8.24. Figure P8.24 shows a finite-length sequence $x[n]$. Sketch the sequences

$$x_1[n] = x[((n-2))_4], \qquad 0 \le n \le 3,$$

and

$$x_2[n] = x[((-n))_4], \qquad 0 \le n \le 3.$$

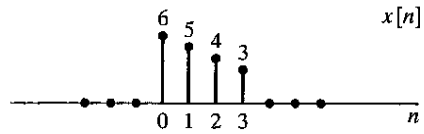

Figure P8.24

8.25. Consider the signal $x[n] = \delta[n-4] + 2\delta[n-5] + \delta[n-6]$.

(a) Find $X(e^{j\omega})$ the discrete-time Fourier transform of $x[n]$. Write expressions for the magnitude and phase of $X(e^{j\omega})$, and sketch these functions.

(b) Find all values of N for which the N-point DFT is a set of real numbers.

(c) Can you find a three-point causal signal $x_1[n]$ (i.e., $x_1[n] = 0$ for $n < 0$ and $n > 2$) for which the three-point DFT of $x_1[n]$ is:

$$X_1[k] = |X[k]| \qquad k = 0, 1, 2$$

where $X[k]$ is the three-point DFT of $x[n]$?

8.26. We have shown that the DFT $X[k]$ of a finite-length sequence $x[n]$ is identical to samples of the DTFT $X(e^{j\omega})$ of that sequence at frequencies $\omega_k = (2\pi/N)k$; i.e., $X[k] = X(e^{j(2\pi/N)k})$ for $k = 0, 1, \ldots, N-1$. Now consider a sequence $y[n] = e^{-j(\pi/N)n}x[n]$ whose DFT is $Y[k]$.

(a) Determine the relationship between the DFT $Y[k]$ and the DTFT $X(e^{j\omega})$.

(b) The result of part (a) shows that $Y[k]$ is a differently sampled version of $X(e^{j\omega})$. What are the frequencies at which $X(e^{j\omega})$ is sampled?

(c) Given the modified DFT $Y[k]$, how would you recover the original sequence $x[n]$?

8.27. The 10-point DFT of a 10-point sequence $g[n]$ is

$$G[k] = 10\,\delta[k].$$

Find $G(e^{j\omega})$, the DTFT of $g[n]$.

8.28. Consider the six-point sequence

$$x[n] = 6\delta[n] + 5\delta[n-1] + 4\delta[n-2] + 3\delta[n-3] + 2\delta[n-4] + \delta[n-5]$$

shown in Figure P8.28.

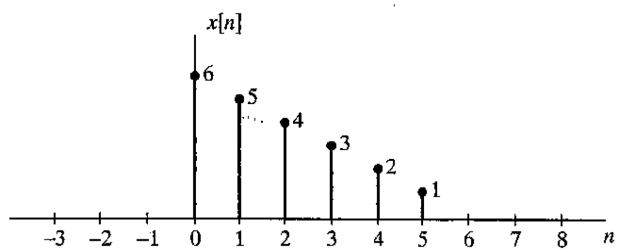

Figure P8.28

(a) Determine $X[k]$, the six-point DFT of $x[n]$. Express your answer in terms of $W_6 = e^{-j2\pi/6}$.

(b) Plot the sequence $w[n]$, $n = 0, 1, \ldots, 5$, that is obtained by computing the inverse six-point DFT of $W[k] = W_6^{-2k} X[k]$.

(c) Use any convenient method to evaluate the six-point circular convolution of $x[n]$ with the sequence $h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$. Sketch the result.

(d) If we convolve the given $x[n]$ with the given $h[n]$ by N-point circular convolution, how should N be chosen so that the result of the circular convolution is identical to the result of linear convolution? That is, choose N so that

$$y_p[n] = x[n] \circledN h[n] = \sum_{m=0}^{N-1} x[m]h[((n-m))_N]$$

$$= x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m] \quad \text{for } 0 \le n \le N-1.$$

(e) In certain applications, such as multicarrier communication systems (see Starr et al, 1999), the linear convolution of a finite-length signal $x[n]$ of length L samples with a shorter finite-length impulse response $h[n]$ is required to be identical (over $0 \le n \le L-1$) to what would have been obtained by L-point circular convolution of $x[n]$ with $h[n]$. This can be achieved by augmenting the sequence $x[n]$ appropriately. Starting with the graph of Figure P8.28, where $L = 6$, add samples to the given sequence $x[n]$ to produce a new sequence $x_1[n]$ such that with the sequence $h[n]$ given in part (c), the ordinary convolution $y_1[n] = x_1[n] * h[n]$ satisfies the equation

$$y_1[n] = x_1[n] * h[n] = \sum_{m=-\infty}^{\infty} x_1[m]h[n-m]$$

$$= y_p[n] = x[n] \circledL h[n] = \sum_{m=0}^{5} x[m]h[((n-m))_6] \quad \text{for } 0 \le n \le 5.$$

(f) Generalize the result of part (e) for the case where $h[n]$ is nonzero for $0 \le n \le M$ and $x[n]$ is nonzero for $0 \le n \le L-1$, where $M < L$; i.e., show how to construct a sequence $x_1[n]$ from $x[n]$ such that the linear convolution $x_1[n] * h[n]$ is equal to the circular convolution $x[n] \circledL h[n]$ for $0 \le n \le L-1$.

8.29. Consider the real five-point sequence

$$x[n] = \delta[n] + \delta[n-1] + \delta[n-2] - \delta[n-3] + \delta[n-4].$$

The deterministic autocorrelation of this sequence is the inverse DTFT of

$$C(e^{j\omega}) = X(e^{j\omega})X^*(e^{j\omega}) = |X(e^{j\omega})|^2,$$

where $X^*(e^{j\omega})$ is the complex conjugate of $X(e^{j\omega})$. For the given $x[n]$, the autocorrelation can be found to be

$$c[n] = x[n] * x[-n].$$

(a) Plot the sequence $c[n]$. Observe that $c[-n] = c[n]$ for all n.

(b) Now assume that we compute the seven-point DFT ($N = 5$) of the sequence $x[n]$. Call this DFT $X_5[k]$. Then, we compute the inverse DFT of $C_5[k] = X_5[k]X_5^*[k]$. Plot the resulting sequence $c_5[n]$. How is $c_5[n]$ related to $c[n]$ from part (a)?

(c) Now assume that we compute the 10-point DFT ($N = 10$) of the sequence $x[n]$. Call this DFT $X_{10}[k]$. Then, we compute the inverse DFT of $C_{10}[k] = X_{10}[k]X_{10}^*[k]$. Plot the resulting sequence $c_{10}[n]$.

(d) Now suppose that we use $X_{10}[k]$ to form $D_{10}[k] = W_{10}^{5k}C_{10}[k] = W_{10}^{5k}X_{10}[k]X_{10}^*[k]$, where $W_{10} = e^{-j(2\pi/10)}$. Then, we compute the inverse DFT of $D_{10}[k]$. Plot the resulting sequence $d_{10}[n]$.

8.30. Consider two sequences $x[n]$ and $h[n]$, and let $y[n]$ denote their ordinary (linear) convolution, $y[n] = x[n] * h[n]$. Assume that $x[n]$ is zero outside the interval $21 \le n \le 31$, and $h[n]$ is zero outside the interval $18 \le n \le 31$.

(a) The signal $y[n]$ will be zero outside of an interval $N_1 \le n \le N_2$. Determine numerical values for N_1 and N_2.

(b) Now suppose that we compute the 32-point DFTs of

$$x_1[n] = \begin{cases} 0 & n = 0, 1, \dots, 20 \\ x[n] & n = 21, 22, \dots, 31 \end{cases}$$

and

$$h_1[n] = \begin{cases} 0 & n = 0, 1, \dots, 17 \\ h[n] & n = 18, 19, \dots, 31 \end{cases}$$

(i.e., the zero samples at the beginning of each sequence are included). Then, we form the product $Y_1[k] = X_1[k]H_1[k]$. If we define $y_1[n]$ to be the 32-point inverse DFT of $Y_1[k]$, how is $y_1[n]$ related to the ordinary convolution $y[n]$? That is, give an equation that expresses $y_1[n]$ in terms of $y[n]$ for $0 \le n \le 31$.

(c) Suppose that you are free to choose the DFT length (N) in part (b) so that the sequences are also zero-padded at their ends. What is the *minimum* value of N so that $y_1[n] = y[n]$ for $0 \le n \le N - 1$?

8.31. Consider the sequence $x[n] = 2\delta[n] + \delta[n-1] - \delta[n-2]$.

(a) Determine the DTFT $X(e^{j\omega})$ of $x[n]$ and the DTFT $Y(e^{j\omega})$ of the sequence $y[n] = x[-n]$.

(b) Using your results from part (a) find an expression for

$$W(e^{j\omega}) = X(e^{j\omega})Y(e^{j\omega}).$$

(c) Using the result of part (b) make a plot of $w[n] = x[n] * y[n]$.

(d) Now plot the sequence $y_p[n] = x[((-n))_4]$ as a function of n for $0 \le n \le 3$.

(e) Now use any convenient method to evaluate the four-point circular convolution of $x[n]$ with $y_p[n]$. Call your answer $w_p[n]$ and plot it.

(f) If we convolve $x[n]$ with $y_p[n] = x[((-n))_N]$, how should N be chosen to avoid time-domain aliasing?

8.32. Consider a finite-duration sequence $x[n]$ of length P such that $x[n] = 0$ for $n < 0$ and $n \ge P$. We want to compute samples of the Fourier transform at the N equally spaced frequencies

$$\omega_k = \frac{2\pi k}{N}, \qquad k = 0, 1, \dots, N-1.$$

Determine and justify procedures for computing the N samples of the Fourier transform using only one N-point DFT for the following two cases:

(a) $N > P$.

(b) $N < P$.