

PROBLEM 1

Given a random process $x(n)$, the autocorrelation $R_x(n_1, n_2)$ is defined as $R_x(n_1, n_2) = E\{x^*(n_1)x(n_2)\}$ and the Power spectral density $S_x(\omega_1, \omega_2) = \sum_{n_1, n_2} R_x(n_1, n_2) e^{-j\omega_1 n_1} e^{-j\omega_2 n_2}$

Show that imposing stationarity condition on $R_x(n_1, n_2)$ so that $R_x(n_1, n_2) = R_x(n_2 - n_1)$ is equivalent to $\omega_1 + \omega_2 = 0$

PROBLEM 2

- Generate a white noise sequence $x(n)$ of length 1024 samples with zero mean and unit variance ($\mu=0, \sigma^2=1$)
- Given $x(n)$, $n=0, \dots, 1023$, generate another sequence $y(n)$ described by the first-order recursive filter

$$y(n) = p y(n-1) + x(n)$$

generate three separate filtered sequences using the values $p=0.95$, $p=0.7$, $p=-0.95$. Plot each of these sequences and the white noise. What differences do you observe?

- Compute the "sample" autocorrelation function for each $y(n)$, $\hat{R}_y(l)$, and define the estimated correlation coefficient $\hat{\rho}$ as

$$\hat{\rho} = \frac{\hat{R}_y(1)}{\hat{R}_y(0)}$$
 How well $\hat{\rho}$ compare with the theoretical value p ?

Problem 3

A complex periodic random signal consists of three components:

$$x[n] = \underbrace{e^{j(\omega_a n + \phi_a)}}_{s_a(n)} + \underbrace{e^{j(\omega_b n + \phi_b)}}_{s_b(n)} + \underbrace{e^{j(\omega_c n + \phi_c)}}_{s_c(n)}$$

The frequencies ω_a , ω_b and ω_c are harmonically related so that

$$\omega_a = \omega_b + \omega_c$$

The phases ϕ_a and ϕ_b are independent and uniformly distributed between $-\pi$ and π

Calculate and draw the Power Spectral Density and the Bispectrum of $x[n]$ for the following two cases.

Case 1: ϕ_c is independent of the other two phases and is also uniformly distributed between $-\pi$ and π .

Case 2: ϕ_a is linearly related to the other two phases i.e., $\phi_a = \phi_b + \phi_c$