Department of Electrical and Computer Engineering - University of Toronto

ECE 431, Fall 2023

Tutorial Assignment 1

1. Stored in the memory of a digital signal processor is one cycle of the sinusoidal signal

$$x(n) = sin\left(\frac{2\pi n}{N} + \theta\right)$$

where $\theta = \frac{2\pi q}{N}$, where q and N are integers.

   (a) Determine how this table of values can be used to obtain values of harmonically related sinusoids having the same phase.

   (b) Determine how this table can be used to obtain sinusoids of the same frequency but different phase.

2. Determine which of the following sinusoids are periodic and compute their fundamental period.

   (a) $cos(0.01\pi n)$

   (b) $cos\left(\pi\frac{30n}{105}\right)$

   (c) $cos(3\pi n)$

   (d) $sin(3n)$

   (e) $sin\left(\pi\frac{62n}{10}\right)$

3. Determine whether or not each of the following signals is periodic. In case a signal is periodic, specify its fundamental period.

   (a) $x_a(t) = 3cos(5t + \pi/6)$

   (b) $x(n) = 3cos(5n + \pi/6)$

   (c) $x(n) = 2exp[j(n/6 - \pi)]$

   (d) $x(n) = cos(n/8)cos(\pi n/8)$

   (e) $x(n) = cos(\pi n/2) - sin(\pi n/8) + 3cos(\pi n/4 + \pi/3)$

4. (a) Show that the fundamental period N_p of the signals

$$s_k(n) = e^{j2\pi kn/N}, \qquad k = 0, 1, 2, \ldots$$

   is given by $N_p = N/GCD(k, N)$, where GCD is the greatest common divisor of k and N.

   (b) What is the fundamental period of this set for $N = 7$?

   (c) What is it for $N = 16$?

5. Consider the analog signal

$$x_a(t) = 3cos(2000\pi t) + 5sin(6000\pi t) + 10cos(12000\pi t)$$

   (a) What is the Nyquist rate for this signal?

   (b) Assume that the signal is sampled at a rate $F_s = 5000$ samples/sec. What is the discrete signal $x(n)$ obtained?

   (c) What is the analog signal $y_a(t)$ reconstructed from $x(n)$ by using ideal interpolation? (i.e., ideal low pass filtering?)