1 ECE302 - INTRODUCTION TO THE COURSE

1.1 BASIC TERMS

- The consideration of a single result of some random phenomenon is termed an *"experiment"* or *"trial"* (e.g. flipping a coin)

- The set of all possible outcomes of the experiment is termed the *"universe"* or *"sample space"* (S) (e.g., $S = \{$ heads, tails $\}$)

- In discussing the outcomes of an experiment we do not restrict ourselves to points in S but more generally deal with sets of points in S which are termed *"events"*

- To every *event* A we assign a real number $P(A)$ called the probability measure of A

Events are those subsets of S to which we have assigned probablities

* *"Elementary event"* or *"single event"* is the event that consists of only one point.

* *"Certain event"* is the event that occurs in every trial.

1.2 THE MEANING OF PROBABILITY AND RANDOMNESS

* The meaning of *"random"*.

Universe S1	Identical prior to the occurence of an event	**Universe S2**	Event occurs in S1 and S2 If S1 and S2 remain identical after the occurence of the event then the event is "deterministic" If S1 and S2 differ then the event is "random"

In practice, we use the idea of "randomness" (i.e., we use "random" or "probabilistic" models) when we don't know all the factors which might determine the outcome of an experiment. If all the factors are known or assumed known we use deterministic models.

Ex.1: "Flipping a coin" or "tossing a die". These experiments are deterministic situations to which random models are appropriate.

Ex.2: "Random number generator". Not random in reality since one number (seed) produces another number (new seed).

PROBABILISTIC MODELS are if nothing else, useful ways of dealing with our ignorance.

DETERMINISTIC OR RANDOM [1]? **(1)**

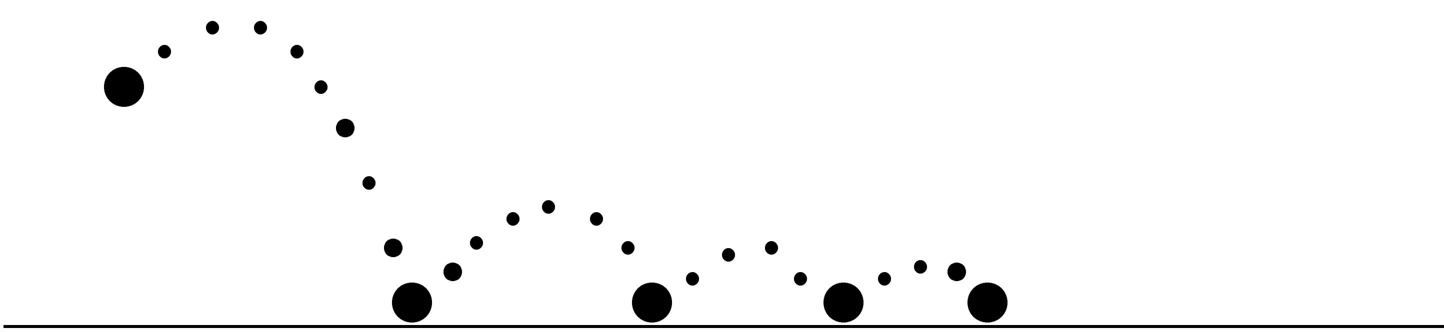
Question: What could more deterministic than the "die throwing"?

1. Assuming the initial impulse is given, then the mechanics of motion (i.e., gravitational pull, air resistance, energy dissipation in shocks and friction) are well understood.

2. Even if initial conditions (position, initial impulse) are exactly known the outcome cannot be fully predicted due to unknown influences.

3. Unpredictability is (or should be ?) the name of the game.

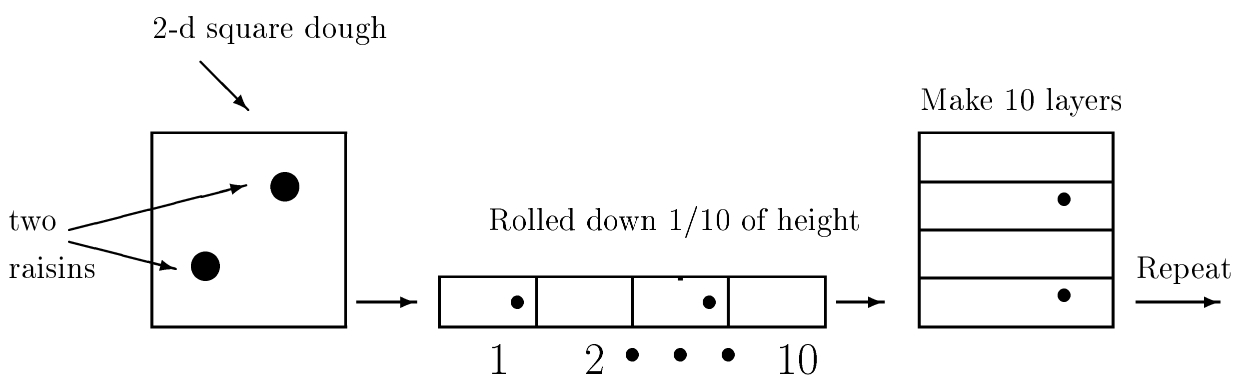
4. The throwing of a die is a completely deterministic situation to which random modeling is appropriate.



[1] Some ideas from the book "Mathematics and the Unexpected" by Ivar Ekeland.

DETERMINISTIC OR RANDOM ? (2)

Experiment 1 The "Bakers transformation".



* After 10 stages 10^{10} layers have been formed.
* Process totally reversible (how ?)

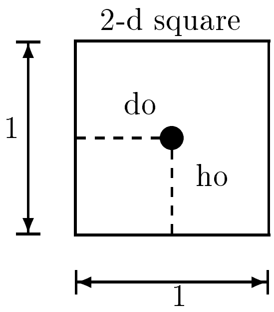
Deterministic situation: The present state totally determines the future. Past and future are contained in the present since the process is reversible.

Random situation: After many repetitions of the experiment there is an equal likelihood in finding the raisins anywhere in the dough.

- Similar situation: shuffling a pack of cards.

DETERMINISTIC OR RANDOM ? (3)

Experiment 2. The “Bernoulli Shift”



Initial point at (do, ho)
let do=0.75637521
ho=0.312795

Remove the first decimal of the number and make it the first decimal of the second

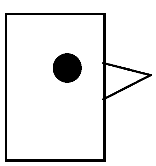
now d1=0.5637521
h1=0.7312795

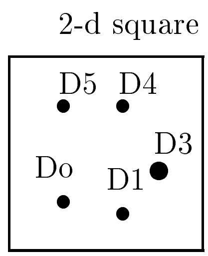
repeat

d2=0.637521
h1=0.573127

*** Situation totally reversible (deterministic)**

2-d observer





Do=(do, ho), D1=(d1,h1), ...

* However, the observer can only see ho,h1,h2...
The movement seems to him random

Randomness appears because the available information though accurate is incomplete. Part of the information is withheld from us.

There are cases where the available information is complete but inaccurate (e.g., chaotic systems, strange attractors). In such cases random models might be appropriate as well.