TUTORIAL #1 ECE 431S

Brief Solutions / answers

1

(a) Let $x_k(n)$ denote the sinusoidal signal sequence

$$x_k(n) = \sin\left(\frac{2\pi nk}{N} + \theta\right)$$

This is a sinusoid with frequency $f_k = k/N$, which is harmonically related to $x(n)$. But $x_k(n)$ may be expressed as

$$x_k(n) = \sin\left[\frac{2\pi(kn)}{N} + \theta\right]$$
$$= x(kn)$$

Thus we observe that $x_k(0) = x(0)$, $x_k(1) = x(k)$, $x_k(2) = x(2k)$, and so on. Hence the sinusoidal sequence $x_k(n)$ can be obtained from the table of values of $x(n)$ by taking every kth value of $x(n)$, beginning with $x(0)$. In this manner we can generate the values of all harmonically related sinusoids with frequencies $f_k = k/N$ for $k/N \le \frac{1}{2}$.

(b) We can control the phase θ of the sinusoid with frequency $f_k = k/N$ by taking the first value of the sequence from memory location $q = \theta N/2\pi$, where q is an integer. Thus the initial phase θ controls the starting location in the table and we wrap around the table each time the index (kn) exceeds N.

.2

a) periodic $f = 1/200$, period = 200
b) periodic $f = 1/7$, period = 7
c) periodic $f = 3/2$, period = 2
d) not periodic $f = 3/(2\pi)$
e) periodic $f = 31/10$, period = 10

.3

a) periodic, period $= 2\pi/5$.
b) non-periodic ($5/2\pi$ not rational)
c) non-periodic ($1/12\pi$ not rational)
d) non-periodic. ($1/16\pi$ not rational)
e) periodic period=least common multiplier (4,16,8)=16

4

a) $\omega = 2\pi k/N$ implies $f = k/N$
let α = GCD of (k,N) i.e. $k = k'\alpha$, $N = N'\alpha$
therefore, $f = k'/N'$ which implies the period
$N' = N/\alpha$ Q.E.D.

b) for N = 7

k =	0	1	2	3	4	5	6	7
period =	1	7	7	7	7	7	7	1

c) for N = 16

k =	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
period =	1	16	8	16	4	16	8	16	2	16	8	16	4	16	8	16

ELE431S Ass.#1 Solutions

(5) a) Frequencies in $X_a(t)$ are $F_1 = 1\ \text{KHz}$, $F_2 = 3\ \text{KHz}$, $F_3 = 6\ \text{KHz}$

Thus the Nyquist rate is $2F_{max} = 2 \cdot 6\ \text{KHz} = 12\ \text{KHz}$.

b) Since we have chosen $F_s = 5\ \text{KHz}$ there is aliasing

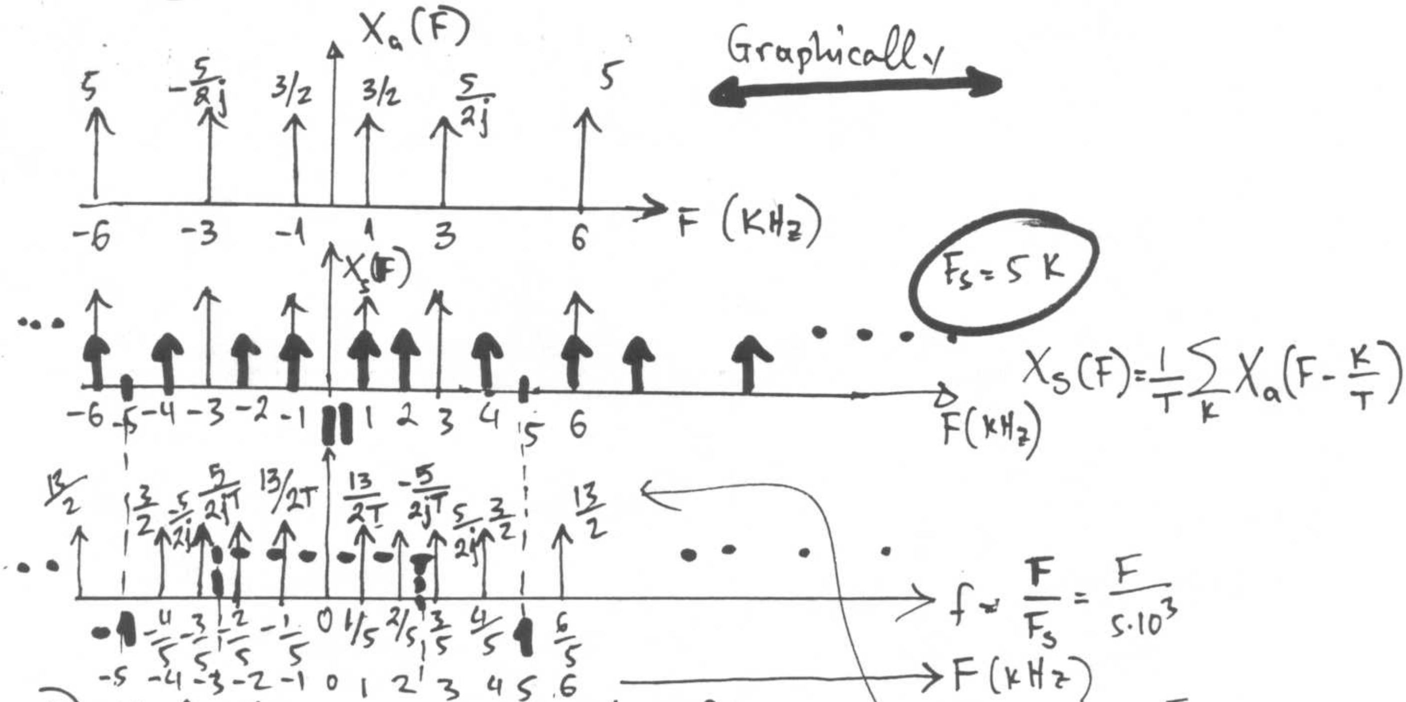
The signal $x(n)$ can be written as

$$x(n) = X_a(nT) = X_a\left(\frac{n}{F_s}\right) = 3\cos\left[\frac{2\pi}{5}n\right] + 5\sin\left[\frac{2\pi \cdot 3}{5}n\right] + 10\cos\left[2\pi\frac{6}{5}n\right]$$

$$= 3\cos\left[\frac{2\pi}{5}n\right] + 5\sin\left[\left(2\pi - \frac{2\pi \cdot 2}{5}\right)n\right] + 10\cos\left[\left(2\pi + \frac{2\pi}{5}\right)n\right]$$

$$= 3\cos\left[\frac{2\pi}{5}n\right] - 5\sin\left[\frac{2\pi 2}{5}n\right] + 10\cos\left[\frac{2\pi}{5}n\right]$$

$$\Rightarrow \boxed{x(n) = 13\cos\left[\frac{2\pi}{5}n\right] - 5\sin\left[\frac{4\pi}{5}n\right]}$$



c) Ideal low pass reconstruction filter cuttof frequency $\frac{F_s}{2} = 2.5\ K$ and amplitude T

Thus

$$y_a(t) = 13\cos\left[2\pi \cdot 1000t\right] - 5\sin\left[2\pi\, 2000t\right]$$

$$= 13\cos\left[2000\pi t\right] - 5\sin\left[4000\pi t\right]$$