Efficient User Selection for Downlink Zero-Forcing based Multiuser MIMO Systems

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***Abstract*—In a downlink multiuser multiple-input and multiple-output (MU-MIMO) system, a base station (BS) communicates with multiple mobile stations (MS) simultaneously in a given spectrum band. The performance of a MU-MIMO system depends on the choice of user selection, power allocation and precoding schemes. In this paper, we study user selection in Zero-Forcing (ZF) precoding based MU-MIMO systems to maximize the sum rate for high data rate applications. We derive analytical results for simultaneous transmission to two MSs with ZF precoding, based on which we propose an extended low-complexity algorithm that jointly considers the noise power, the channel gain of the candidate MSs and orthogonality with respect to the selected MSs' channels. Analysis shows that the proposed scheme requires much lower complexity than current schemes. Simulation results demonstrate that the proposed algorithm outperforms existing algorithms in terms of higher throughput.**

I. Introduction

Multiple-input multiple-output (MIMO) technology is shown to achieve high spectral efficiency and diversity gain and thus has become an important part of modern wireless communication standards such as Long Term Evolution (LTE) and High Speed Packet Access (HSPA) [1]. In a downlink multiuser MIMO (MU-MIMO) system, a base station (BS) communicates simultaneously with K mobile stations (MS) in a given spectrum band. Assuming N transmitter antennas at BS and M receiver antennas at each MS, the sum capacity increases linearly with $\min(N, MK)$ under perfect channel side information (CSI) at both transmitter and receiver. A MU-MIMO channel is also referred as a MIMO broadcast channel (MIMO-BC), where dirty paper coding (DPC) has been shown to achieve the sum capacity [2], [3], whereas the complexity of DPC is very high and suboptimal linear precoding schemes are of interest for practical consideration [4], [5].

Zero forcing (ZF) is a typical linear precoding technique widely used in practice [1]. ZF precoding forcefully eliminates at the transmitter the interference from other streams, some of which may be intended to the same user. ZF precoding can be applied to both multiuser multiple-input single-output (MU-MISO) and MU-MIMO scenarios. Under a given precoding technique, the performance of a MU-MIMO system depends on user selection and power allocation schemes. Several user selection algorithms have been proposed for downlink MU-MIMO system [5]–[7]. In [5], a semiorthogonality-based user selection algorithm has been proposed. The performance of the semiorthogonal user selection (SUS) algorithm is shown to asymptotically achieve the sum capacity of DPC, as the number of users goes to infinity. However, in practice, there typically exists less than 100 rather than a very large number of users. For example, 10 users are recommended for LTE system level evaluation in packet transmission [8]. In addition, the system performance in [5] depends on an empirical orthogonality factor. In [7], a Frobenius norm-based selection algorithm has been proposed for MU-MIMO systems to maximize sum throughput. The scheduler only considers the norm of the channel gain vector, whereas the orthogonality between the selected channel vectors also impacts the system performance. In [6], a measure of sum orthogonality between a candidate eigenvector and the set of selected eigenvectors is evaluated and the user with minimum sum orthogonality is selected. The scheduler in [6] only considers the orthogonality of the candidate user with all the selected users, whereas the norm of the channel vector also contributes to the sum rate of a MU-MIMO system and needs to be jointly considered for user selection. In [9], a forward and backward projection-based scheme is proposed, which can be regarded as an implicit measure of orthogonality of user channels. This scheme requires to calculate null space and aggregate correlation at each iteration, which still requires high complexity for large number of users.

In this paper, we investigate user selection to maximize sum rate in downlink MU-MIMO systems, which is important for high data rate applications. We assume the BS has full CSI of all MSs while a MS only has its own CSI knowledge and does not know CSIs of other MSs. We propose an efficient user selection algorithm by jointly considering channel norm and orthogonality among the channels of the selected MSs. Simulation results show that the proposed algorithm outperforms the existing algorithms with higher sum rate.

The rest of the paper is organized as follows. The system model and background are introduced in section II. We present the proposed user selection scheme for ZF based MU-MIMO systems in section III and give complexity analysis in section IV. Simulation results are given in section V followed by conclusions in section VI.

Notation In this paper, uppercase boldface letters denote matrices, and lowercase boldface letters denote vectors. $\mathbf{A}^*$ and $\mathbf{A}^\dagger$ denote respectively the conjugate transpose and the pseudo inverse of matrix $\mathbf{A}$. $\mathrm{tr}(\mathbf{A})$ and $\det(\mathbf{A})$ represent the trace and determinant of matrix $\mathbf{A}$ respectively. $\|\mathbf{A}\|$ denotes the Frobenius norm of $\mathbf{A}$. $< \mathbf{x}, \mathbf{y} >$ denotes the inner produce of $\mathbf{x}$ and $\mathbf{y}$. $|C|$ denotes the cardinality of set C.

II. System Model and Zero Forcing Precoding

A. System Model

We consider a single-cell with a base station and K mobile stations as shown in Fig.1. The BS is equipped with N transmitter antennas and each MS with M_k receiver antennas. The out-of-cell interference together with background noise at MS k is modeled as a complex additive white Gaussian random variable with power σ_k^2. Let $\Omega = \{1, \cdots, K\}$ represent the user set of all the MSs. Denote $\mathbf{H} = [\mathbf{H}_1^*, \cdots, \mathbf{H}_K^*]^*$, where $\mathbf{H}_k$ is the channel matrix of user k with $k \in \{1, \cdots, K\}$. In practice, $N > \min_k M_k$, which implies a BS can support multiple MSs for simultaneous transmission. Let $\mathcal{S}$ denote the set of selected users with $\mathcal{S} \subset \Omega$ and J as the number of selected MSs, then we have $\sum_{k=1}^{J} L_k \leq N$, where L_k represents the number of streams transmitted to the kth selected MS with $k \in \{1, \cdots, J\}$. In this paper, we study how to select the user set $\mathcal{S}$ from Ω for simultaneous transmission to maximize sum rate R. Let $\mathcal{S}'$ be any subset of Ω, the optimum user selection problem can be formulated as

$$\mathcal{S} = \arg\max_{\mathcal{S}' \subset \Omega} R(\mathcal{S}') \tag{1}$$

Let $\mathbf{H}(\mathcal{S}) = [\mathbf{H}_1^*(\mathcal{S}), \cdots, \mathbf{H}_J^*(\mathcal{S})]^*$ be the channel matrix of the user set $\mathcal{S}$, where $\mathbf{H}_k(\mathcal{S})$ is the channel matrix of the kth selected user. For simplicity of notation, we denote $\mathbf{H}_k(\mathcal{S})$ as $\mathbf{H}(k)$ in the rest of the paper. The received signal of a flat-fading MIMO channel at kth selected MS can be written as

$$\mathbf{y}(k) = \mathbf{H}(k)\mathbf{W}(k)\sqrt{\mathbf{P}_k}\mathbf{x}(k) + \sum_{j \neq k} \mathbf{H}(k)\mathbf{W}(j)\sqrt{\mathbf{P}_j}\mathbf{x}(j) + \mathbf{z}(k), \tag{2}$$

where $\mathbf{x}(k) \in \mathbb{C}^{L_k \times 1}$ is the transmitted signal vector to the kth selected MS with the corresponding input covariance matrix $\mathbf{P}_k = \mathrm{E}\{\mathbf{x}(k)\mathbf{x}(k)^*\}$, $\mathbf{W}(k) \in \mathbb{C}^{N \times L_k}$ is the precoding matrix and $\mathbf{z}(k) \in \mathbb{C}^{M_k \times 1}$ is complex additive white Gaussian noise (AWGN), i.e., $\mathbf{z}(k) \sim \mathcal{CN}(0, \sigma^2(k)\mathbf{I})$, with $\sigma^2(k)$ denoting power of noise at kth selected user. The transmitter has a total power constraint $\sum_k \mathrm{tr}(\mathbf{P}_k) \leq P$, where P is the maximum allowable transmitter power. To eliminate inter-user interference, the precoder should be designed such that

$$\mathbf{H}(k)\mathbf{W}(j) = 0, \text{for all } j \neq k \text{ and } k, j \in \mathcal{S}. \tag{3}$$

Throughout this paper, we assume that the channel matrix is perfectly known at the transmitter. It is also assumed that each MS only has its own CSI and no coordination exists among MSs. To aid the discussion of user selection, in the following subsection we briefly review zero-forcing precoding.

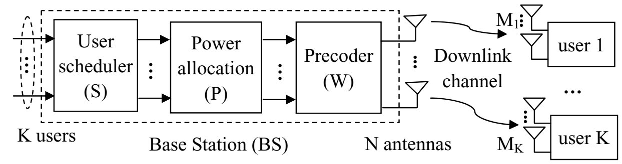

Fig. 1. MU-MIMO downlink system with N transmit antennas and J users selected from K users, each with M_k receive antennas $N > M_k$

B. ZF Precoding

ZF precoding forces the interference from other users or antennas to be zero. In a MU-MISO system ($M_k = 1, \forall k$), let $\mathbf{w}(k) \in \mathbb{C}^{N \times 1}$ and $\mathbf{h}(k) \in \mathbb{C}^{1 \times N}$ represent respectively the precoding vector and the channel vector at the kth MS. For a set of selected users $\mathcal{S}$, we have $\mathbf{H}(\mathcal{S}) = [\mathbf{h}^*(1), \cdots, \mathbf{h}^*(J)]^*$. As shown in [5], ZF based precoding matrix $\mathbf{W}(\mathcal{S})$ can be chosen as the pseudo inverse of $\mathbf{H}(\mathcal{S})$

$$\mathbf{W}(\mathcal{S}) = \mathbf{H}(\mathcal{S})^\dagger = \mathbf{H}(\mathcal{S})^*[\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^*]^{-1}. \tag{4}$$

Let P_i be the transmit power allocated to the ith selected user. The sum rate of the ZF based MU-MIMO system is

$$R_{ZF}(\mathcal{S}) = \max_{\sum_i P_i \leq P} \sum_{i \in \mathcal{S}} \log_2(1 + \frac{g_i P_i}{\sigma^2(i)}), \tag{5}$$

with

$$g_i = \frac{1}{\|\mathbf{w}(i)\|^2} = \frac{1}{[(\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^*)^{-1}]_{i,i}}, \tag{6}$$

which can be interpreted as the effective channel power gain to the ith selected user. Let γ_i denote the signal-to-noise ratio (SNR) at the ith selected MS, then we have $\gamma_i = \mathbf{g}_i P_i / \sigma^2(i)$.

Water-filling (WF) and equal power allocation are two typical power allocation schemes. WF is the optimal power allocation scheme to maximize system throughput when CSI are known at transmitter. Under WF scheme, P_i can be obtained by

$$P_i = \begin{cases} \frac{1}{\mu} - \frac{\sigma^2(i)}{g_i}, & \text{if } \frac{1}{\mu} \geq \frac{\sigma^2(i)}{g_i} \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

with the cutoff value $\frac{1}{\mu} = \left(\sum_i \sigma^2(i)/g_i + P\right)/J$.

Equal power scheme is optimal when the CSI is unknown at the transmitter. Even with CSI, equal power allocation may also be used due to per-antenna power constraints in practice [10], then we have, $P_i = P/J, \forall i$.

ZF precoding can also be applied to MU-MIMO systems ($M_k > 1$), where each antenna is treated as a separate user without coordination; therefore, the same results hold for ZF based MU-MIMO systems except that $\mathcal{S}$ is chosen over multiple antennas of all users. However this simple extension to MU-MIMO might incur significant losses.

III. User selection for ZF based MU-MIMO

In this section, we first investigate the potential factors affecting the sum rate of ZF based MU-MISO systems for $J < 3$ selected users; a generalized low-complexity user selection algorithm is then developed.

A. Analysis of Sum Rate in MU-MISO Systems

As shown in (5), the sum rate of a MU-MISO system is directly related to the effective channel gain g_i in (6). To obtain g_i, we first have to calculate

$$\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^* = \begin{bmatrix} \mathbf{h}(1)\mathbf{h}(1)^* & \cdots & \mathbf{h}(1)\mathbf{h}(J)^* \\ \mathbf{h}(2)\mathbf{h}(1)^* & \cdots & \mathbf{h}(2)\mathbf{h}(J)^* \\ \vdots & \ddots & \vdots \\ \mathbf{h}(J)\mathbf{h}(1)^* & \cdots & \mathbf{h}(J)\mathbf{h}(J)^* \end{bmatrix} \tag{8}$$

(1) When $|\mathcal{S}| = J = 1$, i.e., only one user is selected, then

$$[(\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^*)^{-1}]_{i,i} = \frac{1}{\mathbf{h}(1)\mathbf{h}(1)^*} = \frac{1}{||\mathbf{h}(1)||^2} \tag{9}$$

and $\gamma_1 = P||\mathbf{h}(1)||^2/\sigma^2(1)$, which implies that the user with maximum SNR should be selected.

(2) When $|\mathcal{S}| = J = 2$, i.e., two users are selected, then

$$\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^* = \begin{bmatrix} \mathbf{h}(1)\mathbf{h}(1)^* & \mathbf{h}(1)\mathbf{h}(2)^* \\ \mathbf{h}(2)\mathbf{h}(1)^* & \mathbf{h}(2)\mathbf{h}(2)^* \end{bmatrix} \tag{10}$$

Let r_i denote the norm of vector $\mathbf{h}(i)$, then $\mathbf{h}(i)\mathbf{h}(i)^* = ||\mathbf{h}(i)||^2 = r_i^2$. Let λ_i denote the ith eigenvalue of $\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^*$, we then have $\det[\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^*] = \lambda_1\lambda_2$, and

$$\begin{aligned} \lambda_1\lambda_2 &= \mathbf{h}(1)\mathbf{h}(1)^*\mathbf{h}(2)\mathbf{h}(2)^* - \mathbf{h}(1)\mathbf{h}(2)^*\mathbf{h}(2)\mathbf{h}(1)^* \\ &= r_1^2 r_2^2 - r_1^2 r_2^2 \cos^2\theta_{12} = r_1^2 r_2^2 \sin^2\theta_{12} \end{aligned} \tag{11}$$

where θ_{12} denotes the angle between the channel vectors of the two selected users. Then we have

$$[\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^*]^{-1} = \frac{1}{r_1^2 r_2^2 \sin^2\theta_{12}} \begin{bmatrix} r_2^2 & -\mathbf{h}(1)\mathbf{h}(2)^* \\ -\mathbf{h}(2)\mathbf{h}(1)^* & r_1^2 \end{bmatrix} \tag{12}$$

Therefore, we have

$$||\mathbf{w}(1)||^2 = [(\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^*)^{-1}]_{1,1} = \frac{1}{r_1^2 \sin^2\theta_{12}}, \tag{13}$$

$$||\mathbf{w}(2)||^2 = [(\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^*)^{-1}]_{2,2} = \frac{1}{r_2^2 \sin^2\theta_{12}}. \tag{14}$$

Then the SNR of the 1st and 2nd selected user are respectively

$$\gamma_1 = \frac{P_1}{||\mathbf{w}_1||^2\sigma^2(1)} = \frac{P_1 r_1^2 \sin^2\theta_{12}}{\sigma^2(1)}, \tag{15}$$

$$\gamma_2 = \frac{P_2}{||\mathbf{w}_2||^2\sigma^2(2)} = \frac{P_2 r_2^2 \sin^2\theta_{12}}{\sigma^2(2)}. \tag{16}$$

It is seen that the SNRs of both users are related to the norm and the angle between their respective channel vectors as well as the noise power. And the angle θ_{12} can be regarded as a measure of orthogonality between the two users. The user selection problem can then be expressed as

$$\begin{aligned} \mathcal{S} &= \underset{i\in\mathcal{S}',\mathcal{S}'\subset\Omega}{\arg\max} \sum_{i=1}^{2} \log_2(1+\gamma_i) \\ &= \underset{i\in\mathcal{S}',\mathcal{S}'\subset\Omega}{\arg\max} \log_2\Bigg(1 + \frac{P_1 r_1^2 \sin^2\theta_{12}}{\sigma^2(1)} + \frac{P_2 r_2^2 \sin^2\theta_{12}}{\sigma^2(2)} \\ &\quad + \frac{P_1 P_2 r_1^2 r_2^2 \sin^4\theta_{12}}{\sigma^2(1)\sigma^2(2)}\Bigg), \end{aligned} \tag{17}$$

where $r_i^2/\sigma^2(i)$ can be regarded as the effective power gain of user i and the term $\sin\theta_{12}$ shows the correlation between the two selected users.

The user selection problem in (17) is a non-linear optimization problem and exhaustive search is generally required to find the optimal solution. However, exhaustive search is of high complexity for large K, and suboptimal algorithms are widely studied.

To simplify the user selection process, user are selected in sequence and the first user s_1 is the one with maximum channel gain according to (9). It should be noted that although equal power allocation is assumed to all selected MSs, other desirable power allocation techniques (e.g. water-filling power allocation) may be applied after the user selection process. Since log function is a monotonic increasing function, user selection problem in (17) reduces to

$$s_2 = \underset{i\in\Omega}{\arg\max}\ f(\mathbf{h}_i|\mathbf{h}(1)) \tag{18}$$

with

$$f(\mathbf{h}_i|\mathbf{h}(1)) = \left(\frac{r_1^2 \sin^2\theta_{12}}{\sigma^2(1)} + \frac{r_2^2 \sin^2\theta_{12}}{\sigma^2(2)} + \frac{P_t r_1^2 r_2^2 \sin^4\theta_{12}}{2\sigma^2(1)\sigma^2(2)}\right) \tag{19}$$

For $J > 2$, the analytic expression for precoding vectors $\mathbf{w}_i$s is very complicated and thus not discussed. In this following subsection, we propose a user selection algorithm which groups the previous selected users as a weighted single user, then the optimal solution (18) for the two users can be applied.

B. Proposed Low-complexity User Selection Algorithm

In this subsection, we propose a suboptimal user selection algorithm based on the above analysis. To reduce the complexity, we combine the selected channels into a single weighted channel $\bar{\mathbf{h}}$, and then the above two-users' selection scheme can be applied. For a general case ($K > 2$), the procedure is as follows.

1) Initialization: $\Omega = \{1, \cdots, K\}, J = 0, \mathcal{S} = \emptyset$.
2) Let s_i be the ith selected user. Then, $s_1 = \underset{i\in\Omega}{\arg\max} ||\mathbf{h}_i||^2/\sigma_i^2$, $J = 1$, $\Omega = \Omega - s_1$, $\mathcal{S} = \mathcal{S} + s_1$ and sum rate $R = \log_2(1 + P||\mathbf{h}(1)||^2/\sigma^2(1))$.
3) For $J = 2 : N$
   a) Define the weighted channel $\bar{\mathbf{h}}$ over the selected users

$$\bar{\mathbf{h}} = \sum_{i=1}^{J} \alpha_i \mathbf{h}(i) \tag{20}$$

   where α_i is the weighted factor for the ith selected user and $\sum_{i=1}^{J}\alpha_i = 1$. Correspondingly, weighted noise power $\bar{\sigma}^2 = \sum_{i=1}^{J}\alpha_i\sigma^2(i)$ and $\bar{r} = ||\bar{\mathbf{h}}||$. In this paper, we consider equal gain for all selected users, $\alpha_i = 1/J$, that is, selected users are treated indiscriminately. Then we have $\bar{\mathbf{h}} = \mathbf{h}(1), \bar{\sigma} = \sigma(1)$ for $J = 1$.

   b) For each candidate user $j \in \Omega$
      i) Calculate the angle $\bar{\theta}_j$ between $\mathbf{h}_j$ and the weighted selected users' channel $\bar{\mathbf{h}}$

$$\bar{\theta}_j = \arccos\frac{|<\mathbf{h}_j, \bar{\mathbf{h}}>|}{r_j \cdot \bar{r}} \tag{21}$$

ii) Calculate $f(\mathbf{h}_j|\bar{\mathbf{h}})$ by (19), where $r_1, \sigma_1, \theta_{12}$ are replaced by $\bar{r}_1, \bar{\sigma}, \bar{\theta}$ respectively.

c) The candidate user $\hat{s} = \arg\max_{i \in \Omega} f(\mathbf{h}_i|\bar{\mathbf{h}})$.

d) Let $\hat{\mathcal{S}} = \mathcal{S} + \hat{s}$ and calculate the sum rate $\hat{R}$ over users' set $\hat{\mathcal{S}}$ by (5)

- *If* $\hat{R} > R$, $R = \hat{R}$, $J = J + 1$, $\Omega = \Omega - \hat{s}$, $\mathcal{S} = \mathcal{S} + \hat{s}$, go to 3);
- *Else*, go to 4).

4) User selection is terminated.

The proposed algorithm for MU-MISO systems can be easily extended to ZF-based MU-MIMO systems, except that the user set Ω becomes the antenna set $\mathcal{A} = \{a_{1,1}, \cdots, a_{1,M_1}, \cdots, a_{K,1}, \cdots, a_{K,M_K}\}$, with $a_{k,i}$ denoting the ith antenna of user k.

IV. Complexity Analysis

In this section, we compare the complexity of the proposed user selection algorithm with several typical existing schemes in terms of required number of Floating-Point Operations per Second (FLOPs). A *real addition, multiplication, division, squared root*, or *comparison* is counted as a FLOP. The complexity of the proposed scheme is summarized as follows.

1) When $J = 1$, the user with maximum norm is selected; therefore the total FLOPs to calculate K users' norms and to choose the maximum norm are $4KN + K - 1$, which is the same as the other schemes.

2) For $J = 2 : N$, the complexity is reduced significantly.

a) Calculating $\bar{\mathbf{h}}$ requires only N_t complex additions and N_t multiplications, due to that previous $J - 1$ selected users are fixed. Then we just need $12N$ FLOPs altogether, with $8N$ FLOPS for $\bar{\mathbf{h}}$ and $4N$ FLOPs for $\|\bar{\mathbf{h}}\|$.

b) For each candidate $j \in \Omega$, the total number of FLOPs for $\bar{\theta}_j$ and $f(\mathbf{h}_j|\bar{\mathbf{h}})$ is $8N + 12$, where $\bar{\theta}_j$ and $f(\mathbf{h}_j|\bar{\mathbf{h}})$ require $8N + 2$ and 10 FLOPs respectively.

3) On selecting the Jth user, we have $K - J + 1$ candidates. The required FLOPs are

$$U_J = 12N + (K - J + 1)(8N + 12) + K - J \approx O(8KN). \quad (22)$$

4) Calculating sum rate in (5) for J users requires FLOPS

$$T_J = J^3 + J^2 + 5J + JN^2 + JN - \frac{N^2}{2} - \frac{N}{2} - 1 \approx O(JN^2), \quad (23)$$

which is independent of K and thus can be negligible.

5) The total FLOPs for J selected users are

$$\mathcal{F}_p = \sum_{j=2}^{J} U_j + 4KN + K - 1 \approx O(8KJN). \quad (24)$$

When $J = N$, the maximum number of FLOPs is $O\left(8KN^2\right)$.

For MU-MIMO systems, we assume that each user has M equal antennas for simplicity of analysis and the maximum number of FLOPs is $O\left(8KMN^2\right)$.

For the exhaustive search scheme, we have to calculate the sum rate of all possible combinations and choose the users with maximum sum rate. From (23), the total FLOPs for MU-MISO systems can be approximated by

$$\mathcal{F}_{ex} = \sum_{J=1}^{N} \binom{K}{J} T_J \overset{(b)}{>} O\left(\binom{K}{N} N^3\right) \quad (25)$$

where (b) holds since only a special case $J = N$ is considered.

For the SUS scheme in [5], the number of candidates depends on the semi-orthogonal factor and we give upper bound complexity analysis here. For each user i in the candidate user set, the BS has to find the Gram-Schmidt orthogonalization (GSO) component of $\mathbf{h}_i$, which requires $14N(J - 1)$ FLOPs. Then the total number of FLOPs for all the candidate users is $14N(J - 1)(K - J + 1) \approx O(14KNJ)$. Calculating the norm of each GSO channel requires $4N(K - J + 1)$ FLOPs. The maximum number of FLOPs can be approximated by

$$\mathcal{F}_{SUS} = \sum_{J=1}^{N} O(14NJK) + 4N(K - J + 1) \approx O(7N^3K) \quad (26)$$

In [6], the sum orthogonality-based scheme first implements SVD operation and then measures the sum correlation of candidates with selected users. The SVD of K users requires $6K(4N^2M + 8NM^2 + 9M^3) = O(24N^2MK)$ FLOPs. The sum correlation to choose Jth user from $K - J + 1$ unselected users requires $((J - 1)(8N - 2) + J - 1)(KM - J) = O(8KMNJ)$. The total required FLOPs are approximately

$$\mathcal{F}_{SO} \approx O(24KMN^2) + \sum_{J=2}^{N} O(8KMNJ) \quad (27)$$

When $N < 6$, the SVD operation dominates and $\mathcal{F}_{SO} \approx O(24KMN^2)$, otherwise, the sum correlation operation dominates and $\mathcal{F}_{SO} \approx O(4KMN^3)$.

The FLOPs of the Frobenius norm-based user selection in [7] can be approximated by

$$\mathcal{F}_{FN} \approx O\left(KJ^2N^3\right) \quad (28)$$

From (24)-(28), we can see that the proposed scheme has much lower complexity, with $O\left(KMN^2\right)$ FLOPs compared to at least $O\left(KMN^3\right)$ FLOPs for the others schemes.

V. Simulation Results

In this section, we evaluate the performance of the proposed scheme in terms of the sum rates of MU-MIMO systems by link-level simulations. Each result is averaged by 10000 Monte Carlo simulations. The main simulation parameters are given in Table. I.

Fig.2 shows the performance of the proposed scheme under equal power allocation with the number of transmit antennas $N = 4$. The sum rates of the user selection algorithms *SUS* [5], *sum orth* [7], *Frobenius norm* [7] and exhaustive search (*ES*), are also plotted under the same channel realizations for comparison. The sum rates under water-filling power allocation have similar results and thus are not plotted.

It is seen that the proposed algorithm outperforms the other suboptimal use selection with higher throughput. In particular,

TABLE I
SIMULATION PARAMETERS

Parameters	Values
Transmitter antenna at BS N	2–12
Receiver antenna at each MS M_i	1
Carrier frequency	2 GHz
Number of MS K	5–50
Cell-level user distribution	uniform
Channel model	Rayleigh fading
Lognormal shadow variance	5 dB
Total transmit Power P	18 dBm
Spectrum band Δf	15 KHz
Noise power spectrum density	-174 dBm/Hz

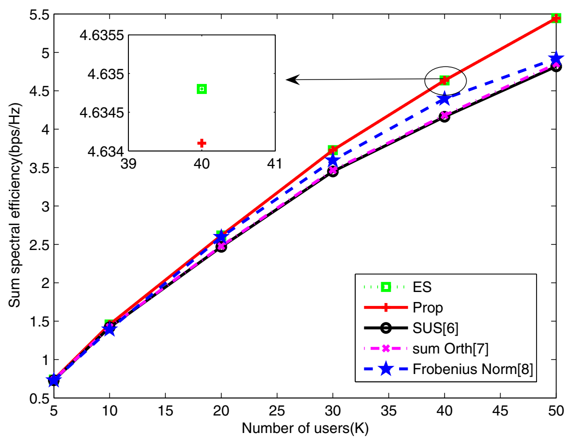

Fig. 2. Sum rate of user selection algorithms versus number of MSs under equal power with $N = 4$

the proposed scheme tightly follows exhaustive search. For example, when $K = 40$ in Fig. 2, there is only 0.02 percent sum rate reduction compared to about 10 percent reduction for the sum orthogonality based scheme. Moreover, the performance of the proposed algorithm is independent of the number of users, whereas the sum rates degradation of the existing algorithms aggravates with number of active users. The reason lies in that the proposed scheme uses a better measure than other schemes, since we jointly considers all possible factors while the other schemes consider only some of the factors or consider them separately. We also notice that the sum rate (spectral efficiency) increases with the number of users for all schemes, which indicates multiuser diversity attributes significantly to system performance.

Fig.3 shows the sum rates of user selection algorithms v.s. the number of transmit antennas,i.e., the maximum number of supported users.

We observe that the sum rate increases with the maximum number of supported users for all schemes, which indicates that user selection has great impact on system throughput when the number of transmit antenna is large.

VI. CONCLUSIONS

In this paper, we have investigated user selection for zero forcing precoding-based MU-MIMO systems to maximize the sum rate. Based on analytical results derived for $K < 3$ users, we propose an efficient low-complexity user selection algorithm, which jointly considers the noise power, the norm of the candidate MS channel gains and orthogonality with respect to selected MSs' channels. Complexity is analyzed in terms of FLOPs and the results show that the proposed scheme requires only $O(KMN^2)$ compared to at least $O(KMN^3)$ for other suboptimal schemes. Simulation results show the proposed algorithm outperforms existing user selection algorithms with higher throughput and is more tight to approximate the bound of the optimum exhaustive search algorithm.

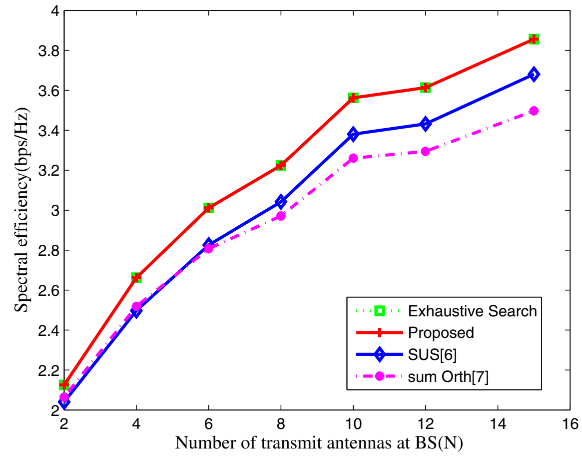

Fig. 3. Sum rate of user selection algorithms versus number of transmit antennas at BS

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