From text by Monson Hayes, "Statistical DSP and modeling"

7.2.3 Noise Cancellation

Another application of Wiener filtering is the problem referred to as *noise cancellation.* As in the filtering problem, the goal of a noise canceller is to estimate a signal $d(n)$ from a noise corrupted observation

$$x(n) = d(n) + v_1(n)$$

that is recorded by a primary sensor. However, unlike the filtering problem, which requires that the autocorrelation of the noise be known, with a noise canceller this information is obtained from a secondary sensor that is placed within the noise field as illustrated in Fig. 7.8. Although the noise measured by this secondary sensor, $v_2(n)$, will be *correlated* with the noise in the primary sensor, the two processes will not be equal. There may be a number of reasons for this, such as differences in the sensor characteristics, differences in the propagation paths from the noise source to the two sensors, and leakage of the signal $d(n)$ into the measurements made by the secondary sensor. Since $v_1(n) \neq v_2(n)$ it is not possible to estimate $d(n)$ by simply subtracting $v_2(n)$ from $x(n)$. Instead, the noise canceller consists of a Wiener filter that is designed to estimate the noise $v_1(n)$ from the signal received by the secondary sensor. This estimate, $\hat{v}_1(n)$, is then subtracted from the primary signal $x(n)$, to form an estimate of $d(n)$, which is given by

$$\hat{d}(n) = x(n) - \hat{v}_1(n)$$

An example of where such a system may be useful is in air-to-air communications between pilots in fighter aircraft or in air-to-ground communications between a pilot and the control tower. Since there is often a large amount of engine and wind noise within the cockpit of the fighter aircraft, communication is often a difficult problem. However, if a secondary microphone is placed within the cockpit of an aircraft, then one may estimate the noise that is transmitted when the pilot speaks into the microphone, and subtract this estimate from the transmitted signal, thereby increasing the signal-to-noise ratio.

The Wiener-Hopf equations for the noise cancellation system in Fig. 7.8 may be derived as follows. With $v_2(n)$ the input to the Wiener filter that is used to estimate the noise $v_1(n)$, the Wiener-Hopf equations are

$$\mathbf{R}_{v_2}\mathbf{w} = \mathbf{r}_{v_1 v_2}$$

where $\mathbf{R}_{v_2}$ is the autocorrelation matrix of $v_2(n)$ and $\mathbf{r}_{v_1 v_2}$ is the vector of cross-correlations between the desired signal $v_1(n)$ and Wiener filter input, $v_2(n)$. For the cross-correlation

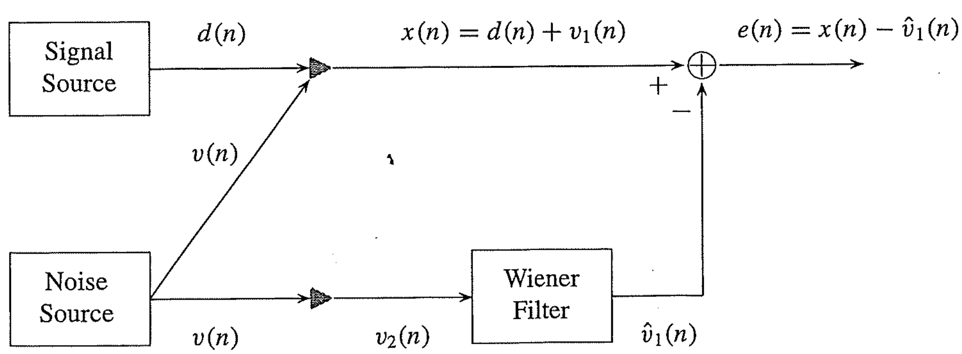

Figure 7.8 *Wiener noise cancellation using a secondary sensor to measure the additive noise $v_1(n)$.*

between $v_1(n)$ and $v_2(n)$ we have

$$r_{v_1v_2}(k) = E\{v_1(n)v_2^*(n-k)\} = E\{[x(n)-d(n)]v_2^*(n-k)\}$$
$$= E\{x(n)v_2^*(n-k)\} - E\{d(n)v_2^*(n-k)\} \tag{7.21}$$

If we assume that $v_2(n)$ is uncorrelated with $d(n)$, then the second term is zero and the cross-correlation becomes

$$r_{v_1v_2}(k) = E\{x(n)v_2^*(n-k)\} = r_{xv_2}(k) \tag{7.22}$$

Therefore, the Wiener-Hopf equations are

$$\mathbf{R}_{v_2}\mathbf{w} = \mathbf{r}_{xv_2} \tag{7.23}$$

We will now look at a specific example.

Example 7.2.6 *Noise Cancellation*

Suppose that the desired signal $d(n)$ in Fig. 7.8 is a sinusoid

$$d(n) = \sin(n\omega_0 + \phi)$$

with $\omega_0 = 0.05\pi$, and that the noise sequences $v_1(n)$ and $v_2(n)$ are AR(1) processes that are generated by the first-order difference equations

$$v_1(n) = 0.8v_1(n-1) + g(n)$$
$$v_2(n) = -0.6v_2(n-1) + g(n)$$

where $g(n)$ is zero-mean, unit variance white noise that is uncorrelated with $d(n)$. Shown in Fig. 7.9*a* is a plot of 200 samples of $x(n) = d(n) + v_1(n)$ with the desired signal, $d(n)$, indicated by the dashed line, and shown in Fig. 7.9*b* is the reference signal $v_2(n)$ that is used to estimate $v_1(n)$. Estimating $r_{v_2}(k)$ using the sample autocorrelation

$$\hat{r}_{v_2}(k) = \frac{1}{N}\sum_{n=0}^{N-1} v_2(n)v_2(n-k)$$

and $r_{xv_2}(k)$ using the sample cross-correlation

$$\hat{r}_{xv_2}(k) = \frac{1}{N}\sum_{n=0}^{N-1} x(n)v_2(n-k)$$

FIR Wiener filters of orders $p = 6$ and $p = 12$ were found by solving Eq. (7.23). Using these filters to estimate $v_1(n)$, the sinusoid $d(n)$ was then estimated by subtracting $\hat{v}_1(n)$ from $x(n)$. The results are shown in Figs. 7.9*c* and *d*.

In typical applications, $d(n)$ and $v_1(n)$ are often found to be non-stationary processes. Therefore, the use of a linear shift-invariant Wiener filter will not be optimum. However, as we will see in Chapter 9, an adaptive Wiener filter that has filter coefficients that are allowed to vary as a function of time may provide effective noise cancellation in nonstationary environments.

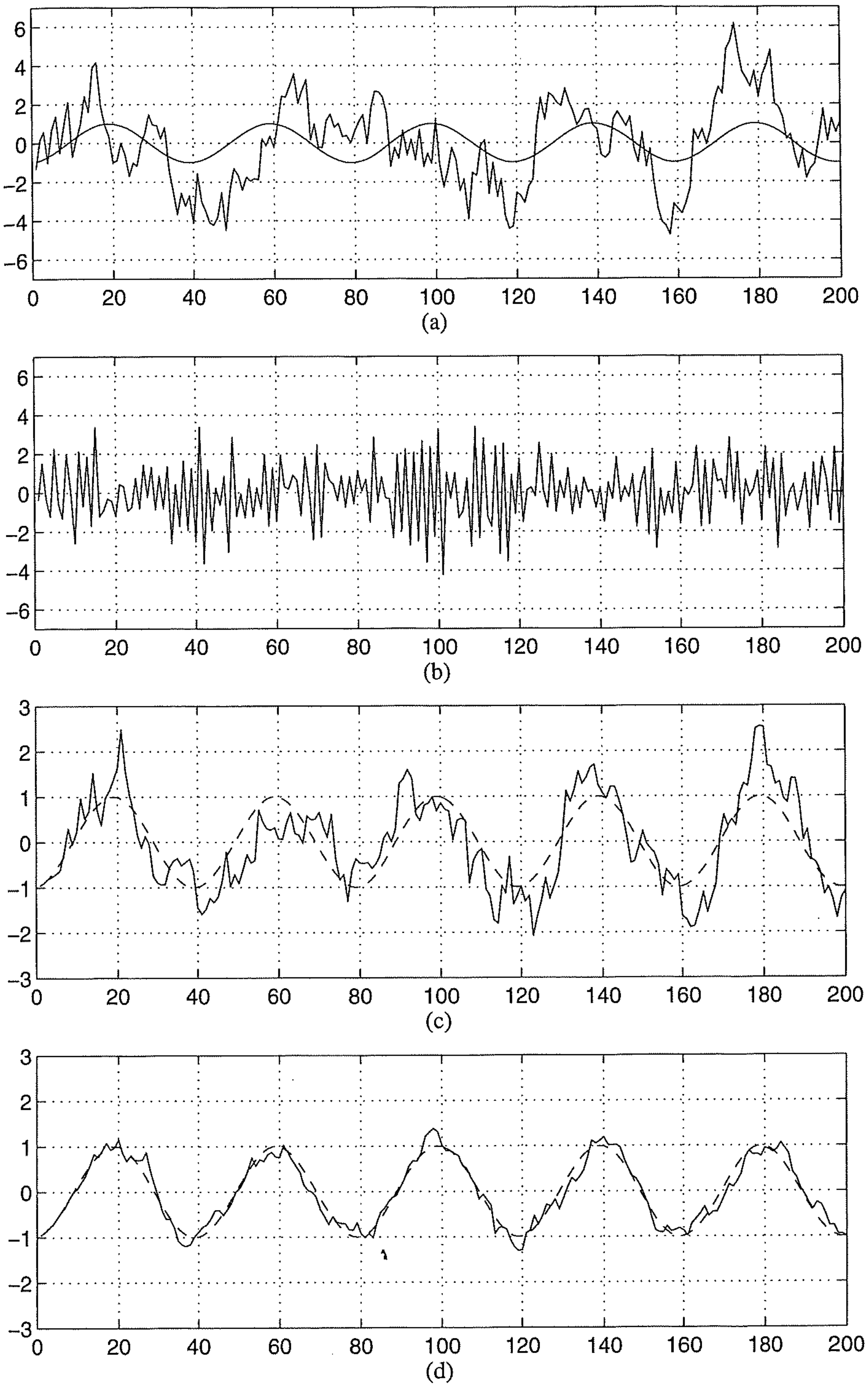

Figure 7.9 *Noise cancellation example. (a) Noise corrupted sinusoid, (b) Reference signal used by secondary sensor, (c) Output of sixth-order Wiener noise canceller, (d) Output of twelth-order Wiener noise canceller.*

ECE 1511, Fall 2019

Hw # 4

Due date: October 22, 2019

PROBLEM

C7.2. In this exercise we look at the noise cancellation problem considered in Example 7.2.6. Let

$$x(n) = d(n) + g(n)$$

where $d(n)$ is the harmonic process

$$d(n) = \sin(n\omega_0 + \phi)$$

with $\omega_0 = 0.05\pi$ and ϕ is a random variable that is uniformly distributed between $-\pi$ and π. Assume that $g(n)$ is unit variance white noise. Suppose that a noise process $v_2(n)$ that is correlated with $g(n)$ is measured by a secondary sensor. The noise $v_2(n)$ is related to $g(n)$ by a filtering operation,

$0.8 \cdot v_2(n-1) + g(n)$

$$v_2(n) = 0.8v_2(n) + g(n)$$

(a) Using MATLAB, generate 500 samples of the processes $x(n)$ and $v_2(n)$.

(b) Derive the Wiener-Hopf equations that define the optimum pth-order FIR filter for estimating $g(n)$ from $v_2(n)$.

(c) Using filters of order $p = 2, 4$, and 6, design and implement the Wiener noise cancellation filters. Make plots of the estimated process $\hat{g}(n)$ and compare the average squared errors for each filter.

(d) In some situations, the desired signal may leak into the secondary sensor. In this case, the performance of the Wiener filter may be severely compromised. To see what effect this has, suppose the input to the Wiener filter is

$$v_0(n) = v_2(n) + \alpha d(n)$$

where $v_2(n)$ is the filtered noise defined above. Evaluate the performance of the Wiener noise canceller for several different values of α for filter orders of $p = 2$, 4, and 6. Comment on your observations.