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this page should be last.

$$\text{Then } \hat{X}(\omega) = \frac{1}{2}\underbrace{\left[H_1(\omega-\pi)H_0(\omega) - H_1(\omega)H_0(\omega-\pi)\right]}_{T(\omega)} X_1(\omega)$$

Then we want

$$\boxed{T(\omega) = C\, e^{-j\omega n_0}}$$

i.e. have a flat response over all frequencies and linear phase

It can be shown that there is no trivial solution to this objective. Usually $H_0(\omega)$, $H_1(\omega)$ are chosen so that amplitude distortion is eliminated and an acceptable level of phase distortion is present.

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Solutions

1. $x(n) \rightarrow \boxed{\downarrow M} \xrightarrow{x(nM)=y(n)}$ From theory $Y(\omega)=\frac{1}{M}\sum_{i=0}^{M-1} X\left(\frac{\omega-2\pi i}{M}\right)$

Easy to show that $a_1 x_1(n) + a_2 x_2(n) \longrightarrow a_1 y_1(n) + a_2 y_2(n)$
So system is linear, Since we not place the output as $Y(\omega)=H(\omega)\cdot X(\omega) \Rightarrow H(\omega)=\frac{Y(\omega)}{X(\omega)}$ system is not time invariant.

Similarly for $\rightarrow \boxed{\uparrow L} \rightarrow$. - - - - - - - -

2. $x(n) \rightarrow \boxed{\downarrow M} \xrightarrow{w(n)} \boxed{\uparrow L} \rightarrow y(n)$

$$\left.\begin{array}{l} Y(\omega)=W(\omega L) \\ W(\omega)=\frac{1}{M}\sum_{i=0}^{M-1} X\left(\frac{\omega-2\pi i}{M}\right) \end{array}\right\} \Rightarrow Y(\omega)=\frac{1}{M}\sum_{i=0}^{M-1} X\left(\frac{\omega L-2\pi i}{M}\right)$$

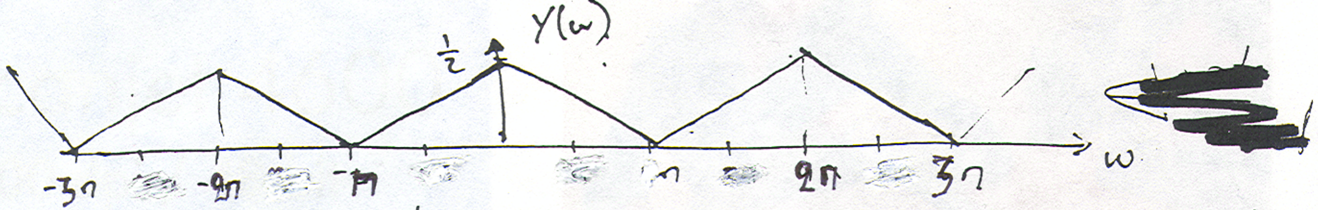
Assuming $x(n)$ is bandlimited to a bandwidth $0 \le |\omega| < \frac{\pi}{M}$ then no aliasing occurs and

$$Y(\omega)=\frac{1}{M} X\left(\frac{\omega L}{M}\right) \quad \text{in} \quad 0 \le |\omega| \le \pi.$$

Then if $\boxed{L=M}$ $\quad Y(\omega)=\frac{1}{M}X(\omega)$ and system is **LTI**

If aliasing occurs or $M \neq L$ system is not LTI

③ $x(n) \rightarrow \boxed{\downarrow 2} \rightarrow y(n)$ $\qquad Y(\omega) = \frac{1}{2}\left[X\left(\frac{\omega}{2}\right) + X\left(\frac{\omega}{2} - \pi\right)\right]$



Aliasing occurs but $x(n)$ can still be reconstructed from $y(n)$ by upsampling by 2 and then bandpass filtering. Note however that we must know a priori that $x(n)$ is high-pass to select the appropriate filter.

④

$$W_0(\omega) = \frac{1}{2}\left[H_0\left(\frac{\omega}{2}\right)X\left(\frac{\omega}{2}\right) + H_0\left(\frac{\omega}{2}-\pi\right)X\left(\frac{\omega}{2}-\pi\right)\right]$$

$$W_1(\omega) = \frac{1}{2}\left[H_1\left(\frac{\omega}{2}\right)X\left(\frac{\omega}{2}\right) + H_1\left(\frac{\omega}{2}-\pi\right)X\left(\frac{\omega}{2}-\pi\right)\right]$$

$$\hat{X}(\omega) = F_0(\omega)\cdot W_0(2\omega) + F_1(\omega)W_1(2\omega) =$$

$$= \frac{1}{2}F_0(\omega)\left[H_0(\omega)X(\omega) + H_0(\omega - \cancel{2\pi})X(\omega - \cancel{2\pi})\right]$$

$$+ \frac{1}{2}F_1(\omega)\left[H_1(\omega)X(\omega) + H_1(\omega-\cancel{2\pi})X(\omega-\cancel{2\pi})\right] =$$

$$= \frac{1}{2}\left[F_0(\omega)H_0(\omega) + H_1(\omega)F_1(\omega)\right]X(\omega)$$

$$+ \underbrace{\frac{1}{2}\left[H_0(\omega-\cancel{2\pi})\cdot F_0(\omega) + H_1(\omega-\cancel{2\pi})F_1(\omega)\right]X(\omega-2\pi)}_{\text{aliasing term}}$$

To remove aliasing term choose: $F_0(\omega) = H_1(\omega - \cancel{2\pi})$, $F_1(\omega) = -H_0(\omega - \cancel{2\pi})$