ECE 1511, Problem set 2 (due Oct 9, 2019)

October 1, 2019

PROBLEM 1

Given a random process $x[n]$, the autocorrelation $R_x(n_1, n_2)$ is defined as $R_x(n_1, n_2) = E\{x^*(n_1)x(n_2)\}$ and the Power spectral density $S_x(\omega_1, \omega_2) = \sum_{n_1}\sum_{n_2} R_x(n_1, n_2)e^{-j\omega_1 n_1}e^{-j\omega_2 n_2}$.

Show that imposing stationarity condition on $R_x(n_1, n_2)$ so that $R_x(n_1, n_2) = R_x(n_2 - n_1)$ is equivalent to $\omega_1 + \omega_2 = 0$

PROBLEM 2

- Generate a white noise sequence $x[n]$ of length 1024 samples. with zero mean and unit variance ($\mu = 0$, $\sigma^2 = 1$)
- Given $x[n]$, $n = 0, \dots, 1023$, generate another sequence $y[n]$ described by the first-order recursive filter

$$y[n] = \rho\, y[n-1] + x[n]$$

generate three separate filtered sequences using the values $\rho = 0.95$, $\rho = 0.7$, $\rho = -0.95$. Plot each of these sequences and the white noise. What differences do you observe?

- Compute the "sample" autocorrelation function for each $y[n]$, $\hat{R}_y(l)$. and define the estimated correlation coefficient $\hat{\rho}$ as $\hat{\rho} = \hat{R}_y(1)/\hat{R}_y(0)$

How well $\hat{\rho}$ compare with the theoretical value ρ?

Problem ~~2.~~ 3

A complex periodic random signal consists of three components:

$$x[n] = \underbrace{e^{j(w_a n + \varphi_a)}}_{s_a(n)} + \underbrace{e^{j(w_b n + \varphi_b)}}_{s_b(n)} + \underbrace{e^{j(w_c n + \varphi_c)}}_{s_c(n)}$$

The frequencies w_a, w_b and w_c are harmonically related so that

$$w_a = w_b + w_c$$

The phases φ_a and φ_b are independent and uniformly distributed between $-\pi$ and π

Calculate and draw the Power Spectral Density and the Bispectrum of $x[n]$ for the following two cases.

Case 1: φ_c is independent of the other two phases and is also uniformly distributed between $-\pi$ and π.

Case 2: φ_a is linearly related to the other two phases, i.e., $\varphi_a = \varphi_b + \varphi_c$