ECE 1511 (2017) Problem Set 1, Brief Solutions

PROBLEM 1

Usually we consider zero mean white processes that is $E\{v_1\}=E\{v_2\}=0$

a) $R_x(k) = E\{x(n)x(n+k)\} = E\{(v_1(n)+3v_2(n-1))(v_1(n+k)+3v_2(n+k-1))\}$

$= E\{v_1(n)v_1(n+k)\} + 3E\{v_1(n)\cdot v_2(n+k-1)\} + 3E\{v_2(n-1)v_1(n+k)\} +$

$+ 9E\{v_2(n-1)v_2(n+k-1)\} = R_{v_1}(k) + 0 + 0 + 9R_{v_2}(k)$

$= R_{v_1}(k) + 9R_{v_2}(k) = 0.5\,\delta(k) + 9\cdot 0.5\,\delta(k) = \underline{5\,\delta(k)}$

Similarly you can show that $R_y(k) = 5\delta(k)$ as well.
Both processes are wide sense stationary because both mean and autocorrelations are time invariant.

b) $R_{xy}(n_1,n_0) = E\{x(n_1)y(n_0)\} =$

$= E\{v_1(n_1)\cdot v_2(n_0+1)\} + 3E\{v_1(n_1)\cdot v_1(n_0+1)\} + 3E\{v_2(n_1+1)v_2(n_0+1)\}$

$+ 9E\{v_2(n_1+1)v_1(n_0+1)\} =$

$= 0 + 3R_{v_1}(n_0-n_1+1) + 3R_{v_2}(n_0-n_1) + 0 =$

$= 3\cdot 0.5\,\delta(\underbrace{n_0-n_1}_{\tau}+1) + 3\cdot 0.5\,\delta(\underbrace{n_0-n_1}_{\tau})$ which indicates that

$R_{x,y}(n_1,n_0)$ is time invariant.

So yes the two processes are marginally and jointly

WSS

PROBLEM 2

a) $S_x(\omega) = \sum_{l=-\infty}^{\infty} R_x(l) e^{-j\omega l} = \sum_{l=-3}^{3} (3-|l|) e^{-j\omega l} = 0 + e^{j2\omega} + 2e^{+j\omega} + 3 + 2e^{-j\omega} + e^{-j2\omega} + 0 =$

$= 3 + \underbrace{e^{j2\omega} + e^{-j2\omega}}_{2\cos 2\omega} + \underbrace{2e^{j\omega} + 2e^{-j\omega}}_{4\cos\omega} = 3 + 2\cos\omega + 4\cos\omega$

Real ✓

Nonnegative ✓ (you need to plot the components and show it graphically)

b) similar to part (a)

PROBLEM 3:

Use the relation $\sum_{n_1=-\infty} \sum_{n_0=-\infty}^{\infty} a^*(n_1) R_x(n_1 - n_0) a(n_0) \geq 0$

with $a(n) = \begin{cases} b & n=0 \\ b^* & n=l \\ 0 & \text{otherwise} \end{cases}$

Then,

$b^* b R_x[0-0] + b^2 R_x(l-0) + (b^*)^2 (0-l) + b b^* R_x[l-l] \geq 0$

$\Rightarrow 2|b|^2 R_x(0) + 2 \operatorname{Re}[b^2 R_x(l)] \geq 0$

Assuming $R_x(l) = |R_x(l)| e^{j\varphi(l)}$ and taking $b = e^{-j\frac{\varphi(l)+\pi}{2}}$

we get $2 R_x(0) + 2 \operatorname{Re}[-|R_x(l)|] \geq 0$

$\Rightarrow R_x(0) - |R_x(l)| \geq 0$ ✓