(1 + P_2(\hat{h}_{AB})|g_A|^2) \right]$

The overall rate is NOT:  $R_T + R_M$

- Power Allocation in  $R_M$  leaks  $\hat{h}_{AB}$  to Eavesdropper
- Without Power Allocation,  $R_M$  is generally zero.

# Proposed Scheme: Randomness Sharing

Khisti '12

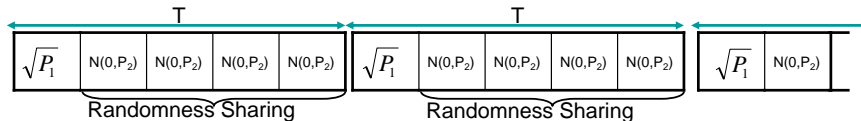


- **Training:**  $x_A(i, 1) = \sqrt{P_1}$
- **Randomness Sharing:**  $x_A(i, t) \sim \mathcal{CN}(0, P_2)$  for  $t = 2, \dots, T$   
 $\mathbf{x}_A(i) = [x_A(i, 2), \dots, x_A(i, T)] \in \mathbb{C}^{T-1}$ .
- **Training:**  $\hat{h}_{AB}(i)$  and  $\hat{h}_{BA}(i)$
- **Correlated Sources:**  
Forward Channel:  $\mathbf{y}_B(i) = h_{AB}(i)\mathbf{x}_A(i) + \mathbf{n}_B(i) \in \mathbb{C}^{T-1}$ ,  
Reverse Channel:  $\mathbf{y}_A(i) = h_{BA}(i)\mathbf{x}_B(i) + \mathbf{n}_A(i) \in \mathbb{C}^{T-1}$ .



# Proposed Scheme: Randomness Sharing

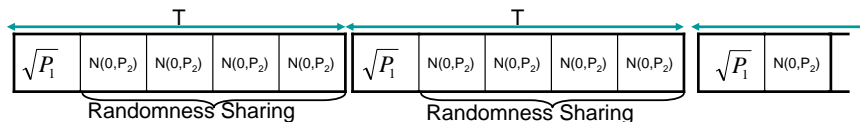
Khisti '12



	$A$	$B$	$E$
Channel State	$\hat{h}_{BA}^K$	$\hat{h}_{AB}^K$	$(\mathbf{g}_A^K, \mathbf{g}_B^K)$
Forward Channel	$\mathbf{x}_A^K$	$\mathbf{y}_B^K$	$\mathbf{z}_{AE}^K$
Reverse Channel	$\mathbf{y}_A^K$	$\mathbf{x}_B^K$	$\mathbf{z}_{BE}^K$

# Proposed Scheme: Randomness Sharing

Khisti '12



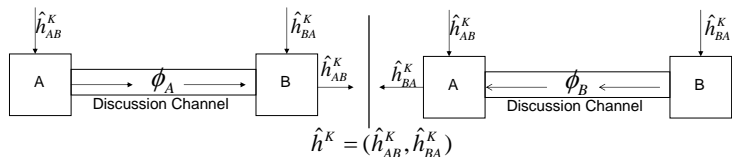
	$A$	$B$	$E$
Channel State	$\hat{h}_{BA}^K$	$\hat{h}_{AB}^K$	$(\mathbf{g}_A^K, \mathbf{g}_B^K)$
Forward Channel	$\mathbf{x}_A^K$	$\mathbf{y}_B^K$	$\mathbf{z}_{AE}^K$
Reverse Channel	$\mathbf{y}_A^K$	$\mathbf{x}_B^K$	$\mathbf{z}_{BE}^K$

Generate a secret-key from these sequences.

# Error Reconciliation

## Public Discussion Channel, Discrete-Valued Sequences

### Channel-Sequence Reconciliation

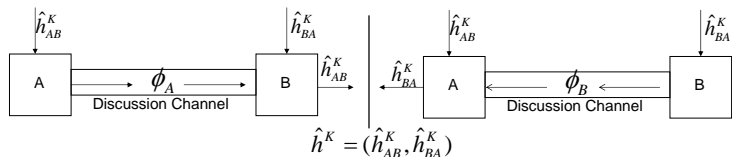


$$H(\phi_A) = H(\hat{h}_{BA} | \hat{h}_{AB}), \quad H(\phi_B) = H(\hat{h}_{AB} | \hat{h}_{BA})$$

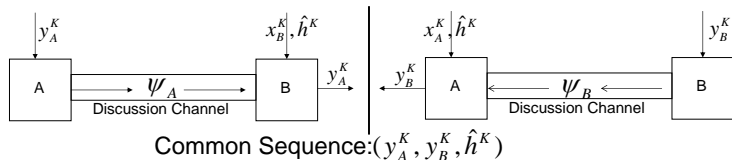
# Error Reconciliation

Public Discussion Channel, Discrete-Valued Sequences

## Channel-Sequence Reconciliation



## Source-Sequence Reconciliation



$$H(\psi_A) \leq H(y_A | x_B, \hat{h}_{AB}, \hat{h}_{BA}), \quad H(\psi_B) \leq H(y_B | x_A, \hat{h}_{AB}, \hat{h}_{BA})$$

# Equivocation Bound

- Public Messages:  $\{\phi_A, \phi_B, \psi_A, \psi_B\}$
- Common Sequences:  $(\mathbf{y}_A^K, \mathbf{y}_B^K, \hat{\mathbf{h}}_{AB}^K, \hat{\mathbf{h}}_{BA}^K)$
- Equivocation Rate:  
$$\frac{1}{T \cdot K} H(\mathbf{y}_A^K, \mathbf{y}_B^K, \hat{\mathbf{h}}_{AB}^K, \hat{\mathbf{h}}_{BA}^K | \phi_A, \phi_B, \psi_A, \psi_B, \mathbf{z}^K, \mathbf{g}^K)$$

# Equivocation Bound

Equivocation-Rate Bound:

$$\begin{aligned} & \frac{1}{T \cdot K} H(\mathbf{y}_A^K, \mathbf{y}_B^K, \hat{\mathbf{h}}_{AB}^K, \hat{\mathbf{h}}_{BA}^K | \phi_A, \phi_B, \psi_A, \psi_B, \mathbf{z}^K, \mathbf{g}^K) \\ & \geq \frac{1}{T \cdot K} \left\{ H(\mathbf{y}_A^K, \mathbf{y}_B^K, \hat{\mathbf{h}}_{AB}^K, \hat{\mathbf{h}}_{BA}^K | \mathbf{z}_A^K, \mathbf{z}_B^K, \mathbf{g}_A^K, \mathbf{g}_B^K) \right. \\ & \quad \left. - \underbrace{H(\phi_A) - H(\phi_B) - H(\psi_A) - H(\psi_B)}_{=\Delta} \right\} \\ & \geq \frac{1}{T \cdot K} \left\{ H(\hat{\mathbf{h}}_{AB}^K, \hat{\mathbf{h}}_{BA}^K) + H(\mathbf{y}_A^K, \mathbf{y}_B^K | \mathbf{z}_A^K, \mathbf{z}_B^K, \mathbf{g}_A^K, \mathbf{g}_B^K, \hat{\mathbf{h}}_{AB}^K, \hat{\mathbf{h}}_{BA}^K) - \Delta \right\} \\ & \geq \frac{1}{T \cdot K} \left\{ H(\mathbf{y}_A^K | \hat{\mathbf{h}}_{BA}^K, \mathbf{z}_B^K, \mathbf{g}^K) + H(\mathbf{y}_B^K | \hat{\mathbf{h}}_{AB}^K, \mathbf{z}_A^K, \mathbf{g}^K) \right. \\ & \quad \left. + H(\hat{\mathbf{h}}_{AB}^K, \hat{\mathbf{h}}_{BA}^K) - \Delta \right\} \end{aligned}$$

# Equivocation Bound

$$\begin{aligned} & \frac{1}{T \cdot K} H(\mathbf{y}_A^K, \mathbf{y}_B^K, \hat{\mathbf{h}}_{AB}^K, \hat{\mathbf{h}}_{BA}^K | \phi_A, \phi_B, \psi_A, \psi_B, \mathbf{z}^K, \mathbf{g}^K) \\ & \geq \left\{ \underbrace{\frac{1}{T} I(\hat{\mathbf{h}}_{AB}; \hat{\mathbf{h}}_{BA})}_{\text{Training}} + \underbrace{\frac{T-1}{T} \left[ I(y_B; x_A, \hat{\mathbf{h}}_{AB}) - I(y_B; z_A, \mathbf{g}_A, h_{AB}) \right]}_{\text{Forward Channel}} \right. \\ & \quad \left. + \frac{T-1}{T} \underbrace{\left[ I(y_A; x_B, \hat{\mathbf{h}}_{BA}) - I(y_A; z_B, \mathbf{g}_B, h_{BA}) \right]}_{\text{Reverse Channel}} \right\} = R_{\text{key}} \end{aligned}$$

# Equivocation Bound

$$\begin{aligned}
 & \frac{1}{T \cdot K} H(\mathbf{y}_A^K, \mathbf{y}_B^K, \hat{\mathbf{h}}_{AB}^K, \hat{\mathbf{h}}_{BA}^K | \phi_A, \phi_B, \psi_A, \psi_B, \mathbf{z}^K, \mathbf{g}^K) \\
 & \geq \left\{ \underbrace{\frac{1}{T} I(\hat{\mathbf{h}}_{AB}; \hat{\mathbf{h}}_{BA})}_{\text{Training}} + \underbrace{\frac{T-1}{T} \left[ I(y_B; x_A, \hat{\mathbf{h}}_{AB}) - I(y_B; z_A, \mathbf{g}_A, h_{AB}) \right]}_{\text{Forward Channel}} \right. \\
 & \quad \left. + \frac{T-1}{T} \underbrace{\left[ I(y_A; x_B, \hat{\mathbf{h}}_{BA}) - I(y_A; z_B, \mathbf{g}_B, h_{BA}) \right]}_{\text{Reverse Channel}} \right\} = R_{\text{key}}
 \end{aligned}$$

$$\begin{aligned}
 R^+ & \leq \frac{1}{T} I(h_{AB}; h_{BA}) + \max_{P(h_{AB}) \in \mathcal{P}} \{ I(y_B; x_A | h_{AB}, z_A, \mathbf{g}_A) \} \\
 & \quad + \max_{P(h_{BA}) \in \mathcal{P}} I(y_A; x_B | h_{BA}, z_B, \mathbf{g}_B)
 \end{aligned}$$



## Theorem

*In the high SNR regime our upper and lower bounds coincide:*

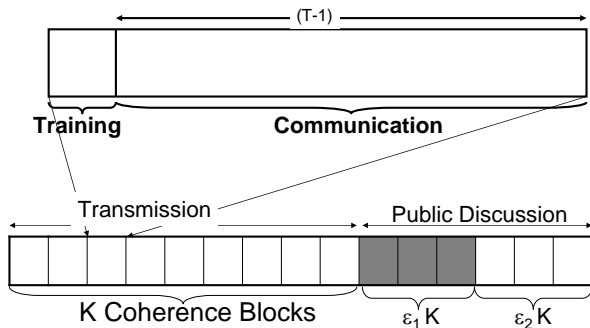
$$\lim_{P \rightarrow \infty} \left\{ R^+(P) - R_{\text{PD}}^-(P) \right\} \leq \frac{c}{T}$$

where

$$c = E \left[ \log \left( 1 + \frac{|h_{AB}|^2}{|g_{AE}|^2} \right) \right] + E \left[ \log \left( 1 + \frac{|h_{BA}|^2}{|g_{BE}|^2} \right) \right]$$

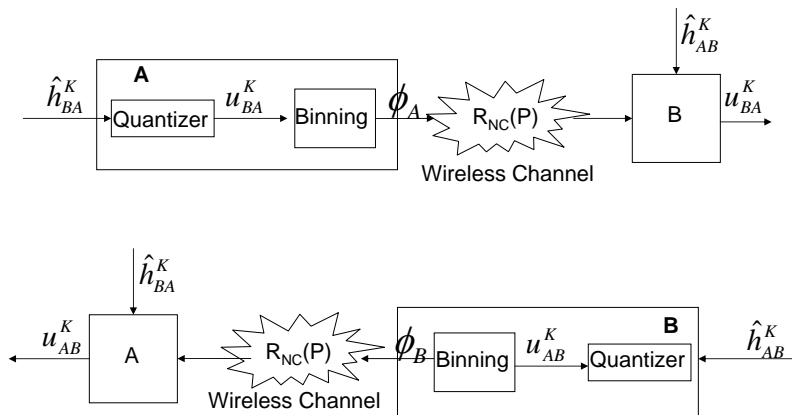
# Separation Scheme

Without Public Discussion



Phase	Coherence Blocks
Probing + Randomness Sharing	$K$
Channel-Sequence Reconciliation	$\epsilon_1 \cdot K$
Source-Sequence Reconciliation	$\epsilon_2 \cdot K$

# Error Reconciliation - Channel Sequences

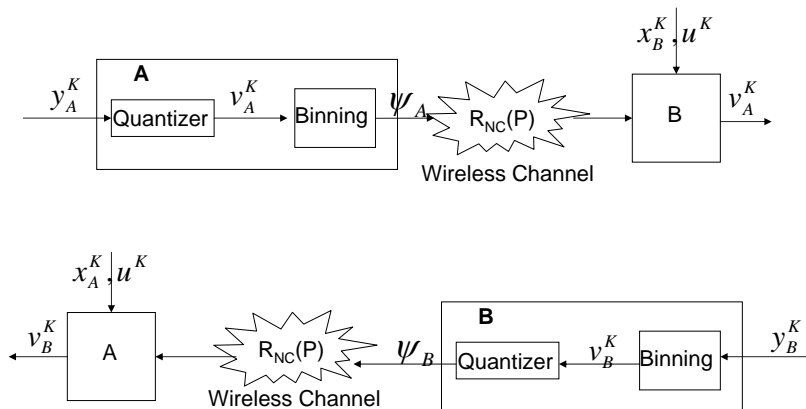


Common Sequence:  $\mathbf{u}^K \triangleq (u_{AB}^K, u_{BA}^K)$ .

Rate Constraints:

- $I(u_{BA}; \hat{h}_{BA} | \hat{h}_{AB}) \leq \varepsilon_1 (T - 1) R_{NC}(P)$
- $I(u_{AB}; \hat{h}_{AB} | \hat{h}_{BA}) \leq \varepsilon_1 (T - 1) R_{NC}(P)$

# Error Reconciliation - Source Sequences



Rate Constraints:

$$I(v_A; y_A | x_B, \mathbf{u}) \leq \varepsilon_2 \cdot R_{NC}(P), \quad I(v_B; y_B | x_A, \mathbf{u}) \leq \varepsilon_2 \cdot R_{NC}(P)$$

# Secret-Key Rate

Without Public Discussion

$$R = \frac{1}{1 + \varepsilon_1 + \varepsilon_2} \left( \frac{1}{T} R_T + \frac{T-1}{T} R_F + \frac{T-1}{T} R_B \right)$$

# Secret-Key Rate

Without Public Discussion

$$R = \frac{1}{1 + \varepsilon_1 + \varepsilon_2} \left( \frac{1}{T} R_T + \frac{T-1}{T} R_F + \frac{T-1}{T} R_B \right)$$

$$R_T = I(\mathbf{u}_{AB}; \hat{\mathbf{h}}_{BA}) + I(\mathbf{u}_{BA}; \hat{\mathbf{h}}_{AB}) - I(\mathbf{u}_{AB}; \mathbf{u}_{BA})$$

$$R_F = I(\mathbf{v}_A; \mathbf{x}_B, \mathbf{u}_{AB}, \mathbf{u}_{BA}) - I(\mathbf{v}_A; \mathbf{z}_B, \mathbf{g}_B, \mathbf{h}_{BA})$$

$$R_B = I(\mathbf{v}_B; \mathbf{x}_A, \mathbf{u}_{AB}, \mathbf{u}_{BA}) - I(\mathbf{v}_B; \mathbf{z}_A, \mathbf{g}_A, \mathbf{h}_{AB})$$

Rate Constraints:

$$I(\mathbf{u}_{BA}; \hat{\mathbf{h}}_{BA} | \hat{\mathbf{h}}_{AB}) \leq \varepsilon_1 (T-1) R_{\text{NC}}(P)$$

$$I(\mathbf{u}_{AB}; \hat{\mathbf{h}}_{AB} | \hat{\mathbf{h}}_{BA}) \leq \varepsilon_1 (T-1) R_{\text{NC}}(P)$$

$$I(\mathbf{v}_A; \mathbf{y}_A | \mathbf{x}_B, \mathbf{u}_{AB}, \mathbf{u}_{BA}) \leq \varepsilon_2 R_{\text{NC}}(P)$$

$$I(\mathbf{v}_B; \mathbf{y}_B | \mathbf{x}_A, \mathbf{u}_{AB}, \mathbf{u}_{BA}) \leq \varepsilon_2 R_{\text{NC}}(P)$$

## Theorem

*In the high SNR regime our upper and lower bounds coincide:*

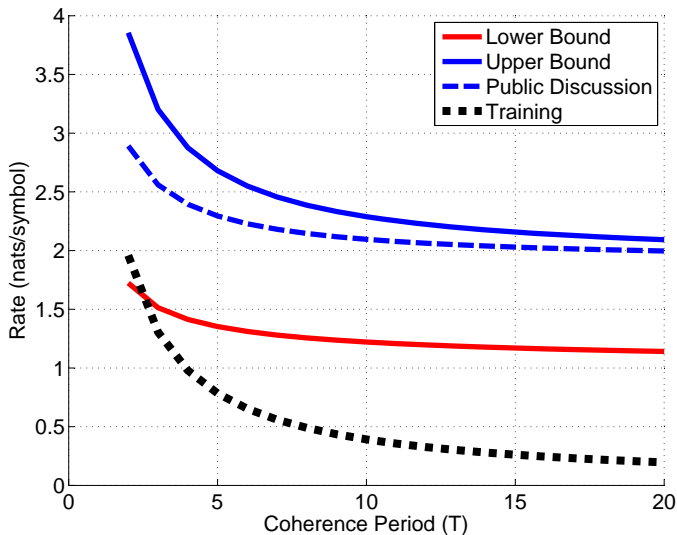
$$\lim_{P \rightarrow \infty} \left\{ R^+(P) - R^-(P) \right\} \leq \frac{c}{T}$$

where

$$c = E \left[ \log \left( 1 + \frac{|h_{AB}|^2}{|g_{AE}|^2} \right) \right] + E \left[ \log \left( 1 + \frac{|h_{BA}|^2}{|g_{BE}|^2} \right) \right]$$

# Numerical Plot

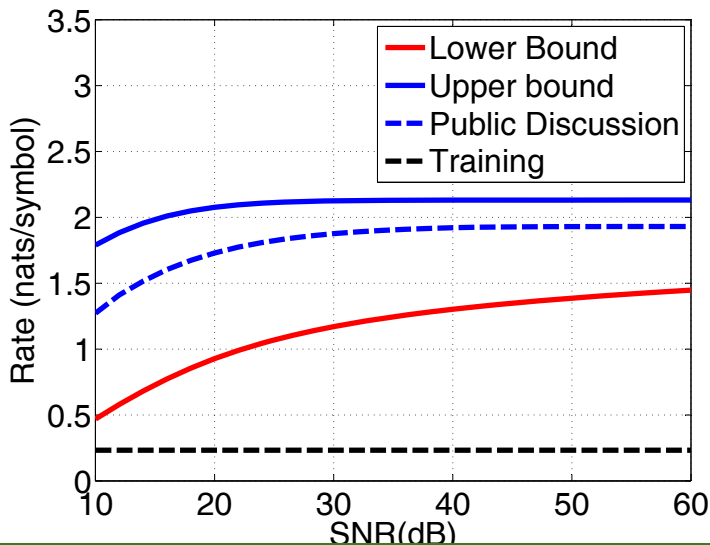
SNR = 35 dB,  $h_1, h_2 \sim \mathcal{CN}(0, 1)$ ,  $\rho = 0.99$ .





# Numerical Plot

$$T = 10, h_1, h_2 \sim \mathcal{CN}(0, 1), \rho = 0.95$$



# Secret-Key Capacity — Upper Bound

Khisti'12

## Theorem

*An upper bound on the secret-key capacity is given by:*

$$R^+ \leq \frac{1}{T} I(\mathbf{h}_{AB}; \mathbf{h}_{BA}) + \max_{P(\mathbf{h}_{AB}) \in \mathcal{P}} \{I(y_B; x_A | \mathbf{h}_{AB}, \mathbf{z}_A, \mathbf{g}_A)\} \\ + \max_{P(\mathbf{h}_{BA}) \in \mathcal{P}} I(y_A; x_B | \mathbf{h}_{BA}, \mathbf{z}_B, \mathbf{g}_B)$$

*where:*  $p_{x_A | h_{AB}} \equiv \mathcal{CN}(0, P(\mathbf{h}_{AB}))$ ,  $p_{x_B | h_{BA}} \equiv \mathcal{CN}(0, P(\mathbf{h}_{BA}))$ .

# Upper Bound - Proof

Maurer '93

$$\begin{aligned}NR &\leq I(k_A; k_B) - I(k_A; \mathbf{z}^N, \mathbf{g}^K) \\ &\leq I(k_A; k_B | \mathbf{z}^N, \mathbf{g}^K) \\ &\leq I(m_A, h_{BA}^N, y_A^N; m_B, h_{AB}^N, y_B^N | \mathbf{z}^N, \mathbf{g}^N)\end{aligned}$$

# Upper Bound - Proof

Maurer '93

$$\begin{aligned}NR &\leq I(k_A; k_B) - I(k_A; \mathbf{z}^N, \mathbf{g}^K) \\ &\leq I(k_A; k_B | \mathbf{z}^N, \mathbf{g}^K) \\ &\leq I(m_A, h_{BA}^N, y_A^N; m_B, h_{AB}^N, y_B^N | \mathbf{z}^N, \mathbf{g}^N) \\ &= I(m_A, h_{BA}^N, y_A^{N-1}; m_B, h_{AB}^N, y_B^{N-1} | \mathbf{z}^N, \mathbf{g}^K) + \\ &\quad I(y_A(N); m_B, h_{AB}^N, y_B^{N-1} | \mathbf{z}^N, \mathbf{g}^K, m_A, h_{BA}^N, y_A^{N-1}) + \\ &\quad I(m_A, h_{BA}^N, y_A^{N-1}; y_B(N) | \mathbf{z}^N, \mathbf{g}^K, m_B, h_{AB}^N, y_B^{N-1}) + \\ &\quad I(y_A(N); y_B(N) | \mathbf{z}^N, \mathbf{g}^K, m_B, h_{AB}^N, y_B^{N-1}, m_A, h_{BA}^N, y_A^{N-1}).\end{aligned}$$

# Upper Bound - Proof

Maurer '93

$$\begin{aligned}NR &\leq I(k_A; k_B) - I(k_A; \mathbf{z}^N, \mathbf{g}^K) \\ &\leq I(k_A; k_B | \mathbf{z}^N, \mathbf{g}^K) \\ &\leq I(m_A, h_{BA}^N, y_A^N; m_B, h_{AB}^N, y_B^N | \mathbf{z}^N, \mathbf{g}^N) \\ &= \underbrace{I(m_A, h_{BA}^N, y_A^{N-1}; m_B, h_{AB}^N, y_B^{N-1} | \mathbf{z}^N, \mathbf{g}^K)}_+ \\ &\quad \leq I(m_A, h_{BA}^N, y_A^{N-1}; m_B, h_{AB}^N, y_B^{N-1} | \mathbf{z}^{N-1}, \mathbf{g}^K) \\ &\quad \underbrace{I(y_A(N); m_B, h_{AB}^N, y_B^{N-1} | \mathbf{z}^N, \mathbf{g}^K, m_A, h_{BA}^N, y_A^{N-1})}_+ \\ &\quad \leq I(x_B(N); y_A(N) | z_B(N), g_B(N), h_{BA}(N)) \\ &\quad \underbrace{I(y_B(N); m_A, h_{BA}^N, y_A^{N-1} | \mathbf{z}^N, \mathbf{g}^K, m_B, h_{AB}^N, y_B^{N-1})}_+ \\ &\quad \leq I(x_A(N); y_B(N) | z_A(N), g_A(N), h_{AB}(N)) \\ &\quad \underbrace{I(y_A(N); y_B(N) | \mathbf{z}^N, \mathbf{g}^K, m_B, h_{AB}^N, y_B^{N-1}, m_A, h_{BA}^N, y_A^{N-1})}_+.\end{aligned}$$

# Upper Bound - Proof

Maurer '93

$$\begin{aligned}NR &\leq I(k_A; k_B) - I(k_A; \mathbf{z}^N, \mathbf{g}^K) \\ &\leq I(k_A; k_B | \mathbf{z}^N, \mathbf{g}^K) \\ &\leq I(m_A, h_{BA}^N, y_A^N; m_B, h_{AB}^N, y_B^N | \mathbf{z}^N, \mathbf{g}^N) \\ &\leq I(h_{AB}^N; h_{BA}^N) + \sum_{i=1}^N I(x_B(i); y_A(i) | z_B(i), \mathbf{g}_B(i), h_{BA}(i)) + \\ &\quad \sum_{i=1}^N I(x_A(i); y_B(i) | z_A(i), \mathbf{g}_A(i), h_{AB}(i))\end{aligned}$$

Optimality of Gaussian Inputs, Power Constraints ...

- Secret-Key Agreement in Two-Way fading channels
- Upper and Lower Bounds on Capacity
- Asymptotic Optimality
- Significant Gains over Training Based Schemes

## Future Work:

- Improved Upper Bounds
- Stationary Fading Channels
- Low SNR Regime
- Stronger Eavesdropper Channels