ECE 1511,
PROBLEM SET **1**(3 problems)

Problem 1

In the following, $x[n]$ is a sample of a real stationary zero-mean random process. The random vector x is the vector of samples

$$x = [x[0]\ x[2]\ x[3]\ x[4]\ x[5]]^T$$

Note that $x[1]$ is missing. Say if the following statements are correct or incorrect and why.
(a) The covariance matrix for this random vector is a *Toeplitz* matrix.
(b) The covariance matrix for this random vector is a *symmetric* matrix.
(c) If the covariance matrix of the vector x is given, we can determine from it the covariance matrix for the vector

$$x' = [x[0]\ x[1]\ x[2]\ x[3]\ x[4]\ x[5]]^T$$

where $x[1]$ is not missing.

Problem 2.

A certain real random process is defined by
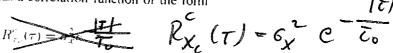
$$x[n] = A\cos\omega_0 n + w[n]$$

where A is a Gaussian random variable with mean zero and variance σ_A^2 and $w[n]$ is a white noise process with variance σ_w^2 independent of A.
(a) What is the correlation function of $x[n]$?
(b) Can the power spectrum of $x[n]$ be defined? If so, what is the power spectral density function?
(c) Repeat parts (a) and (b) in the case when the cosine has an independent random phase uniformly distributed between $-\pi$ and π.

Problem 3.

A bandlimited random signal is sampled at $T = 1$ ms.
(a) If this signal is used to form a discrete random sequence, what is the highest frequency in hertz that can be represented in the power density spectrum $S_x(e^{j\omega})$?
(b) What must be the maximum bandwidth of the process if the continuous-time signal is to be reconstructed from the discrete-time sequence?
(c) If the continuous random process has a correlation function of the form

$$R_{x_c}(\tau) = \sigma_x^2\, e^{-\frac{|\tau|}{\tau_0}}$$

what is the smallest value of the parameter τ_0 such that the continuous spectrum at the bandlimit corresponding to the sampling interval above is 40 dB below its value at $f = 0$? This gives a measure of the "correlation time" for a random process that can be adequately represented by samples at the given sampling interval.