ARRAY PROCESSING

- Introduction to Array Processing

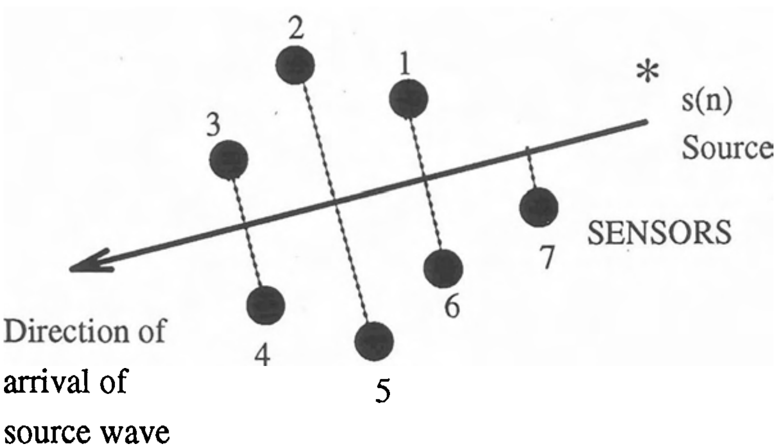
  Signal subspace and noise subspace Array Processing methods

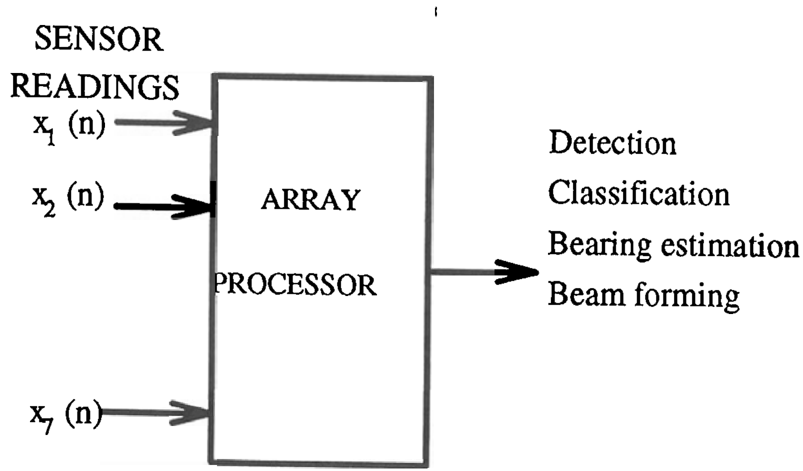
- Methods for resolving coherent sources

  Wideband Array Processing methods

INTRODUCTION TO ARRAY PROCESSING (1)

- Array is a set of sensors. The output signals from the sensors are combined appropriately to produce a desired result.





$\{x_1(n),\ldots,x_7(n)\}$ is a "snapshot"

INTRODUCTION TO ARRAY PROCESSING (2)

Narrowband Array Processing　　Wideband Array Processing

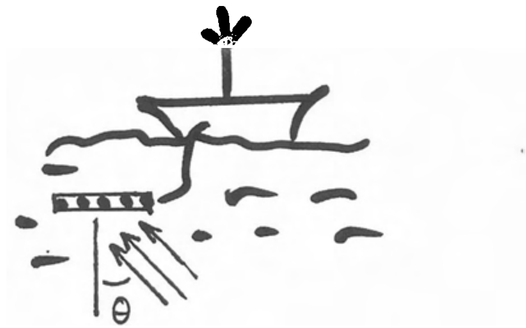
■ APPICATIONS

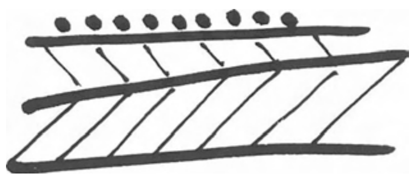
1) SONAR

2) RADAR

3) GEOPHYSICS

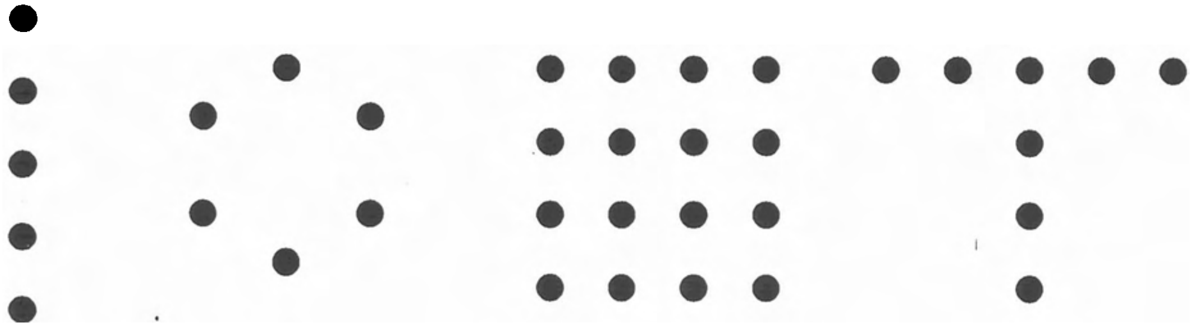
4) BIOMEDICINE





INTRODUCTION TO ARRAY PROCESSING (3)

- **ARRAY CONFIGURATIONS**



LINEAR CIRCULAR PLANAR T-SHAPE

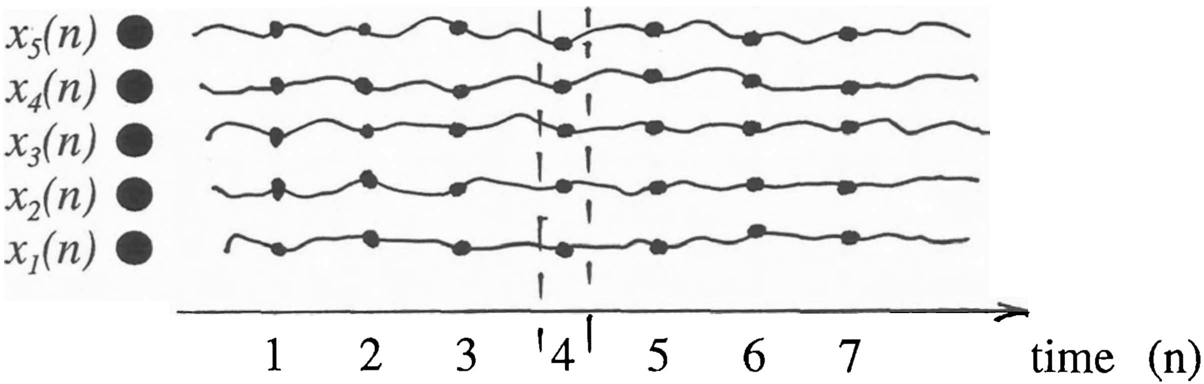
- "SNAPSHOT": is a set of sensor readings (outputs) at time instant n

ARRAY PROCESSING: is the Power Spectral Analysis along snapshots (instead of time)

1-d ARRAY PROCESSING: A snapshot is in the form of 1-d signal

INTRODUCTION TO ARRAY PROCESSING (4)

■ 1-d ARRAY PROCESSING



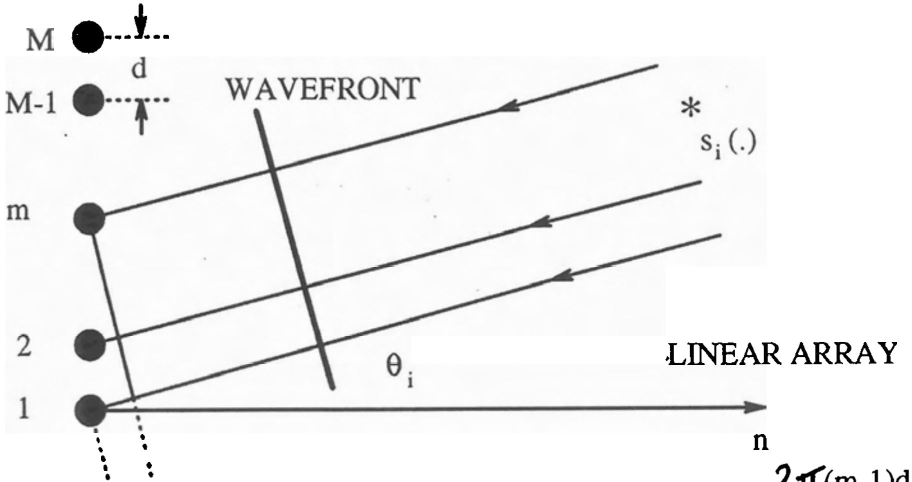
4th snapshot

<u>kth snapshot</u> $[x_1(k),\ x_2(k),\ x_3(k),\ x_4(k),\ x_5(k)]$

■ Difficult spectral estimation problem since the number of sensors is usually small. However, we can use "ensemble" averaging over different snapshots.

INTRODUCTION TO ARRAY PROCESSING (5)

■ 1-d ARRAY PROCESSING



$(m-1)d \sin\theta_i \longrightarrow$ phase difference if $u(i,m) = \dfrac{2\pi(m-1)d\sin\theta}{\lambda_i}$

d: spacing betwenn sensors, λ: wavelength of source $s_i(i)$, θ_i: bearing of source $s_i(.)$

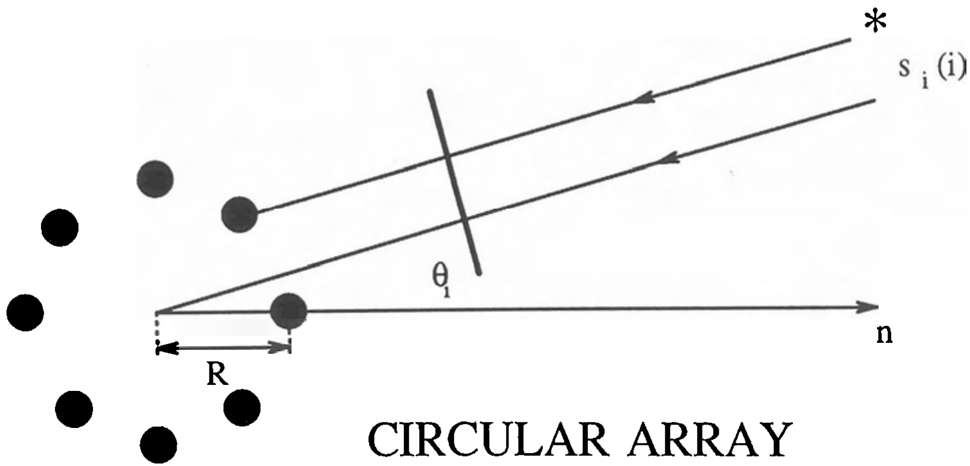
■ MODEL EQUATION [I incoherent sources (i.e., with different $\dfrac{\sin\theta_i}{\lambda_i}$)]

$$x_m(n) = \sum_{i=1}^{I} s_i[u(i,m) + \phi(i,n)] + w_m(n)$$

m[th] sensor at time n phase difference random phase noise

INTRODUCTION TO ARRAY PROCESSING (6)

■ 1-d ARRAY PROCESSING



CIRCULAR ARRAY

■ MODEL EQUATION (I incoherent sources)

$$x_m(n) = \sum_{i=1}^{I} s_i[u(i,m) + \phi(i,n)] + w_m(n)$$

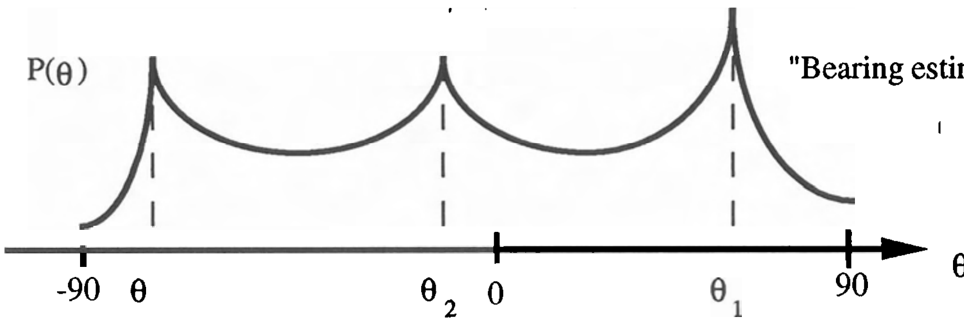
m^{th} sensor at time n   phase difference   random phase   noise

$$u(i,m) = \frac{2\pi R}{\lambda_i}\left[\cos\left(\frac{2\pi m}{M} - \theta_i\right) - \cos\left(\frac{2\pi m}{M}\right)\right]$$

INTRODUCTION TO ARRAY PROCESSING (7)

1-d ARRAY PROCESSING

- **Objective:** Given $\{x_m(n)\}$, n=1,...,N, m=1,...,M, estimate $\{\theta_i\}$, i=1,...,I



DUALITY BETWEEN PSE and AP

1) $P(f),\ -\frac{1}{2} \le f \le \frac{1}{2} \ \leftrightarrow\ P(\theta),\ -90 \le \theta \le 90°$

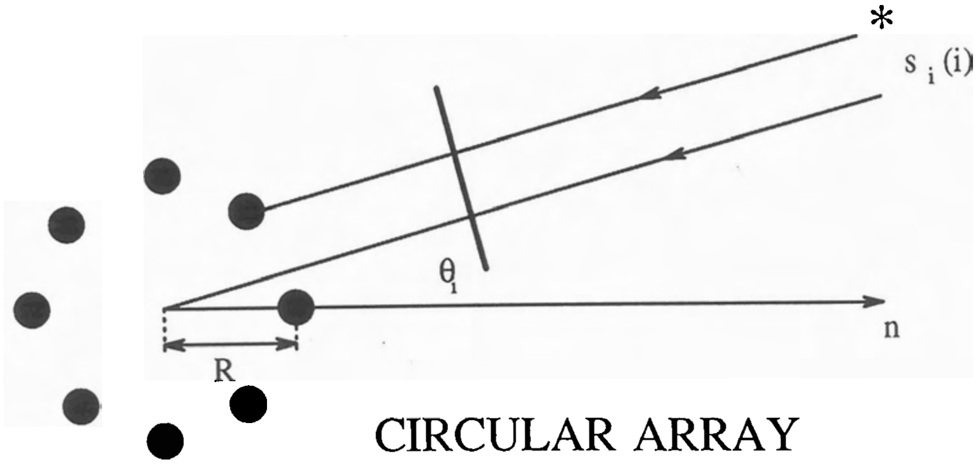
2) Frequency, $f \leftrightarrow$ Angle, $\sin\theta$

3) Sampling period, $T \leftrightarrow$ sensor spacing, d

4) Aliasing $\left[f_o > \frac{1}{2T}\right] \leftrightarrow$ Aliasing $[\lambda<2d]$

INTRODUCTION TO ARRAY PROCESSING (6)

■ 1-d ARRAY PROCESSING



CIRCULAR ARRAY

■ MODEL EQUATION (I incoherent sources)

$$x_m(n) = \sum_{i=1}^{I} s_i[u(i,m) + \phi(i,n)] + w_m(n)$$

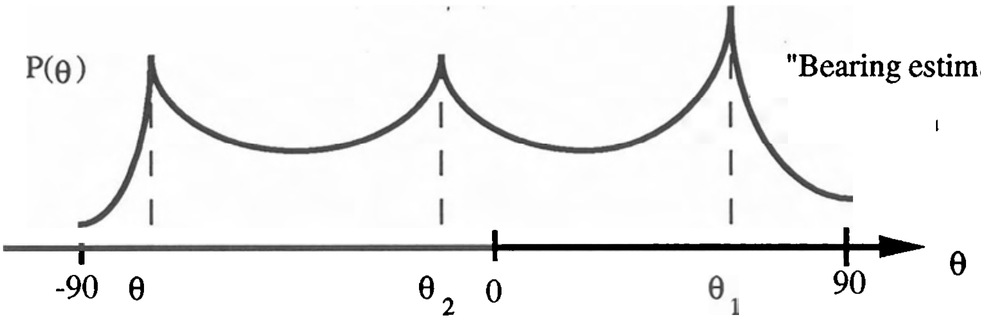
m^{th} sensor at time n phase difference random phase noise

$$u(i,m) = \frac{2\pi R}{\lambda_i}\left[\cos\left(\frac{2\pi m}{M} - \theta_i\right) - \cos\left(\frac{2\pi m}{M}\right)\right]$$

INTRODUCTION TO ARRAY PROCESSING (7)

■ 1-d ARRAY PROCESSING

- **Objective:** Given $\{x_m(n)\}$, n=1,...,N, m=1,...,M, estimate $\{\theta_i\}$, $i=1,\ldots,I$



■ DUALITY BETWEEN PSE and AP

1) $P(f)$, $-\frac{1}{2} \le f \le \frac{1}{2}$ $\leftrightarrow$ $P(\theta)$, $-90 \le \theta \le 90°$

2) Frequency, $f \leftrightarrow$ Angle, $\sin\theta$

3) Sampling period, $T \leftrightarrow$ sensor spacing, d

4) Aliasing $\left[f_o > \frac{1}{2T}\right] \leftrightarrow$ Aliasing $[\lambda<2d]$

■ 1-d ARRAY PROCESSING

Estimate P(θ)~θ or equivalently find the distribution of power for $\{x_m(n)\}$ on the exponentials $\left\{e^{\frac{j2\pi(m-1)d\sin\theta_i}{\lambda}}\right\}$, $m = 1,2,\ldots,M$

Notice the similarity with Fourier transform exponentials

$\{e^{j2\pi fnT}\}$, $n = 1,2,\ldots.$ Actually array processing is nothing else but

evaluation of the Fourier transform along snapshots ofthe array.

■ **AMBIGUITY:** To avoid distributing the power on $e^{\frac{\pm j2\pi(m-1)d\sin\theta_i}{\lambda}}$

(sign ambiguity) compute first the analytical signal for each sensor signal

$\{x_m(n)\}$, $n=1,2,\ldots,N$

INTRODUCTION TO ARRAY PROCESSING (9)

COMPLEX ANALYTIC SIGNAL FOR SENSOR m

$$y_m(n) = x_m(n) + jH\left[x_m(n)\right] \qquad \text{where } H[.] \text{ is the Hilbert transform}$$

■ COMPUTATION USING FFT

Given $\{x_m(n)\}$, $n=1,2,\ldots,N$

1) Obtain: $X_m(k) = \mathrm{FFT}[x_m(n)]$, $k=1,\ldots,N$

2) Form: $$Y_m(k) = \begin{cases} X_m(k), & k = 2,3,\ldots,\frac{N}{2} \\ X_m(k)/2, & k = 1, \frac{N}{2}+1 \\ 0, & k = \frac{N}{2}+2,\ldots,N \end{cases}$$

3) Obtain: $y_m(n) = \mathrm{IFFT}\,[Y_m(k)]$, $n=1,\ldots,N$

■ SPATIAL COVARIANCE MATRIX OF SNAPSHOTS

$$\underline{R} = E\left\{\underline{Y}(n)\,\underline{Y}^H(n)\right\}$$

M×M M: # of sensors

$$\underline{Y}^T(n) = \left[y_1(n)\,,\, y_2(n)\,,\ldots,\, y_M(n)\right]^T$$

■ ESTIMATED COVARIANCE MATRIX

$$\hat{\underline{R}} = \frac{1}{N}\sum_{n=1}^{N} \underline{Y}(n)\,\underline{Y}^H(n)$$

M×M

(If N<M, then $\hat{\underline{R}}$ is singular)

ARRAY PROCESSING METHODS (1)

- **CONVENTIONAL (BEAMFORMING)**

$$P(\theta) = \underline{c}^{H}\hat{\underline{R}}\,\underline{c}$$

- **ML OF CAPON**

$$P(\theta) = \frac{1}{\underline{c}^{H}\hat{\underline{R}}^{-1}\,\underline{c}}$$

- **AR**

$$P(\theta) = \frac{1}{\left|\underline{u}^{T}\hat{\underline{R}}^{-1}\,\underline{c}\right|^{2}}$$

- **THERMAL NOISE**

$$P(\theta) = \frac{1}{\left|\underline{u}^{T}\hat{\underline{R}}^{-H}\,\hat{\underline{R}}^{-1}\,\underline{c}\right|^{2}}$$

ARRAY PROCESSING METHODS (2)

- $\underline{u}^T = [1, 0, 0, \ldots, 0]$ (1×M) (Unit steering vector)

- $\underline{c} = \left[1,\ e^{j\phi(1)},\ ,e^{j\phi(2)},\ \ldots,\ e^{j\phi(M-1)}\right]^T$ (Steering vector)

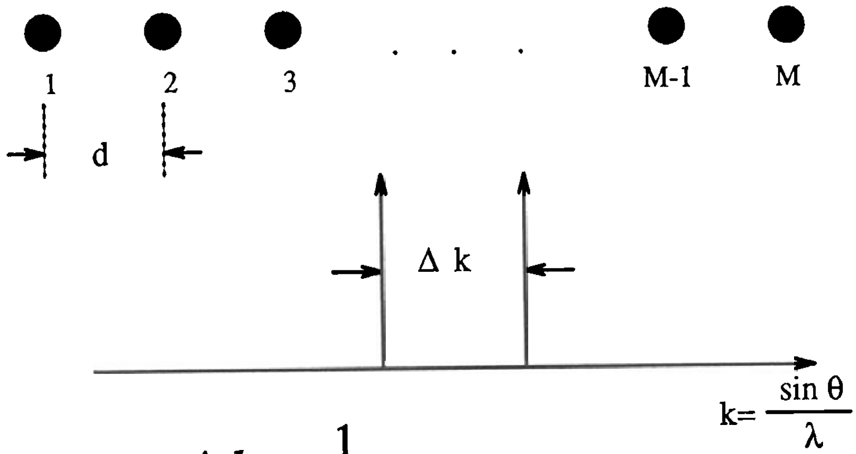
$$\phi(m) = \frac{2\pi d}{\lambda}(m-1)\sin\theta \qquad \text{(LINEAR ARRAY)}$$

$$\phi(m) = \frac{2\pi R}{\lambda}\left[\cos\left(\frac{2\pi m}{M} - \theta\right) - \cos\left(\frac{2\pi m}{M}\right)\right] \qquad \text{(CIRCULAR ARRAY)}$$

$$\phi(m) = m 2\pi f T \qquad \text{(SPECTRUM ESTIMATION)}$$

ARRAY PROCESSING METHODS (3)

RESOLUTION

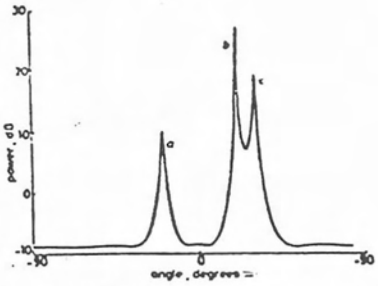


CONV $$\Delta k \geq \frac{1}{dM}$$

ML $$\Delta k \geq \frac{1}{\sqrt{SNR}\, dM}$$

■ **AR** $$\Delta k \geq \frac{1}{SNR\, dM}$$

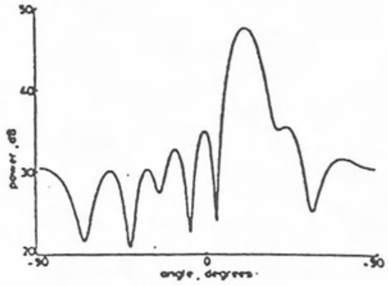
ARRAY PROCESSING METHODS (4) EXAMPLE: 3 SOURCES



ML

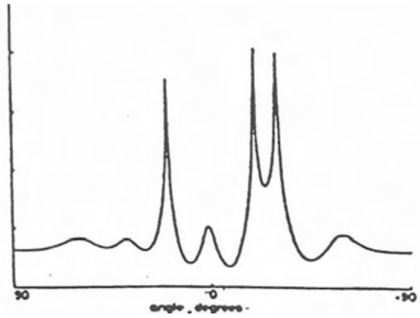
Estimate of angular spectrum (MLM)

a 10 dB source
b 30 dB source
c 20 dB source



CONVENTIONAL

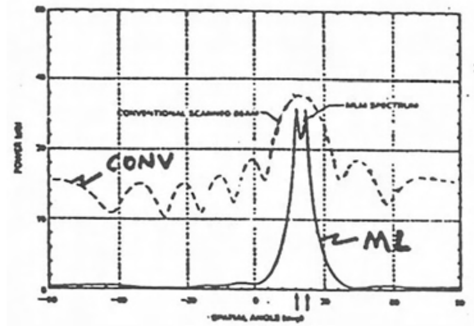
Conventional beamformer estimate

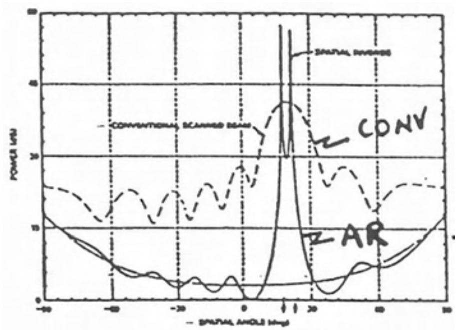


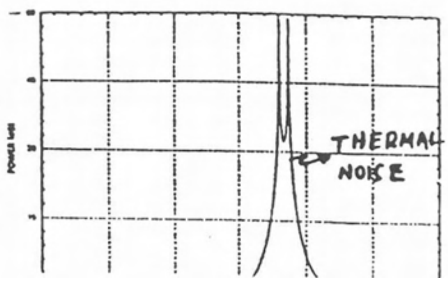
AR

ARRAY PROCESSING METHODS (5)

EXAMPLE: M=8, N=1024, I=2, BEARING 18° and 22°

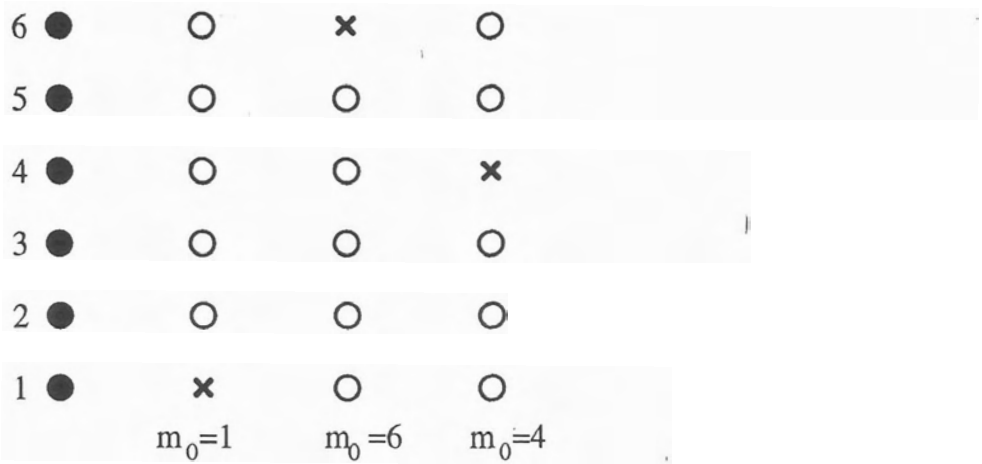






ARRAY PROCESSING METHODS (6)

EFFECT OF THE CHOICE OF UNIT STEERING VECTOR



$\underline{u}^T = [\,1, 0, 0, 0, 0, 0\,]$ (m_0=1) $\underline{u}^T = [\,0, 0, 0, 0, 0, 1\,]$ (m_0=6)

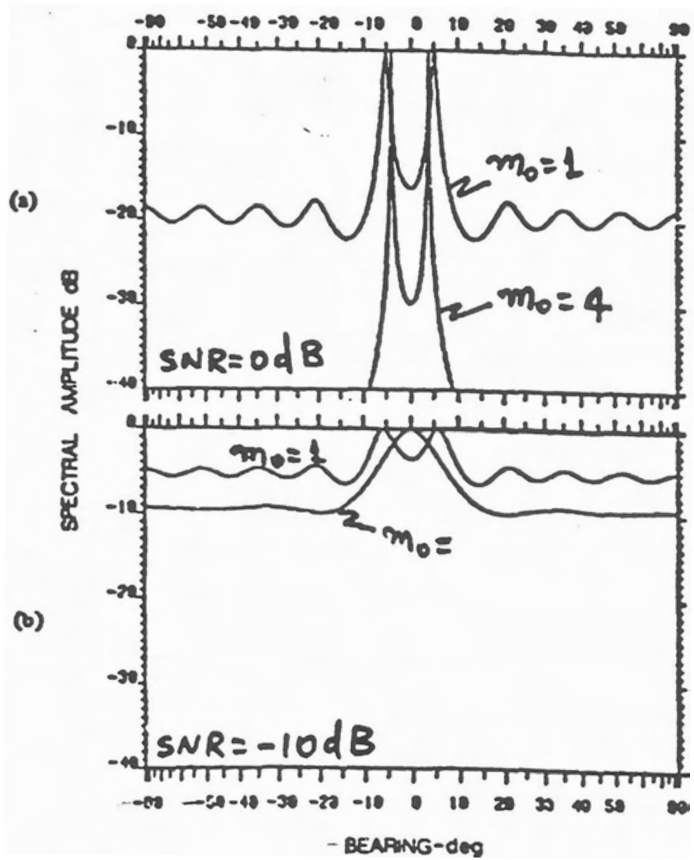
$\underline{u}^T = [\,0, 0, 0, 1, 0, 0\,]$ (m_0=4)

LINEAR PREDICTION FORMULA $\hat{x}_{m_0}(n) = \sum\limits_{m,\, m \neq m_0} a_m x_m(n)$,

$$\text{minimize } E\left\{\left|x_{m_0}(n) - \hat{x}_{m_0}(n)\right|^2\right\}$$

ARRAY PROCESSING METHODS (7)

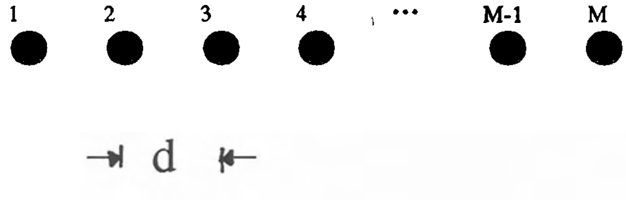
EFFECT OF DIFFERENT CHOICES FOR UNIT STEERING VECTOR



ARRAY PROCESSING METHODS (8)

PROPERTIES OF SPATIAL COVARIANCE MATRIX

1) LINE ARRAY:



2) I narrowband plane waves of frequency $w_0 = 2\pi f_0$ and bearings

$\theta_1, \theta_2, \ldots, \theta_I$, I<M; $d < \dfrac{\lambda_0}{2}$ where $\lambda_0 = \dfrac{c}{f_0}$; c: the speed of propagation

3) Sensor analytic signals (Linear array)

$$y_m(n) = \sum_{i=1}^{I} s_i(n)\, e^{-j\omega_0 (m-1) \frac{\sin\theta_i}{c} . d} + w_m(n)$$

$s_i(.)$ is the signal of i[th] wavefront

$w_i(.)$ is additive noise with variance σ^2, uncorrelated with each other noise sources and with signals

ARRAY PROCESSING METHODS (9) (noise sources assumed spatially uncorrelated)

4) In vector form $\underline{Y}(n) = \underline{A}\,\underline{S}(n) + \underline{W}(n)$

M×1 M×I I×1 M×1

$$\underline{Y}(n) = [y_1(n)\,,\, y_2(n)\,, \ldots,\, y_M(n)]^T \qquad (\text{M}\times 1)$$

$$\underline{S}(n) = [s_1(n)\,,\, s_2(n)\,, \ldots,\, s_I(n)]^T \qquad (\text{I}\times 1)$$

$$\underline{A}(n) = [\underline{\alpha}_1(\theta_1)\,,\, \underline{\alpha}_2(\theta_2)\,, \ldots,\, \underline{\alpha}_M(\theta_I)]^T \qquad (\text{M}\times \text{I})$$

where $\underline{\alpha}(\theta_i) = \left[1,\, e^{-j\phi_i(1)},\, \ldots,\, e^{-j\phi_i(M-1)}\right]^T$

$$\phi_i(m) = \omega_0 \cdot \frac{d}{c} \cdot\; m \;\cdot\; \sin\theta_i = 2\pi \;\; m \;\cdot\; d \cdot \frac{\sin\theta_i}{\lambda_0}$$

$$\underline{W}(n) = [w_1(n)\,,\, w_2(n)\,, \ldots,\, w_M(n)]^T$$

With M sensors we can only identify up to M incoherent sources

ARRAY PROCESSING METHODS (10)

SPATIAL COVARIANCE MATRIX

$$\underline{R} \triangleq E\left\{\underline{Y}(n)\,\underline{Y}^H(n)\right\}$$

Following a procedure similar to narrowband PSE method

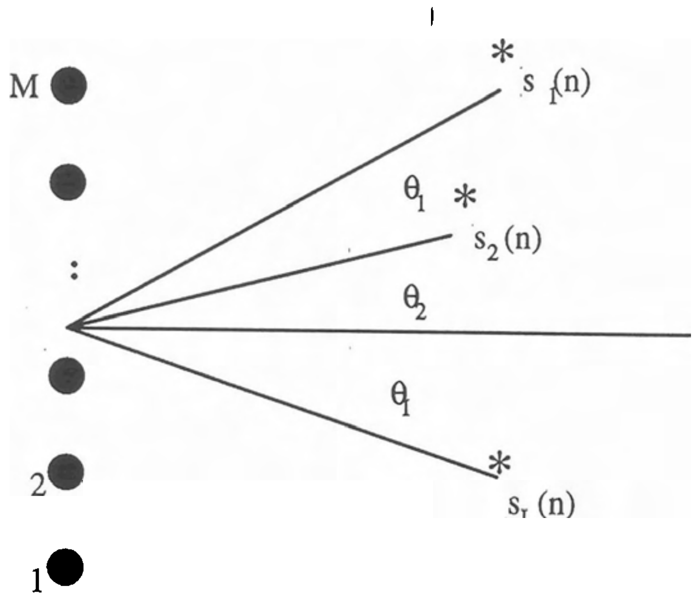
$$\underset{M\times M}{\underline{R}} = \underset{\substack{M\times M\\ \text{signal}}}{S_M} + \underset{\substack{M\times M\\ \text{noise}}}{W_M}$$

where, $\underset{M\times I}{S_M} = \underset{I\times I}{A}\ \underset{I\times M}{S}$

$$\underline{S} = E\left\{\underline{S}(n)\,\underline{S}^H(n)\right\}$$

$I\times I$ (cross correlation matrix of source signals)

$W_m = \sigma^2 I_m$ (White noise assumption)



ARRAY PROCESSING METHODS (11)

SPATIAL COVARIANCE MATRIX

Thus, for $\underline{\boldsymbol{R}} = \underline{\boldsymbol{S}}_M + \underline{\boldsymbol{W}}_M$; $\underline{W}_m = \sigma^2 \underline{I}_M$

M×M

We have:

Rank($\underline{\boldsymbol{S}}_M$) = I (NUMBER OF SOURCES)

Rank($\underline{\boldsymbol{W}}_M$) = M (NUMBER OF ELEMENTS)

- Rank($\underline{\boldsymbol{R}}$) = M

If there is no noise ($\sigma^2 = 0$), then $\underline{\boldsymbol{R}} = \underline{\boldsymbol{S}}_M$ and therefore Rank($\underline{\boldsymbol{R}}$)=I

ARRAY PROCESSING METHODS (12)

SPATIAL COVARIANCE MATRIX

$\{s_i(n)\}, \quad i = 1,\ldots,I$	$\underline{S} = E\{\underline{S}(n)\,\underline{S}^H(n)\}$ I×I
1) Uncorrelated	Diagonal non-sigular
2) Partially correlated	Non-diagonal, non-singular
3) $\underbrace{\text{Coherent}}_{\left(\frac{\sin\theta_i}{\lambda_i} = \frac{\sin\theta_j}{\lambda_j}\right)}$, fully correlated	Non-diagonal, singular

ARRAY PROCESSING METHODS (13)

■ ORTHOGONAL DECOMPOSITION OF R

$$\underline{R} = \sum_{i=1}^{M} \rho_i \underline{V}_i \underline{V}_i^H \; ; \quad \underline{V}_i^H \underline{V}_i = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

- SIGNAL + NOISE EIGENVECTORS

$$\left\{ \underline{V}_1, \underline{V}_2, \ldots, \underline{V}_I \right\} \qquad \rho_i = \lambda_i + \sigma^2 ; \quad i = 1,\ldots,I$$

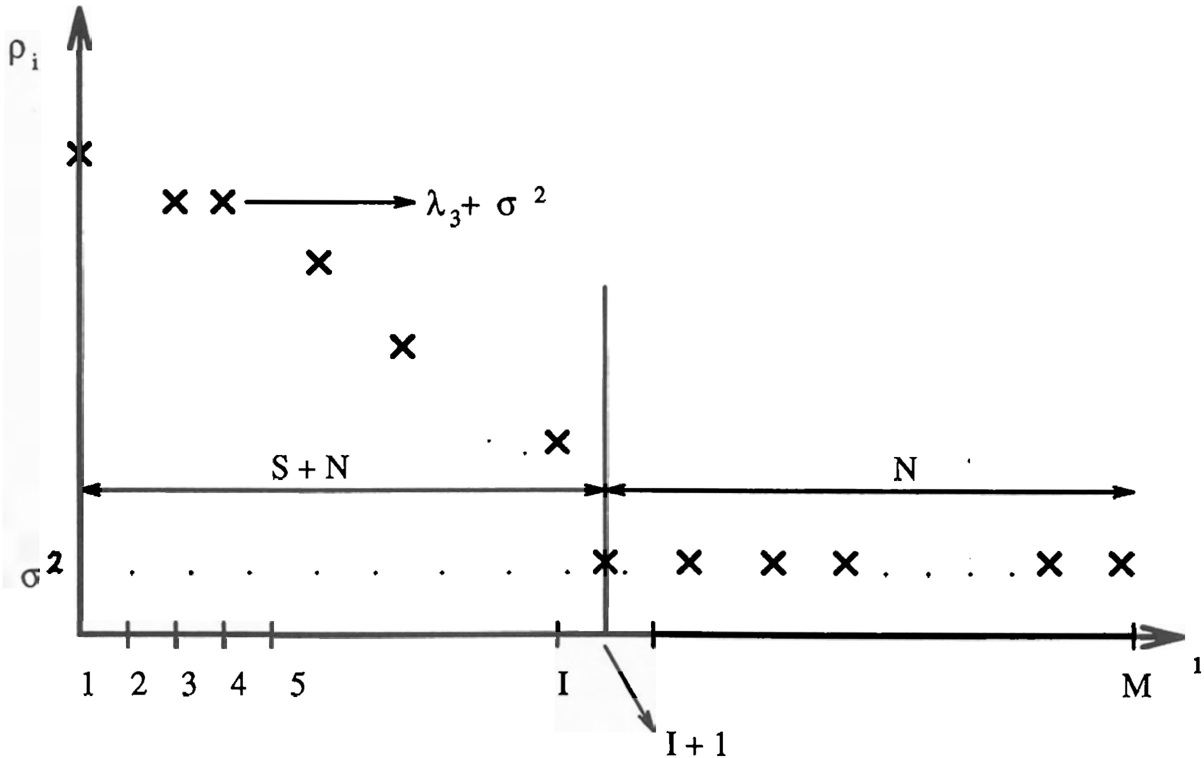
- NOISE EIGENVECTORS

$$\left\{ \underline{V}_{I+1}, \underline{V}_{I+2}, \ldots, \underline{V}_M \right\} \qquad \rho_i = \sigma^2 ; \quad i = I+1,\ldots,M$$

In other words: $$\underline{R} = \underbrace{\sum_{i=1}^{I} (\lambda_i + \sigma^2) \underline{V}_i \underline{V}_i^H}_{\underline{S}_M} + \underbrace{\sigma^2 \sum_{i=1}^{M} \underline{V}_i \underline{V}_i^H}_{\underline{W}_M}$$

ARRAY PROCESSING METHODS (14)

EIGENVALUES OF *R*



ARRAY PROCESSING METHODS (15)

SIGNAL SUBSPACE

$$\{\underline{V}_1, \underline{V}_2, \cdots \underline{V}_I\}$$

$$\{\rho_1, \rho_2, \cdots \rho_I\}$$

- CONVENTIONAL
- ML
- AR
- THERMAL NOISE

NOISE SUBSPACE

$$\{\underline{V}_{I+1}, \underline{V}_{I+2}, \cdots \underline{V}_M\}$$

$$\{\rho_{I+1}, \rho_{I+2}, \cdots \rho_M\}$$

- MUSIC
- EIGENVECTOR
- PISARENKO

ARRAY PROCESSING METHODS (16)

SIGNAL SUBSPACE METHODS

- RECONSTRUCT THE "NOISE FREE" SPATIAL COVARIANCE MATRIX

$$\underset{M\times M}{\underline{\tilde{R}}} = \sum_{i=1}^{I} \rho_i \underline{V}_i \underline{V}_i^H \qquad \text{and} \qquad \underset{M\times M}{\underline{\tilde{R}}^{-1}} = \sum_{i=1}^{I} \frac{1}{\rho_i} \underline{V}_i \underline{V}_i^H$$

- METHODS

CONV: $P(\theta) = \underline{c}^H \underline{\tilde{R}}\, \underline{c}$

ML: $P(\theta) = \dfrac{1}{\underline{c}^H \underline{\tilde{R}}^{-1} \underline{c}}$

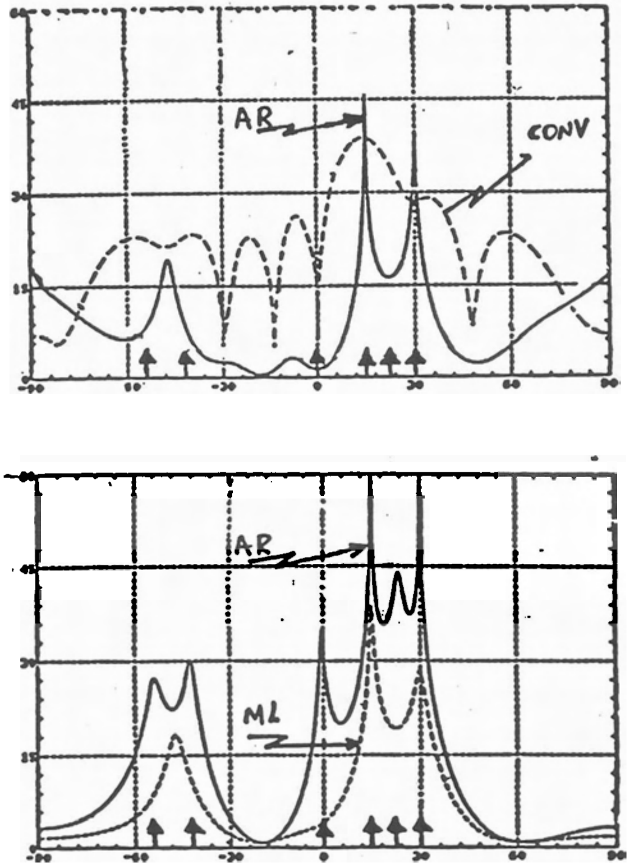
AR: $P(\theta) = \dfrac{1}{\left|\underline{u}^T \underline{\tilde{R}}^{-1} \underline{c}\right|^2}$

TH.N: $P(\theta) = \dfrac{1}{\left|\underline{u}^T \underline{\tilde{R}}^{-H} \underline{\tilde{R}}^{-1} \underline{c}\right|^2}$

ARRAY PROCESSING METHODS (17)

EXAMPLE Sources at: $\underbrace{-54°, -42°}_{\text{10dB,}}$, $\underbrace{0°, 15°, 22°, 30°}_{\text{20dB}}$,

POWER IN dB



SIGNAL SUBSPACE

BEARING

<u>ARRAY PROCESSING METHODS (18)</u>

■ NOISE SUBSPACE METHODS

- Basic estimator

$$\boxed{P(\theta) = \frac{1}{\sum_{i=I+1}^{M} q_i \, |\underline{c}^H(\theta)\, \underline{V}_i|^2}} \quad , \quad -90° \le \theta \le 90°$$

where $\underline{c}(\theta)$ is the steering vector

- MUSIC: $q_i = 1$ for all $\{i\}$
- EIGENVECTOR: $q_i = 1/\rho_i$
- PISARENKO: $q_{I+1} = \ldots = q_{M-1} = 0 \;, \quad q_M = 1$

ARRAY PROCESSING METHODS (19)

■ USING SVD

- Obtain SVD of covariance matrix

$$\underline{R} = \sum_{i=1}^{M} \rho_i \underline{V}_i \underline{S}_i^H = \sum_{i=1}^{I} (\lambda_i + \sigma^2) \underline{V}_i \underline{S}_i^H + \sum_{i=I+1}^{M} \sigma^2 \underline{V}_i \underline{S}_i^H, \qquad \rho_1 \geq \rho_2 \geq \ldots \geq \rho_M$$

signal + noise noise

■ SIGNAL SUBSPACE METHODS

$$\underset{M\times M}{\tilde{\underline{R}}} = \sum_{i=1}^{I} \rho_i \underline{V}_i \underline{S}_i^H, \qquad \underset{M\times M}{\tilde{\underline{R}}^{-1}} = \sum_{i=1}^{I} \frac{1}{\rho_i} \underline{S}_i \underline{V}_i^H$$

■ NOISE SUBSPACE METHODS

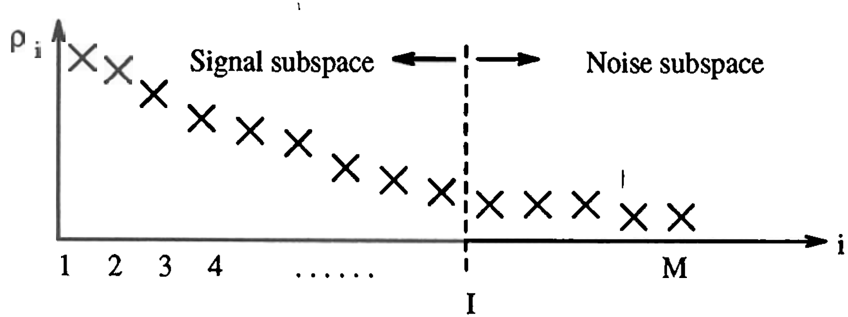
$$\hat{P}(\theta) = \frac{1}{\sum_{i=I+1}^{M} q_i \left[\underline{c}^H(\theta) \underline{V}_i \underline{S}_i^H c(\theta) \right]}$$

ARRAY PROCESSING METHODS (20)

USING SVD

1) Obtain SVD of M×M covariance matrix $\underline{\boldsymbol{R}}$

2) $\rho_1 \geq \rho_2 \geq \ldots \geq \rho_M$



In practice it might not be easy to distinguish noise from signal subspace as above

3) Minimium discription length (MDL) criterion

$$MDL(i) = -\log\left[\frac{\prod_{k=i+1}^{M}\rho_k}{\left(\frac{1}{M-i}\sum_{k=i+1}^{M}\rho_k\right)^{M-i}}\right]^N + \frac{1}{2}i(2M-i)\log N$$

$i = 0, 1, \ldots, M-1$

4) Pick I: $MDL_{min}(i)=MDL(I-1)$

5) if I>(M-I) use signal subspace methods

RESOLVING COHERENT SOURCES OR TARGETS (1)

Very difficult problem: Resolve fixed-phase coherent sources (RF in radar) which are spatially separated by less than the beamwidth of the array sampling aperture.

If the coherent sources maintain their fixed-phase relationship and if the array elements do not move then the signal covariance matrix has one unique eigenvalue: $\underset{M\times M}{\underline{S}_M} = \underset{M\times I}{\underline{A}}\ \underset{I\times I}{\underline{S}}\ \underset{I\times M}{\underline{A}^H}$, Rank$[S_M]=I=1$

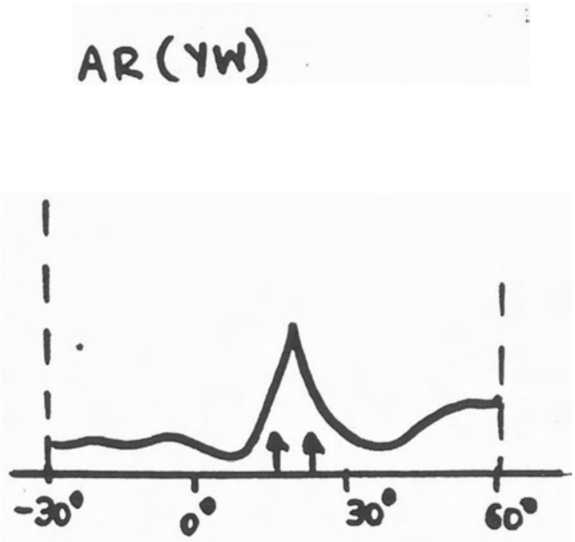
All coherent sources map into one eigenvalue.

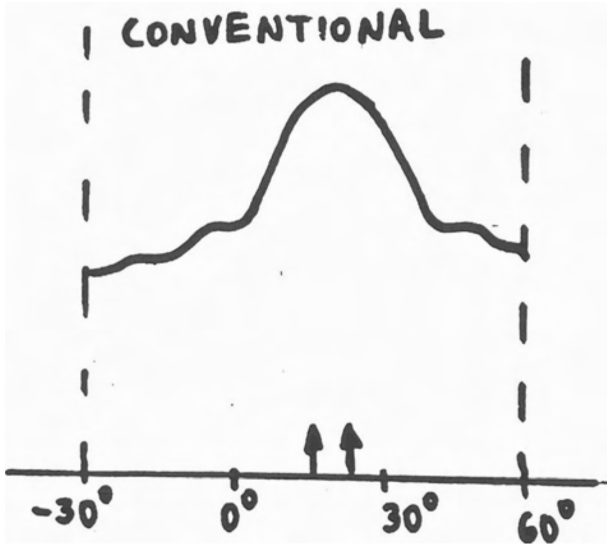
$\underline{S}$ is non-diagonal and singular.

- $\underline{S}_M$ is non-Toeplitz (spatial signal is not "stationary")

RESOLVING COHERENT SOURCES OR TARGETS (2)

Example (1): M=8 element array with half-wavelength spacing, two equal-strength 30dB coherent sources located at 16° and 24° with fixed-phase difference, N=1024 snapshots.



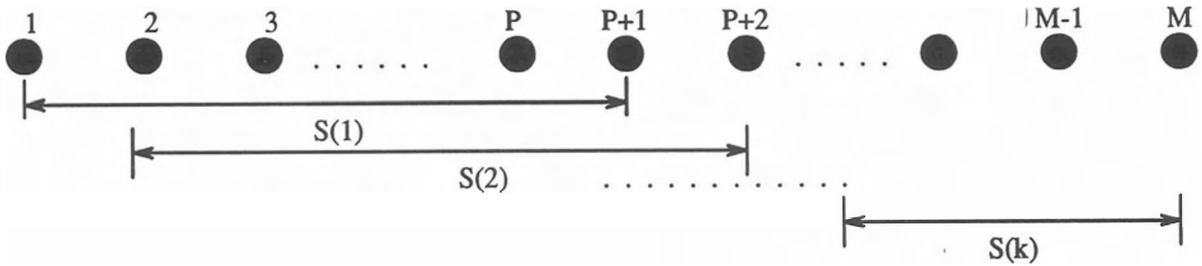


Example (2): in radar the direct and specular component behave like coherent sources.

RESOLVING COHERENT SOURCES OR TARGETS (3)

SPATIAL SMOOTHING METHOD

1) Divide linear array into overlapping subarrays



Subarrays: $s(1)$: {1,2,...,p}
$s(2)$: {2,3,...,p+1}

.

K=M-p+1 $s(K)$=s(M-p+1): {M-P+1,...,M}

RESOLVING COHERENT SOURCES OR TARGETS (4)

2) Estimate the spatial covariance matrix of each subarray

$$\underset{p\times p}{\underline{R}_i}: \quad i = 1,2,\ldots,K \qquad \text{where} \qquad \underline{R}_i = E\left\{\underline{Y}_i(n)\,\underline{Y}_i^H(n)\right\}$$

$\underline{Y}_i(n)$ is the vector of reveived signals at the i[th] subarray

Thus:

$$\boxed{\underset{p\times p}{\underline{R}_i} = \underset{p\times I}{\underline{A}}\left(\underset{I\times I}{\underline{D}^{(i-1)}}\ \underset{I\times I}{\underline{S}}\ \underset{I\times I}{\left[\underline{D}^{(i-1)}\right]^H}\right)\underset{I\times p}{\underline{A}^H} + \underset{p\times p}{\underline{W}_i}}$$

$$\underline{D} = diag\left[e^{-j\omega_0\tau_1}, \ldots, e^{-j\omega_0\tau_I}\right] \qquad \text{where} \qquad \tau_i = \frac{d}{c}\sin\theta_i$$

RESOLVING COHERENT SOURCES OR TARGETS (5)

3) Average the subarray covariance matrices

$$\underset{p\times p}{\bar{R}} = \frac{1}{K}\sum_{i=1}^{K} \underline{R}_i$$

It follows that: $\underline{\bar{R}} = \underline{A}\,\underline{\bar{S}}\,\underline{A}^H + \sigma^2 \underline{I}_p$ where $\bar{S} = \frac{1}{K}\sum_{i=1}^{K} \underline{D}^{(i-1)}\underline{S}\left[D^{i-1}\right]^H$

$\bar{S}$ is nonsingular regardless of the coherence of the signals

CONCLUSION: In resolving coherent sources, apply "incoherent sources techiques" on $\bar{R}$

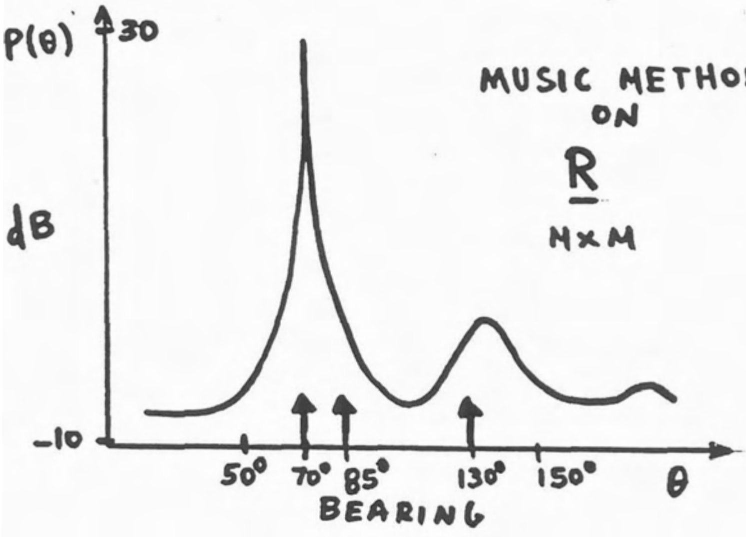
LIMITATION:

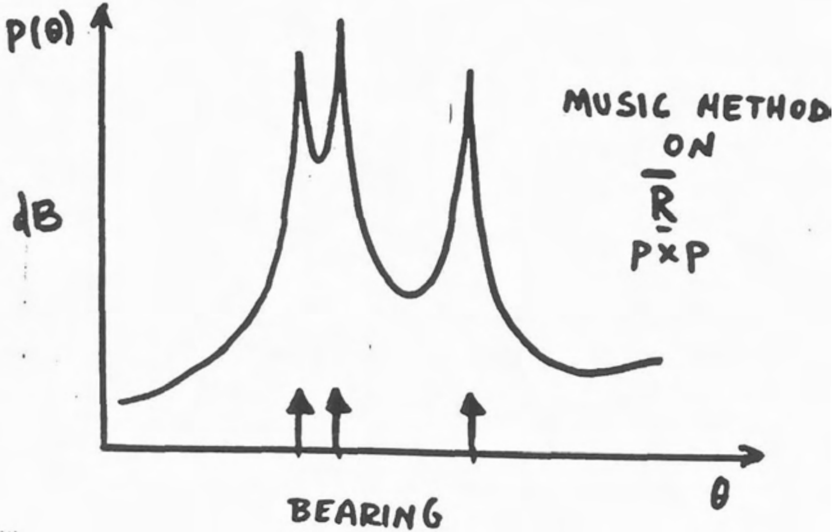
1) Resolution is smaller

2) The number of sources that can be detected is less than p

RESOLVING COHERENT SOURCES OR TARGETS (6)

Example: SNR=3dB, N=600, coherent sources at 70°, 85°, incoherent sources at 130°

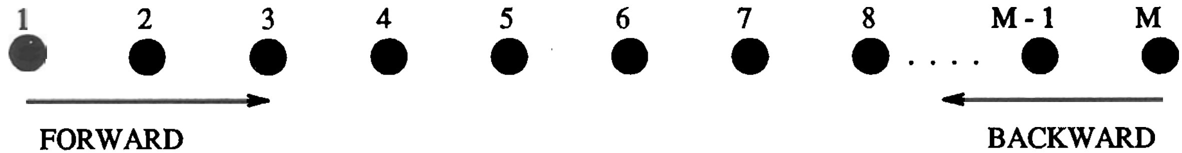




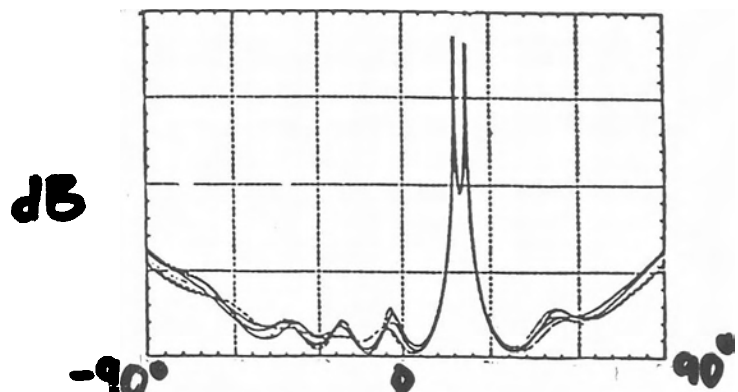
RESOLVING COHERENT SOURCES OR TARGETS (7)

THE FORWARD - BACKWARD LEAST SQUARES AR METHOD

Given the data array



Apply the FBLS AR method on each one of the snapshots independently. We may also use the burg technique, CLS method etc.



- Good performance if model order is accurately estimated.

- Modified FBLS (Kumaresan, Tufts)

BEAMFORMING (1)

- Definition: A beamformer is a processor that in conjunction with an array of sensors provides a form of spatial filtering

  Objective: Given a # of desired signals and a # of interferers (incoherent from the signals), then reject the interferers.

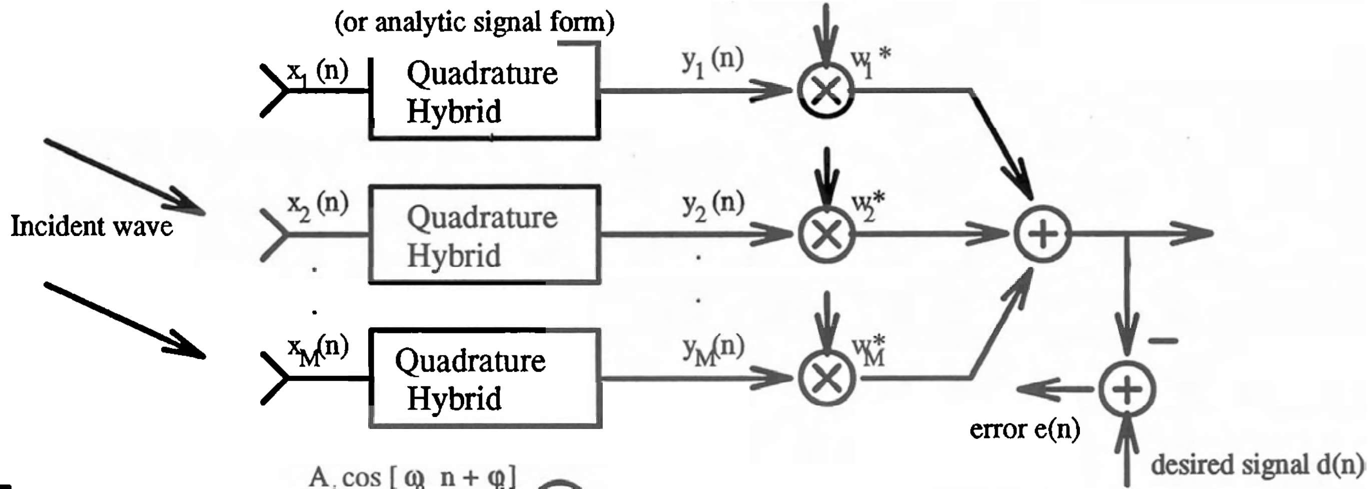
  Case I : D.O.A. for desired signal is known

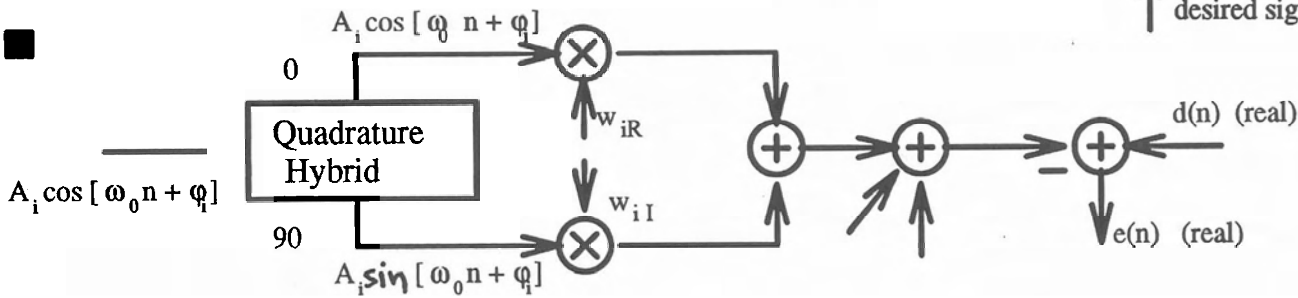
  Case II: D.O.A. for the desired signal is not known

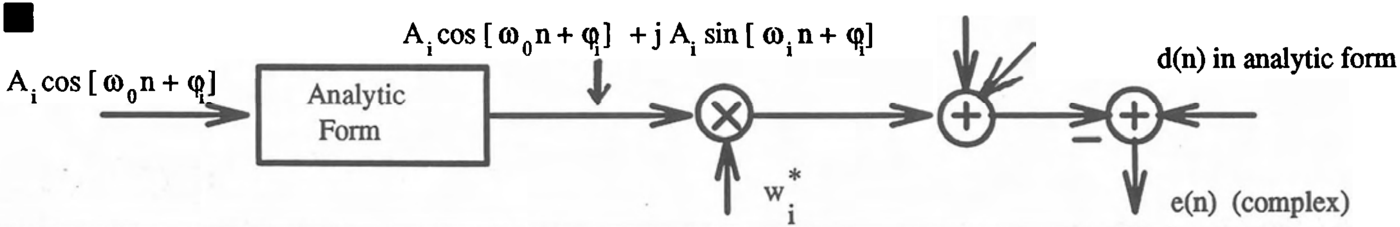
  Implementation: Shape the sensor array pattern by appropriately weighting the sensor outputs so that maximum gain is placed at the direction of "desired signal" and minimum gain (nulls) at the direction of interferers

BEAMFORMING (2)

BEAMFORMER CONFIGURATION (Narrowband)

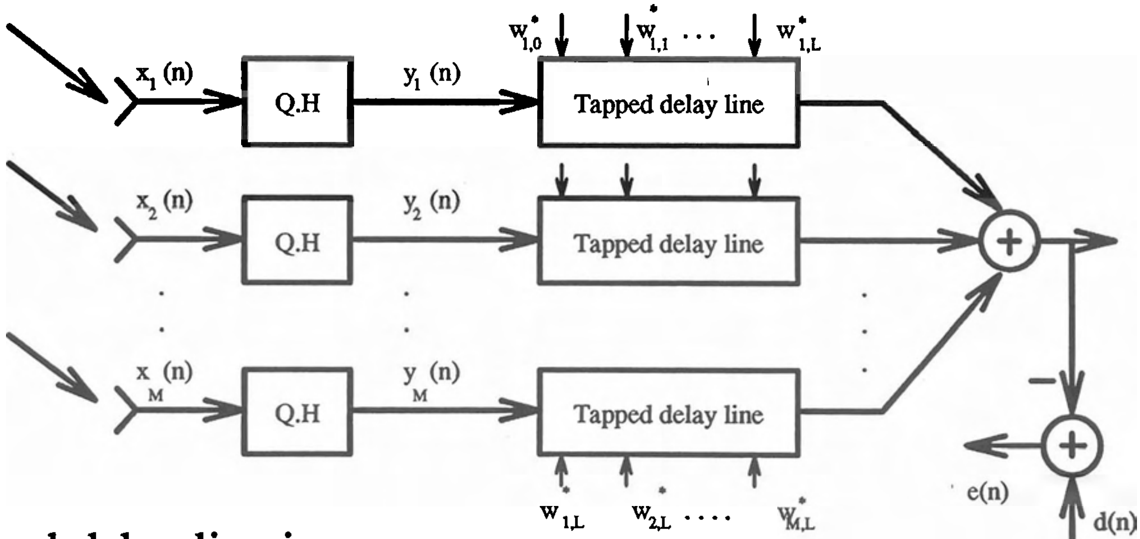




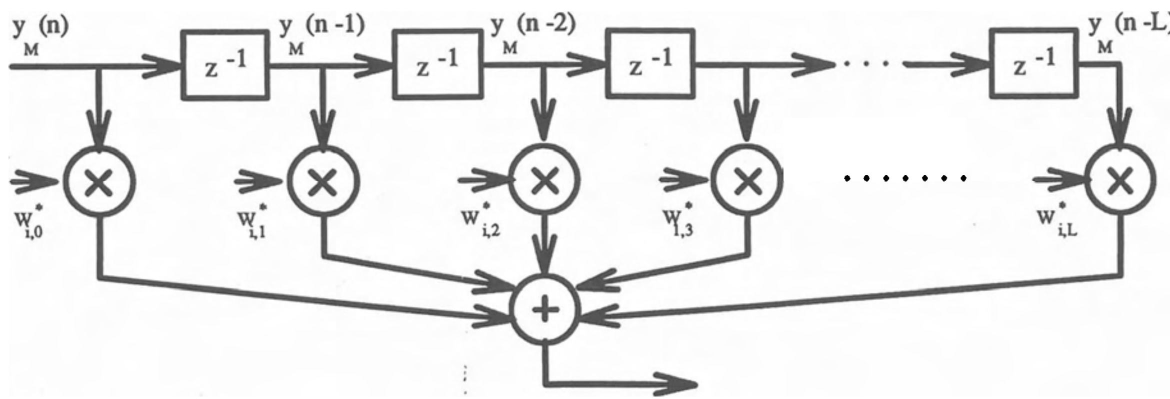


BEAMFORMING (3)

BEAMFORMER CONFIGURATION (Wideband)



Tapped delay line i



BEAMFORMING (4)

SOLUTION

MMSE solution:

Minimize: $E\{|e(n)|^2\}$ w.r.t. $\{w_{i,k}\}$

Solution: $$\boxed{\underline{W}_{opt} = \underline{R}_y^{-1} \cdot \underline{r}_{yd}}$$

where: $\underline{R}_y = \frac{1}{N} \sum_{n=1}^{N} \underline{Y}(n)\, \underline{Y}^H(n)$ and $\underline{r}_{yd} = \frac{1}{N} \sum_{n=1}^{N} \underline{Y}(n)\, d^*(n)$

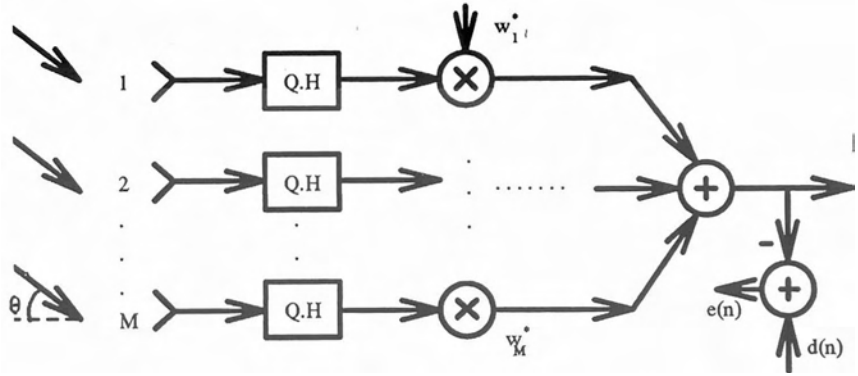
$$\underline{Y}(n) = [y_1(n), y_1(n-1), \dots, y_1(n-L), y_2(n), \dots, y_2(n-L), \dots, y_M(n), \dots y_M(n-L)]^T$$

$$\underline{W}_{opt} = [w_{1,0}, \dots, w_{1,L}, w_{2,L}, \dots, w_{2,L}, \dots, w_{M,1}, \dots, w_{M,L}]^T$$

BEAMFORMING (5)

■ ARRAY PATTERN (Narrowband)

Assuming reference node on top



Array pattern: $$\boxed{F(\theta,\lambda) = \sum_{i=1}^{M} w_i^* . e^{-j\frac{2\pi d}{\lambda}(i-1)\sin\theta}}$$

or $\quad F(\theta,\lambda) = \underline{W}^H . \underline{c}(\theta,\lambda)$

$\underline{c}(\theta,\lambda)$: steering vector at wave λ

BEAMFORMING (6)

LINEAR CONSTRAINED MINIMUM VARIANCE BEAMFORMING (LCMV)

Given the spatial covariance matrix $\underline{R}_y = \frac{1}{N}\sum_{n=1}^{N} \underline{Y}(n)\,\underline{Y}^H(n)$

Minimize $\left[\underline{W}^H \underline{R}_y \underline{W}\right]$, s.t.c. $\underline{W}^H \underline{c}(\theta_d, \lambda_d) = 1$, w.r.t. $\{w_i\}$, $i = 1,\ldots,M$

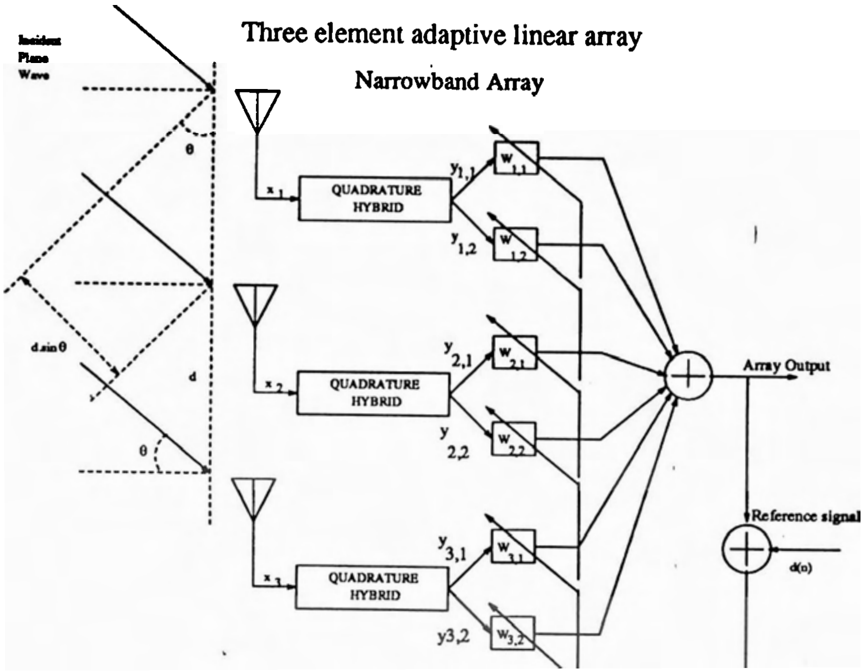
where: θ_d, λ_d: D.O.A., wavelength of desired signal respectively

SOLUTION

$$\underline{W} = \frac{\underline{R}_y^{-1}\,\underline{c}(\theta,\lambda)}{\underline{c}^H(\theta,\lambda)\,\underline{R}_y^{-1}\,\underline{c}(\theta,\lambda)}$$

ADAPTIVE BEAMFORMING (1)

LINEAR ADAPTIVE ARRAY (1)



Note:

- Use LMS, RLS or . . . adaption rules
- Reference (desired) signal must be known
- DOA for the desired signal is not known
- M sensor can be rejected up to M-1 interferers

ADAPTIVE BEAMFORMING (2)

LINEAR ADAPTIVE ARRAY (2) (3 sensors)

- LMS ALGORITHM

1) Initialization: $\underline{W}(0) = \underline{0}$

2) Update equation: $\underline{W}(n) = \underline{W}(n-1) + \mu\,\underline{Y}(n-1)\,e(n-1)$

where $e(n) = d(n) - \underline{W}^H(n)\,\underline{Y}(n)$

3) $0 < \mu < \dfrac{2}{\underline{Y}^H(n)\,\underline{Y}(n)}$

where d(n), e(n) are real

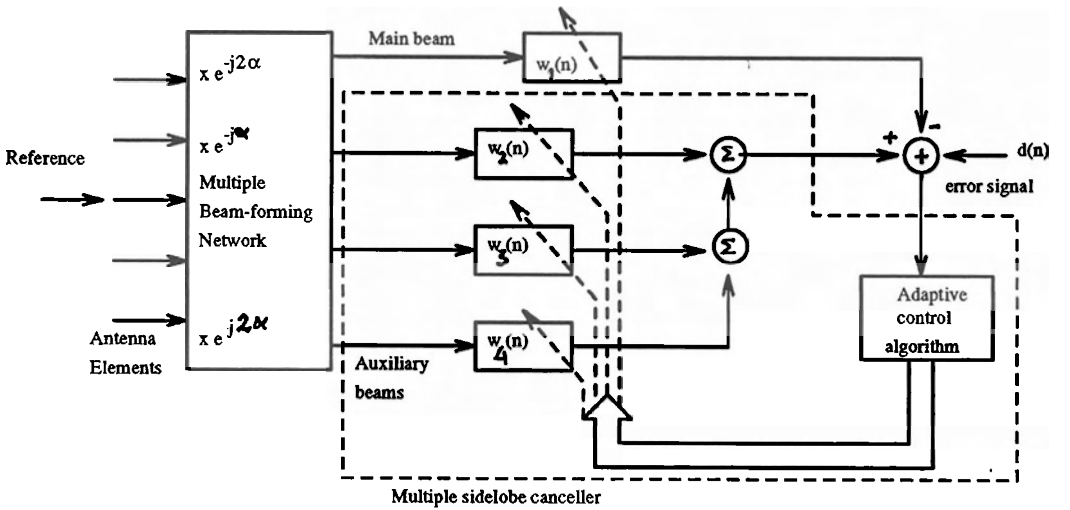
$$\underline{W}(n) = \left[w_{1,1}(n), w_{1,2}(n), w_{2,1}(n), w_{2,2}(n), \ldots, w_{M,1}(n), w_{M,2}(n)\right]^T$$

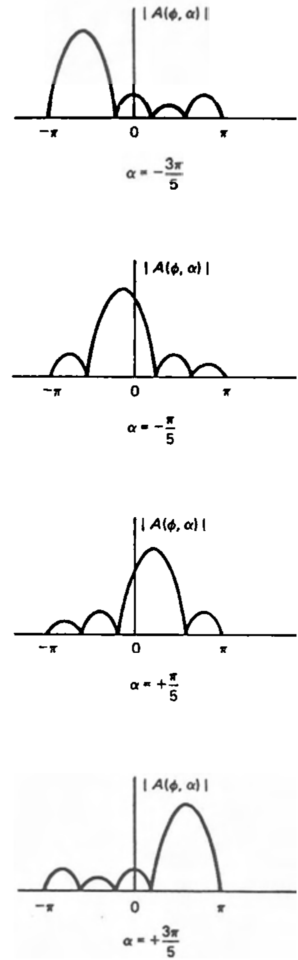
$$\underline{Y}(n) = \left[y_{1,1}(n), y_{1,2}(n), y_{2,1}(n), y_{2,2}(n), \ldots, y_{M,1}(n), y_{M,2}(n)\right]^T$$

4) $F_n(\theta,\lambda) = \sum^{M} (w_{i,1} - j w_{i,2})\, e^{-j\frac{2\pi d}{\lambda}(i-1)\sin\theta}$

ADAPTIVE BEAMFORMING (3)

ADAPTIVE SIDELOBE CANCELLER (1) (Narrowband)





- Beamforming network forms a set of orthogonal beams
- Consider as reference sensor the one in the middle
- Assume D.O.A. for desired signal is known
- M sensors can reject up to M-2 interferers

5 sensors

4 orthogonal beams

ADAPTIVE BEAMFORMING (4)

ADAPTIVE SIDELOBE CANCELLER (2)

- Assume for the moment that outputs of the sensors are equally weighted and have a uniform phase. Then,

Response of array to an incident phase ϕ

$$\boxed{A(\theta,\alpha) = \sum_{n=-\frac{M-1}{2}}^{\frac{M-1}{2}} e^{jn\phi}\, e^{-jn\alpha} = \frac{\sin\left[\frac{1}{2}(M+1)(\phi-\alpha)\right]}{\sin\left[\frac{1}{2}(\phi-\alpha)\right]}}$$

for $\alpha = \frac{\pi}{M}k, \quad k = \pm 1, \pm 3, \ldots, \pm M-2$

M-1 orthogonal array beams can be generated

- $\phi = \pm\frac{2\pi d}{\lambda}\sin\theta$, special case ($d = \frac{\lambda}{2}, \quad -\pi \le \phi \le \pi, \quad -90° \le \theta \le 90°$)

ADAPTIVE BEAMFORMING (5)

■ ADAPTIVE SIDELOBE CANCELLER (3)

1) Given M sensors, form M-1 orthogonal beams by combining appropriately the sensor outputs.

2) Place one of the beams at the direction of the desired signal. Then the other auxiliary beams will place a null at that direction.

3) Weight each beam with the weights $\{w_i(n)\}$, $i=1,\ldots,M$. Due to the symmetry of the problem the weights are real.

4) Update $\{w_i(n)\}$, $i=1,\ldots,M$ using LMS algorithm.

5) The array beam at instant n is

$$F_n(\theta,\lambda) = \sum_{i=1}^{M-1} w_i(n) \frac{\sin\left[\frac{1}{2}(M+1)(\phi-\alpha_i)\right]}{\sin\left[\frac{1}{2}(\phi-\alpha_i)\right]}$$

ADAPTIVE BEAMFORMING (6)

Example: INTERFERENCE REJECTION (1)

Given an adaptive array with M=3 elements uniformly spaced with d=1 ($\lambda_d = 2d$). The data received by the array are described by the equation

$$x_m(n) = \cos\left[2\pi f_d(n-1) + \phi_1\right] + I\cos\left[2\pi f_I(n-1) + \phi_2\right] + w(n)$$

$m=1,2,3; \quad n=1,2,\ldots,512$

where:

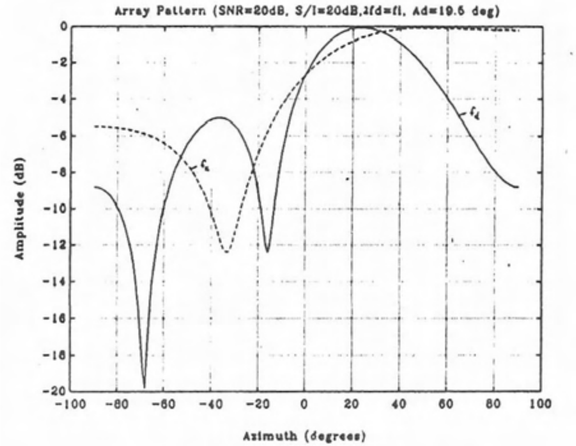
- f_d is the frequency of desired signal

- f_I is the frequency of an interferer

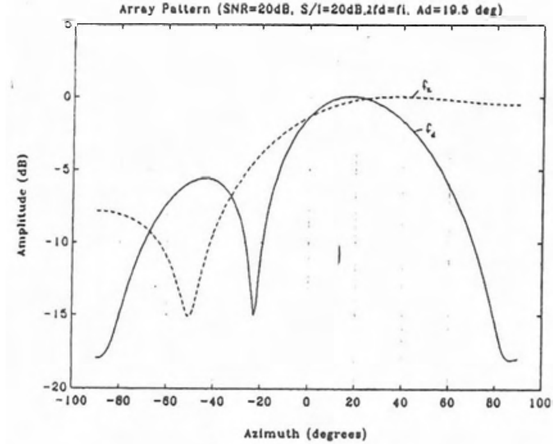
- w(n) if AWGN

ADAPTIVE BEAMFORMING (7)

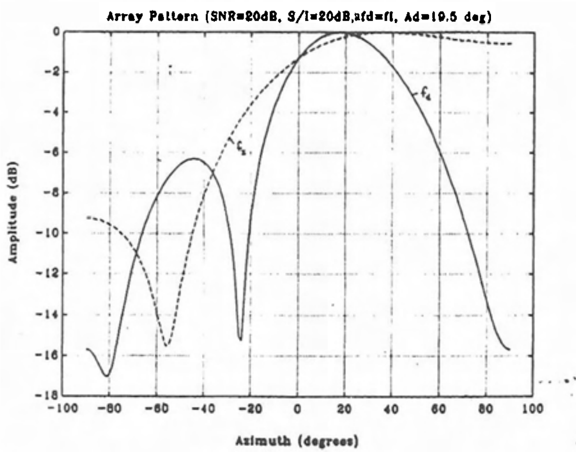
Example: INTERFERENCE REJECTION (2)



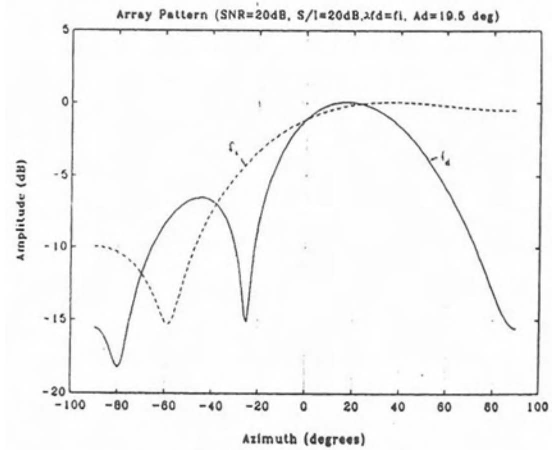
Interferer at 19.5 degrees



Interferer at -60 degrees



Interferer at -50 degrees



Interferer at -80 degrees